The Higgs boson mixes with an SU(2) septet

PRESENTATION

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Koji TSUMURA

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The Higgs boson mixes with an SU(2) septet
J. Hisano, K. Tsumura
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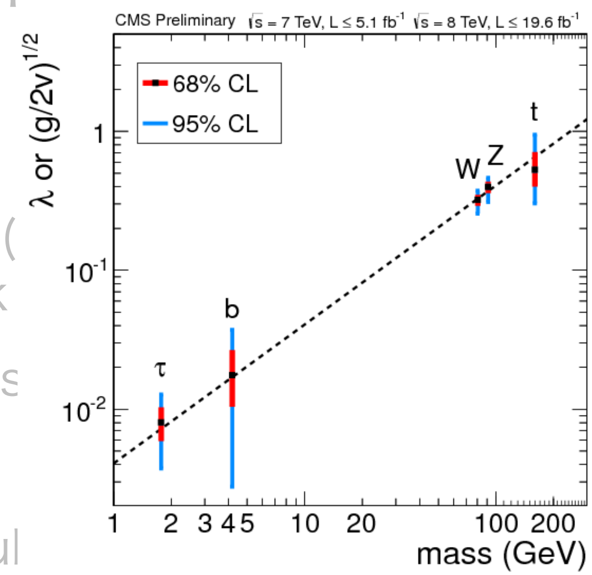
The Higgs boson

"the" = Higgs boson (h) in the SM

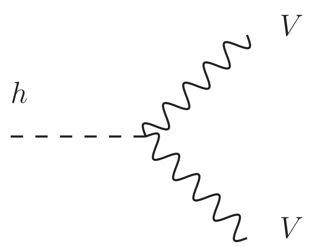


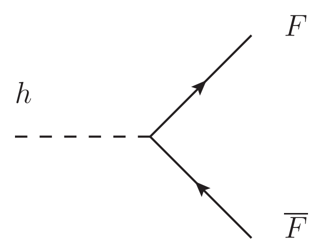
❖ Minima for the Higgs boson

- ✧ The vacuum expectation value [VEV] (
  the Higgs boson triggers electroweak … B]
  - → generate weak gauge boson mass
- ✧ Fermion masses are generated via Yul

