From Little Higgs to Little Flavor

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Spacetime as a topological insulator
Phys. Rev. Lett. 108 (2012) 181807
David B. Kaplan, S.S.
Little Flavor
arXiv:1303.1811
S.S., David B. Kaplan, Ann E. Nelson

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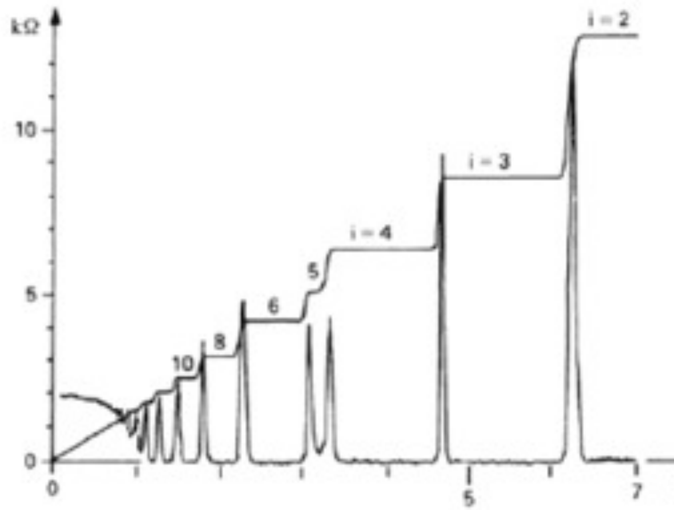
SUSY 2013

A flavor scenario to give rise three families

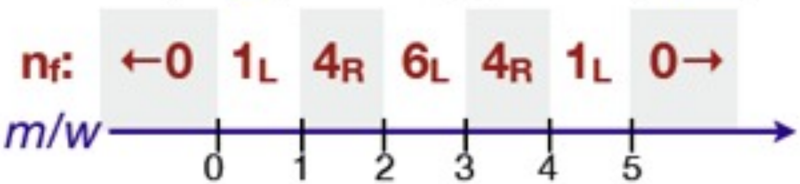
Spacetime as a topological insulator ,David B kaplan and S.S, highlighted in an APS Physics Synopsis.

- “3”, fermion generation number as Chern–number
- Same universal physics behind:
  - Domain wall fermion
  - Quantum Hall effect /topological insulator
  - Chiral fermion in lattice simulation



E.g, d=5 lattice: *lattice derivatives*

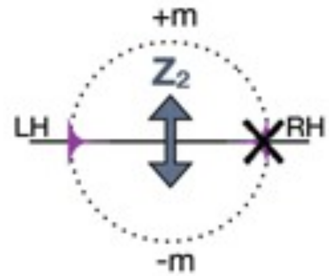
$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - m \bar{\psi}\psi + w\, \bar{\psi} \partial^2 \psi$$



$$\sigma_{xy} = n \frac{e^2}{h}$$

A flavor scenario to give rise three families II

- Three zero mode stuck at one “brane”With different profile.
- Could be implemented into RS. Might having some trouble
 with gauge field.
- Could also put it on a discretized Z2 orbifold.
- Topology in momentum space----need UV completion
  - eg: ABJM theory, full string theory

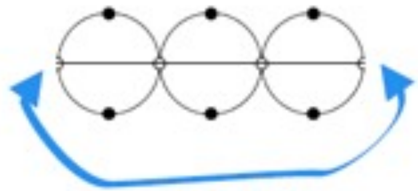


Orbifold projection rather trivial on discretized manifold.

Find index theorem:

# of LH-RH zeromodes = # fixed points under Z_2

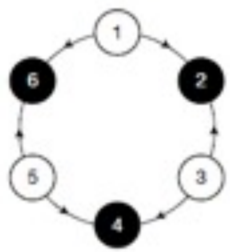
To get 3 families out, need to build in 3 Z_2 fixed pts

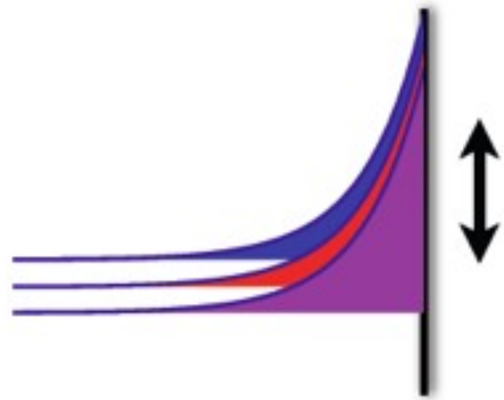


For three families led to bizarre multiply-connected extra dimension, reduced to 9 points

Leads to 4d moose diagram:

- white sites = chiral fermions
- black sites = Dirac fermions





Flavor puzzle in standard model:

- hierarchical structure in flavor parameters
  - couplings: gauge ~ Higgs ~ top Yukawa ~ O(1)
    CP violating phase~O(1)
  - angles: $V_{us} \sim 2\mathrm{x}10^{-1}$, $V_{cb} \sim 4\mathrm{x}10^{-2}$, $V_{ub} \sim 2\mathrm{x}10^{-3}$
  - masses: $b/t \sim 5\mathrm{x}10^{-2}$, $c/t \sim 10^{-2}$, $s/t \sim 10^{-3}$, $u/t \sim d/t \sim 10^{-5}$
- flavor changing neutral currents (FCNC)
  - EW higgs sector, dark matter suggest new TeV physics
  - Absence of FCNC *seems* to require much higher scale physics....
- There is another Hierarchical problem in SM, Higgs hierarchy....

Deconstruction Arkani-Hamed, Cohen, Georgi / Hill, Pokorski, Wang (2001)

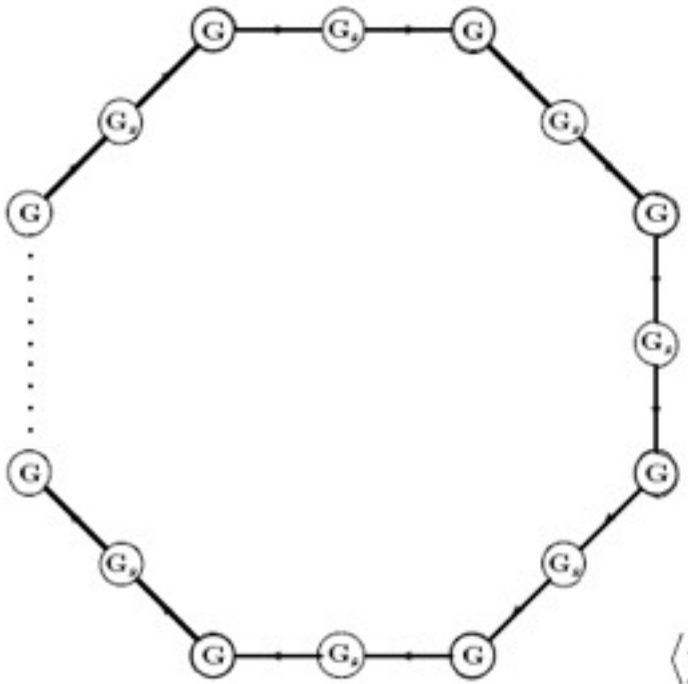
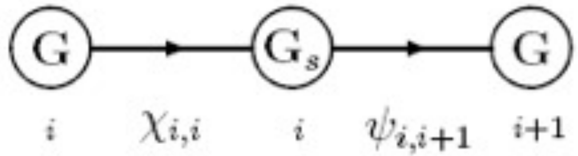


Figure 1: A moose diagram.

$$SU_i(m) \times SU_i(n) \times SU_{i+1}(m)$$



$\chi_{i,i}$ transforming as $(m, \bar{n}, 1)$

$\psi_{i,i+1}$ transforming as $(1, n, \bar{m})$

$$\langle \chi_{i,i}\, \psi_{i,i+1} \rangle \sim 4\pi f_s^3\, U_{i,i+1} \qquad i = 1, \ldots, N$$

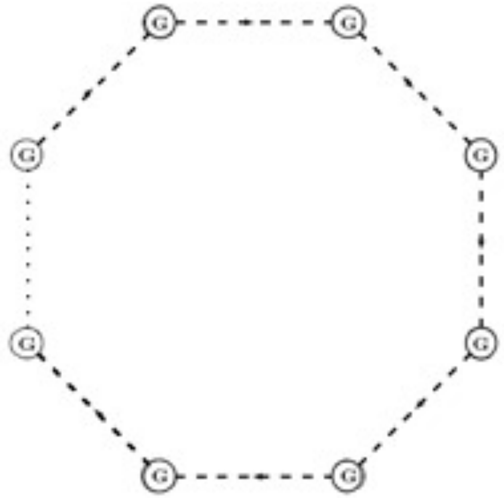


Figure 2: A condensed moose diagram

$$a = \frac{1}{g f_s}, \qquad R = Na\ .$$

$$S = \int d^4x \left(-\frac{1}{2g^2} \sum_{j=1}^{N} \mathrm{tr}\, F_j^2 + f_s^2 \sum_{j=1}^{N} \mathrm{tr} \left[(D_\mu U_{j,j+1})^\dagger D^\mu U_{j,j+1} \right] + \cdots \right)$$

$$D_\mu U_{j,j+1} \equiv \partial_\mu U_{j,j+1} - i A_\mu^j U_{j,j+1} + i U_{j,j+1} A_\mu^{j+1}$$

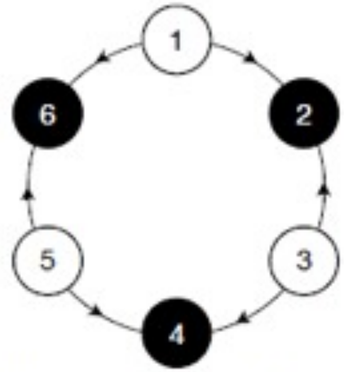
- link field could be parameterized as below, protected by large globe symmetry:

$$X_j = \exp(2 i x_j / f) \qquad x = \begin{pmatrix} \varphi_x + \eta_x & h_x \\ h_x^\dagger & -2\eta_x \end{pmatrix}$$

Deconstruction and Little Higgs

- Composite Higgs   Kaplan, Georgi, (1984)
- Deconstruction   Arkani-Hamed, Cohen, Georgi / Hill, Pokorski, Wang (2001)
- Little Higgs   Arkani-Hamed, Cohen, Katz, Gregoire, A.N., Wacker (2002)
- A latticized, compact new dimension= 4D model with non linear sigma model + product gauge group G×G×..
- Higgs models with no n-loop quadratic divergences n arbitrarily large, although n=1 is "good enough" since there is a cutoff at scale $\Lambda \sim 4\pi f$

Combining little hig with flavor model?



- white sites = chiral fermions
- black sites = Dirac fermions

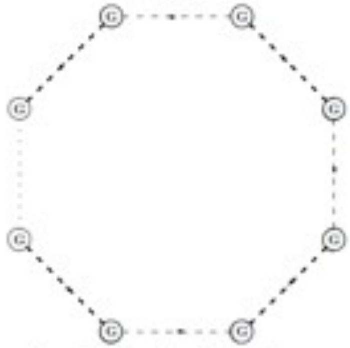


Figure 2: A condensed moose diagram

Little Higgs:

(Arkani-Hamed, Cohen, Georgi (2001); Arkani-Hamed, Cohen, Katz, Nelson (2002))

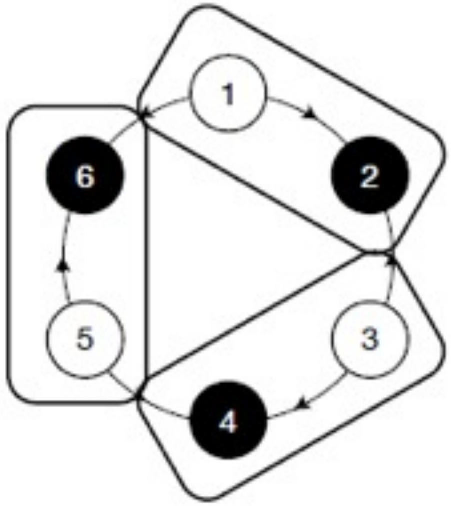
large symmetry group + sparse symmetry breaking spurions
= unusually large natural hierarchy between EW scale and UV (eg $1/\alpha^2$)

Flavor models:

(e.g.: Frogatt-Nielsen (1979))

large flavor symmetry group + sparse symmetry breaking spurions
= natural hierarchy between quark masses & mixing angles

The model (for quarks)



- 3 cells
- on each **black** site:
  - gauge group G_b = *SU(2) x U(1)*
  - 4 **Dirac** fermions:

$$\Psi = \begin{pmatrix} u \\ d \\ U \\ D \end{pmatrix}$$

SU(2) doublet (u, d)

SU(2) singlets (U, D)

- on each **white** site:
  - gauge group G_w = *SU(2) x U(1)*
  - 4 **Chiral** fermions:

$$\psi_L = \begin{pmatrix} u \\ d \\ 0 \\ 0 \end{pmatrix}_L \qquad \psi_R = \begin{pmatrix} 0 \\ 0 \\ U \\ D \end{pmatrix}_R$$

SU(2) doublet

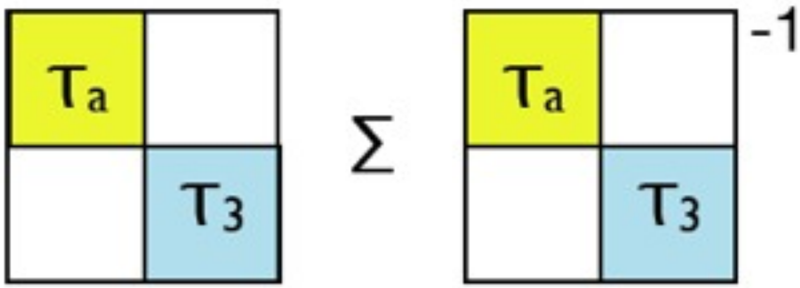
SU(2) singlets

- on each **link**:
  - *SU(4)xSU(4)/SU(4)* nonlinear sigma field

$$\Sigma = \xi \Sigma_H \xi \qquad \Sigma_H = \exp\left[\left(\frac{i\sqrt{2}}{f}\right)\begin{pmatrix} 0 & \Phi^\dagger \\ \Phi & 0 \end{pmatrix}\right]$$

$$\Phi = \begin{pmatrix} H_u^T \\ H_d^T \end{pmatrix} \qquad \xi = \exp\left[(i/2f)\begin{pmatrix} \vec{\pi}' \cdot \vec{\sigma} + \eta/\sqrt{2} & 0 \\ 0 & \vec{\pi} \cdot \vec{\sigma} - \eta/\sqrt{2} \end{pmatrix}\right]$$

- $G_w \times G_b = [SU(2) \times U(1)]^2$ gauge group is embedded in *SU(4) x SU(4)*



$G_w = SU(2) \times U(1)$ $\qquad$ $G_b = SU(2) \times U(1)$

- diagonal *SU(2) x U(1)* will be SM gauge group
- π'=*SU(2)* triplet
- $\pi^\pm$, π^0, η = *SU(2)* singlets
- H_u, H_d = Higgs doublets

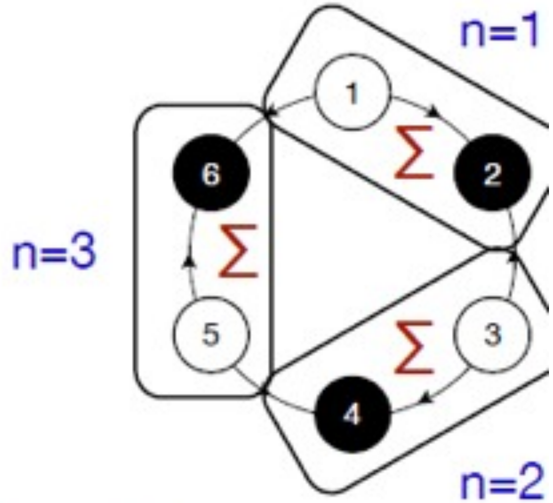
Fermion mass and Yukawa interactions:

U(3) x SU(4) symmetric terms

$$\Psi = \begin{pmatrix} u \\ d \\ U \\ D \end{pmatrix} \quad \psi_L = \begin{pmatrix} u \\ d \\ 0 \\ 0 \end{pmatrix}_L \quad \psi_R = \begin{pmatrix} 0 \\ 0 \\ U \\ D \end{pmatrix}_R$$

SU(2) doublet

SU(2) singlets



$$\mathcal{L}_{\rm sym} = \sum_{n=1}^{3} \left[M \bar{\Psi}_n \Psi_n + \lambda f \left(\bar{\psi}_{L,n} \Sigma \Psi_{R,n} - \bar{\Psi}_{L,n} \Sigma^\dagger \psi_{R,n} \right) \right]$$

- Gives a mass M~5 TeV to black Dirac fermions
- Σ (including Higgs) couples black Dirac fermions to white chiral fermions; f~ 1.5 TeV
- exact U(3) symmetry (acts on index n)
- exact SU(4) symmetry (acts on black Dirac fermions and Σ)

$$\mathcal{L}_{\rm sym} = \sum_{n=1}^{3} \left[M \bar{\Psi}_n \Psi_n + \lambda f \left(\bar{\psi}_{L,n} \Sigma \Psi_{R,n} - \bar{\Psi}_{L,n} \Sigma^\dagger \psi_{R,n} \right) \right]$$

Expand to give Higgs couplings:

$$i\sqrt{2}\,\lambda \left[\left((\bar{u}_{w,n}, \bar{d}_{w,n})_L \Phi^\dagger \begin{pmatrix} U_{b,n} \\ D_{b,n} \end{pmatrix}_R - (\bar{u}_{b,n}, \bar{d}_{b,n})_L \Phi \begin{pmatrix} U_{w,n} \\ D_{w,n} \end{pmatrix}_R \right) \right]$$

$$\Phi^\dagger = (H_u^*, H_d^*)$$

- *Looks* like a Φ (Higgs) vev would give all fermions a mass...
- ...but not true: even with SU(2) x U(1) breaking, still have 3 massless chiral families of quarks + 3 massive Dirac families

3 massless families :
Integrate out the vector-like

$$\mathcal{L}_{\rm sym} = \sum_{n=1}^{3} \left[M\bar{\Psi}_n \Psi_n + \lambda f \left(\bar{\psi}_{L,n} \Sigma \Psi_{R,n} - \bar{\Psi}_{L,n} \Sigma^\dagger \psi_{R,n} \right) \right]$$

$$\Rightarrow \quad \sum_{n=1}^{3} (\lambda f)^2 \left(\bar{\psi}_{L,n} \Sigma \left[\frac{1}{\not{p} + M} \right] \Sigma^\dagger \psi_{R,n} \right) + h.c.$$

$$= \quad \sum_{n=1}^{3} (\lambda f)^2 \left(\bar{\psi}_{L,n} \left[\frac{1}{\not{p} + M} \right] \psi_{R,n} \right) + h.c.$$

+ derivative Higgs couplings.

$$\psi_L = \begin{pmatrix} u \\ d \\ 0 \\ 0 \end{pmatrix}_L \qquad \psi_R = \begin{pmatrix} 0 \\ 0 \\ U \\ D \end{pmatrix}_R$$

Fermion mass and Yukawa interactions:

add U(3) x SU(4) symmetry breaking terms

$$\mathcal{L}_{\rm asym} = \sum_{m,n=1}^{3} \bar{\Psi}_{m,L} \left(M^u X_u + M^d X_d\right)_{mn} \Psi_{n,R} + h.c.$$

$$\Psi = \begin{pmatrix} u \\ d \\ U \\ D \end{pmatrix}$$

SU(2) doublet (u, d)

SU(2) singlets (U, D)

- Acts only on black-site Dirac fermions
- M^u, M^d break the U(3) symmetry ⇒ $U(1)_B$ *(particular texture chosen)*

$$M^u = \begin{pmatrix} \mathcal{M}^u_{11} & \mathcal{M}^u_{12} & 0 \\ 0 & \mathcal{M}^u_{22} & 0 \\ \mathcal{M}^u_{31} & 0 & \mathcal{M}^u_{33} \end{pmatrix} \quad , \qquad M^d = \begin{pmatrix} \mathcal{M}^d_{11} & 0 & 0 \\ \mathcal{M}^d_{21} & \mathcal{M}^d_{22} & 0 \\ 0 & \mathcal{M}^d_{32} & \mathcal{M}^d_{33} \end{pmatrix}$$

- X_u, X_d break the SU(4) symmetry ⇒ different *SU(3)* subgroups

$$X_u = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -3 & \\ & & & 1 \end{pmatrix} , \qquad X_d = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$$

Normal "little Higgs" Mechanism
Plus different generations mainly live on different cells, to explain flavor.

- $<\Sigma>$ breaks $(SU(2)\times U(1))^2$ to diagonal $SU(2)\times U(1)$
  - Identify diagonal $SU(2)\times U(1)$ as SM gauge group
  - symmetry breaking scale is $f\sim 1.5$ TeV
  - $g\,f \sim$ new gauge boson masses
  - Orientation of Σ parameterized by pNGB s
  - π', π^0 are eaten by heavy "axial" $SU(2)\times U(1)$ bosons
  - H doublets can act as Higgs to further break SM $SU(2)\times U(1) \Rightarrow U(1)$ electromagnetic

Peculiar symmetry structure ensures Little Higgs mechanism in the fermion sector:

If M is the full fermion mass matrix, then

- Tr $M^\dagger M$ is independent of H vevs
- Tr $(M^\dagger M)^2$ is independent of H vevs

So there are neither quadratic nor log divergent contributions to the Higgs potential from fermions at one loop

There will be a finite Coleman-Weinberg contribution, Tr $(M^\dagger M)^2 \ln(M^\dagger M)$. To avoid fine tuning of the Higgs potential, there needs to be a Dirac top-partner at ~ 1 TeV.

At this level there is a Peccei-Quinn symmetry protecting against flavor violating Higgs couplings...to be softly broken in Higgs potential

What do FCNC look like in a phenomenological fit to quark masses (RG scaled to 1 TeV) and CKM angles?

$$M = 5000\ \text{GeV}\,, \qquad f = 1500\ \text{GeV}\,, \qquad \tan\beta = \frac{v_u}{v_d} = 1$$

$$\lambda = 1.49794$$

$$M^u = \begin{pmatrix} 1189.54 & 15.4904 & 0 \\ 0 & 6.96490 & 0 \\ 3.50799 e^{-i1.224428} & 0 & 0.01441071 \end{pmatrix}\,, \qquad M^d = \begin{pmatrix} 45.7769 & 0 & 0 \\ -1.60269 & 0.600984 & 0 \\ 0 & 0.137582 & 0.0336607 \end{pmatrix} \quad \text{(GeV)}$$

Yields quark masses

$$\begin{array}{lll} m_t = 153.2 & m_c = 5.32 \times 10^{-1} & m_u = 1.10 \times 10^{-3} \\ m_b = 2.45 & m_s = 4.69 \times 10^{-2} & m_d = 2.50 \times 10^{-3} \end{array} \quad \text{(GeV)}$$

and angles:

$$|V_{\rm CKM}| = \begin{pmatrix} 0.974 & 0.226 & 0.00385 \\ 0.226 & 0.973 & 0.0423 \\ 0.00892 & 0.0415 & 0.998 \end{pmatrix} \qquad \sin(2\alpha) = 0.052\,, \qquad \sin(2\beta) = 0.72\,, \qquad \sin(2\gamma) = 0.68$$

New exotic particles and couplings:

- *W* (80 GeV):
  RH current ~ LH current *x* 10^{-3}

- *W'*(1.4 TeV):
  LH current: *W* x 0.05
  RH current: *W* x 2

- *Z'*(750 GeV), *Z"*(1.4 TeV) ... (next slide)

- heavy quark partners:
  lightest is top partner at 2.6 TeV
  (7% fine-tuning for 126 GeV Higgs)

- Other heavy u, d quarks: 5.4-6.6 TeV

- 3 exotic pseudo-scalars η, $\pi^{\pm}$

Flavor dependence of neutral gauge boson couplings (Z, Z', Z")

$$M_{Z'} = 750\ \text{GeV}\ , \qquad M_{Z''} = 1400\ \text{GeV}$$

$$|\mathcal{L}^u_Z| = \begin{pmatrix} 2.6\times10^{-1} & 0 & 1.9\times10^{-6} \\ 0 & 2.6\times10^{-1} & 9.7\times10^{-6} \\ 1.9\times10^{-6} & 9.7\times10^{-6} & 2.6\times10^{-1} \end{pmatrix}\ , \qquad |\mathcal{R}^u_Z| = \begin{pmatrix} 1.1\times10^{-1} & 0 & 2.3\times10^{-6} \\ 0 & 1.1\times10^{-1} & 1.0\times10^{-5} \\ 2.3\times10^{-6} & 1.0\times10^{-5} & 1.1\times10^{-1} \end{pmatrix}\ ,$$

$$|\mathcal{L}^d_Z| = \begin{pmatrix} 3.2\times10^{-1} & 1.0\times10^{-6} & 5.0\times10^{-6} \\ 1.0\times10^{-6} & 3.2\times10^{-1} & 2.3\times10^{-5} \\ 5.0\times10^{-6} & 2.3\times10^{-5} & 3.2\times10^{-1} \end{pmatrix}\ , \qquad |\mathcal{R}^d_Z| = \begin{pmatrix} 5.5\times10^{-2} & 0 & 0 \\ 0 & 5.5\times10^{-2} & 3.6\times10^{-6} \\ 0 & 3.6\times10^{-6} & 5.5\times10^{-2} \end{pmatrix}\ ,$$

$$|\mathcal{L}^u_{Z'}| = \begin{pmatrix} 2.6\times10^{-3} & 0 & 0 \\ 0 & 2.6\times10^{-3} & 3.4\times10^{-5} \\ 0 & 3.4\times10^{-5} & 3.8\times10^{-3} \end{pmatrix}\ , \qquad |\mathcal{R}^u_{Z'}| = \begin{pmatrix} 1.4\times10^{-2} & 0 & 4.0\times10^{-4} \\ 0 & 1.5\times10^{-2} & 1.7\times10^{-3} \\ 4.0\times10^{-4} & 1.7\times10^{-3} & 3.7\times10^{-1} \end{pmatrix}$$

$$|\mathcal{L}^d_{Z'}| = \begin{pmatrix} 5.\times10^{-3} & 1.9\times10^{-5} & 8.9\times10^{-5} \\ 1.9\times10^{-5} & 4.9\times10^{-3} & 4.1\times10^{-4} \\ 8.9\times10^{-5} & 4.1\times10^{-4} & 3.7\times10^{-3} \end{pmatrix}\ , \qquad |\mathcal{R}^d_{Z'}| = \begin{pmatrix} 6.7\times10^{-3} & 0 & 2.6\times10^{-5} \\ 0 & 6.6\times10^{-3} & 2.0\times10^{-4} \\ 2.6\times10^{-5} & 2.0\times10^{-4} & 8.8\times10^{-3} \end{pmatrix}$$

$$|\mathcal{L}^u_{Z''}| = \begin{pmatrix} 1.9\times10^{-2} & 0 & 7.9\times10^{-5} \\ 0 & 1.9\times10^{-2} & 2.8\times10^{-4} \\ 7.9\times10^{-5} & 2.8\times10^{-4} & 2.9\times10^{-2} \end{pmatrix}\ , \qquad |\mathcal{R}^u_{Z''}| = \begin{pmatrix} 1.4\times10^{-3} & 0 & 0 \\ 0 & 1.4\times10^{-3} & 0 \\ 0 & 0 & 1.3\times10^{-3} \end{pmatrix}$$

$$|\mathcal{L}^d_{Z''}| = \begin{pmatrix} 2.0\times10^{-2} & 1.0\times10^{-4} & 5.0\times10^{-4} \\ 1.0\times10^{-4} & 1.9\times10^{-2} & 2.3\times10^{-3} \\ 5.0\times10^{-4} & 2.3\times10^{-3} & 2.9\times10^{-2} \end{pmatrix}\ , \qquad |\mathcal{R}^d_{Z''}| = \begin{pmatrix} 1.6\times10^{-3} & 0 & 0 \\ 0 & 1.6\times10^{-3} & 0 \\ 0 & 0 & 9.7\times10^{-4} \end{pmatrix}$$

Can read off ΔS = 2 dim 6 operators from Z, Z', Z" exchange:

$$\frac{1\times10^{-12}}{M_Z^2} \simeq \frac{1}{(10^5\ \mathrm{TeV})^2}\,, \qquad \frac{4\times10^{-10}}{M_{Z'}^2} \simeq \frac{1}{(4\times10^4\ \mathrm{TeV})^2}\,, \qquad \frac{1\times10^{-8}}{M_{Z''}^2} \simeq \frac{1}{(1.3\times10^4\ \mathrm{TeV})^2}$$

...all safe from FCNC, even though:

- flavor physics is at the few TeV scale
- full theory does not have a $U(3)^3$ approximate chiral symmetry (for Q,U,D), such as found in minimal flavor violation models, where all flavor symmetry breaking is due to quark mass matrix (Chivukula, Georgi, 1987)

- Direct detection of Z’(750 GeV) or Z”(1.4 TeV)?
  - Production rate of Z’ is down by 10^-3 compared to Z-like couplings

• Production rate of Z” is down by 5 x10^-3 compared to Z-like couplings (Benefited from Moose of LH, delocalization of fermion and gauge boson.) (Far from ruled out.)

• Both RH and LH flavor off-diagonal coupling. (possible explanation for LHCb anomaly result, without running into constraint of BBbar mixing)

• Leptonic partial width not computable in this model (paper working in progress)

• No flavor-off diagonal Higgs coupling (Unusual in vector like extra quark model!)

• RH W coupling give rise to 4% correction to b->sgamma matrix element.

Take home message from “Little flavor”

- NO BSM finding LHC @current energy scale:might imply that nature is a little bit UNUSUAL, e.g:The possible same origin of FLAVOR physics and EW symmetry breaking !
- A brand new model building direction: intertwining two sub-fields..
- Benefits for both side of theories:
  - Bring down the scale of flavor theory without running into constraint of FCNC (“breaking of collective Flavor symmetry”)
  - less find-tuning, looking better for EW precision constraint, etc, comparing to normal LH theory.
- How seriously are we taking a SUSY alternative?
  - fermion partner vs fermion
  - Higgs partner “PGB pions”
  - Global symmetry vs supersymmetry

More development since then:

- Bad radiative correction to fermion masses:

Enlarging Gb to SU(2) x SU(2) to delay radiative correction from gauge bosons at two-loops

- Include leptons: Neutrino see-saw from lepton partners.
- Explore collider phenomenology: Top-partner decay to higgs-like "pions".
- Theory of U(3) symmetry breaking : mass/angle predictions, neutrinos might have large flavor violation process beyond PMNS matrix
- more sophisticated extra-dimension model, related to string compactification.