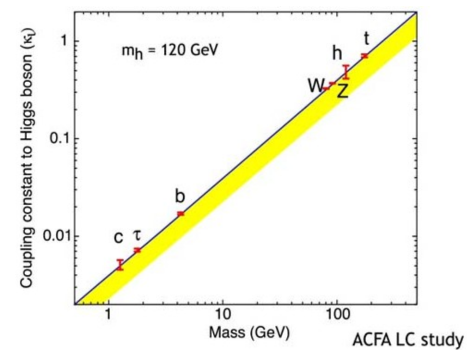
$$m_F = \frac{Y_F}{\sqrt{2}} v$$

- ✧ **"Mass" and "Coupling" relation**

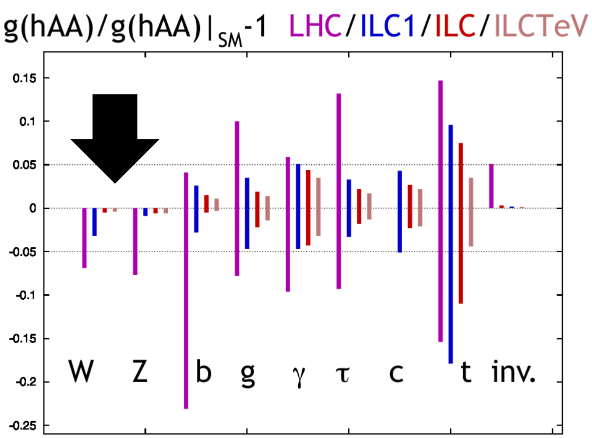




$$\lambda_{hVV} = 2m_V^2/v \qquad \lambda_{hF\overline{F}} = m_F/v$$



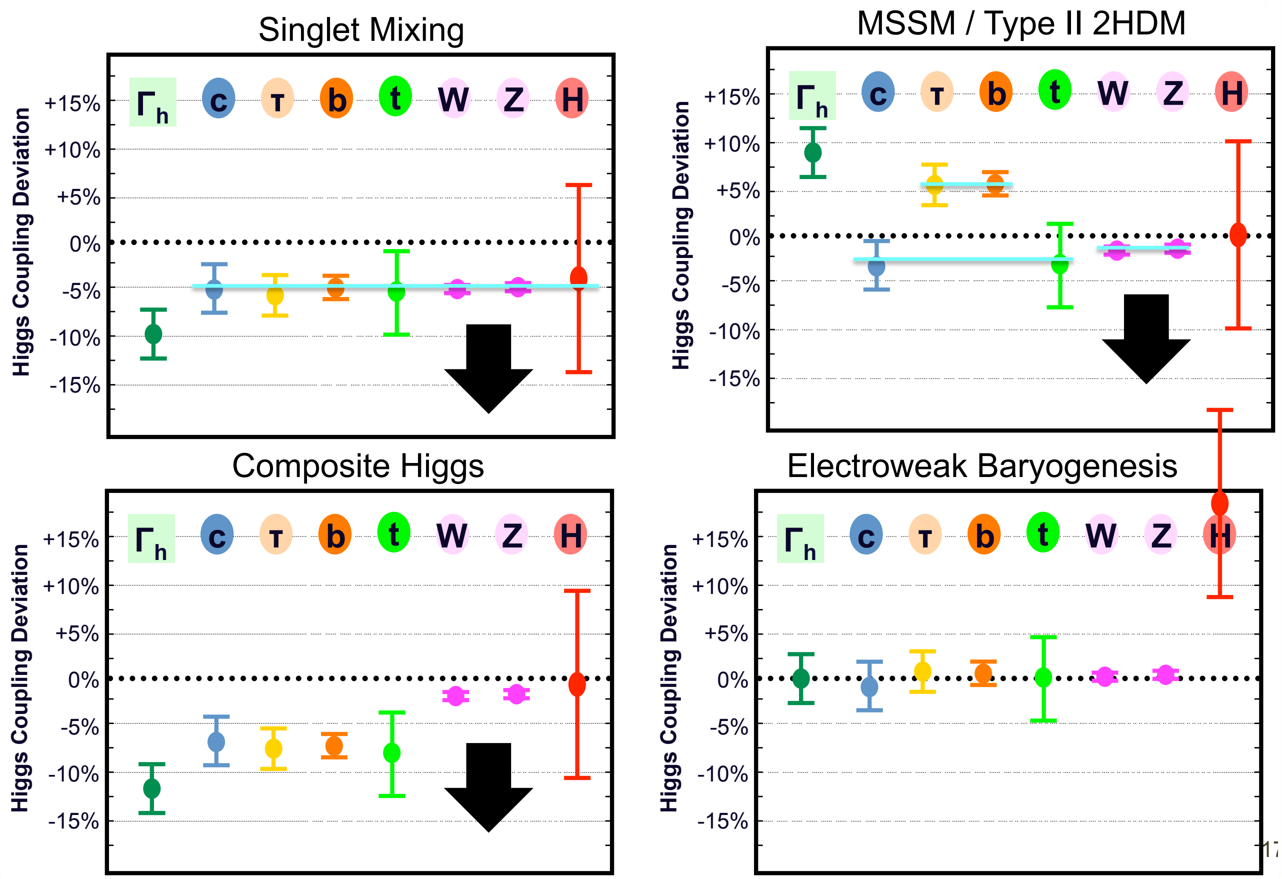
Precision coupling measurement



M. Peskin arXiv:1207.2516

Deviations = New Physics

Is the deviation in hVV always negative?



How can we extend the SM?

- Electroweak (EW) ρ parameter

$$\rho_{\text{tree}} = \frac{\sum_\alpha [I_\alpha(I_\alpha+1) - Y_\alpha^2] v_\alpha^2}{\sum_\beta 2Y_\beta^2 v_\beta}$$

  - For an arbitrary number of Higgs field with an isospin (I_α), a $U(1)_Y$ hypercharge (Y_α) and a VEV (v_α)

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$$\rho^{\rm SM}_{\rm tree} = \frac{m_W^2}{c_W^2\, m_Z^2} = 1 \quad \text{for} \quad I_\alpha = 1/2, Y_\alpha = 1/2, v_\alpha = (\sqrt{2}G_F)^{-1/2}$$

$SU(2)_L$ doublet **in the SM**

Fermi constant

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

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  - Very accurately measured & consistent with the SM

$$\rho_0 = (\rho/\rho_{\rm SM}) = 1.0004^{+0.0003}_{-0.0004}$$

Most important test of the SM [$SU(2)_L \times U(1)_Y$ structure]

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$$\rho_{\rm tree}^{\rm triplet} = \frac{m_W^2}{c_W^2\, m_Z^2} = \frac{1}{2} \quad \text{for} \quad I_\alpha = 1, Y_\alpha = 1$$

↖ SU(2)$_L$ triplet **in a triplet model** w/o the doublet

obviously different from unity

$$\Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}$$

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$$\rho_{\text{tree}}^{\text{SM}} = \frac{m_W^2}{c_W^2\, m_Z^2} = 1 \quad \text{for} \quad I_\alpha = 1/2, Y_\alpha = 1/2, v_\alpha = (\sqrt{2}G_F)^{-1/2}$$

$$\rho_{\text{tree}}^{\text{triplet}} = \frac{m_W^2}{c_W^2\, m_Z^2} = \frac{1}{2} \quad \text{for} \quad I_\alpha = 1, Y_\alpha = 1$$

↖ $SU(2)_L$ triplet **in a triplet model** w/o the doublet

$$\rho_{\text{tree}}^{\text{HTM}} = \frac{m_W^2}{c_W^2\, m_Z^2} = \frac{1+2x^2}{1+4x^2} \approx 1 - 2x^2 \quad \text{with} \quad x = \frac{\langle \Delta^0 \rangle}{\langle \phi^0 \rangle}$$

the SM doublet w/ a $SU(2)_L$ triplet **(Higgs triplet model [HTM])**

$\langle\Delta^0\rangle$ has to be very small

(less contributions to EWSB)

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  - Very accurately measured & consistent with the SM

$$\rho_0 = (\rho/\rho_{\rm SM}) = 1.0004^{+0.0003}_{-0.0004}$$

Most important test of the SM [$SU(2)_L \times U(1)_Y$ structure]

ρ=1 seems to be a good guideline to construct Beyond the SM

$$\rho_{\text{tree}} = \frac{[I(I+1) - Y^2]}{2Y^2} = 1$$

Redefine to make them integers

$$x = 2I + 1, \quad y = 2Y$$

⬇

$$x^2 - 3y^2 = 1$$

Why Higgs septet?

- ρ=1 leads Pell's equation (in Number theory)

  with $x = (2I+1), y = 2Y, n = 3$

$$x^2 - ny^2 = 1$$

  - Trivial solution: (x,y)=(1,0) for arbitrary n

The SM singlet real scalar

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with $x = (2I+1), y = 2Y, n = 3$

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  - Fundamental sol.: $(x_1,y_1)=(2,1)$ for n=3 ["the" Higgs field in the SM]

SU(2) doublet w/ Y=1/2

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    SU(2) doublet w/ Y=1/2

  - General sol.: $$x_k = \frac{1}{2}[(x_1 + y_1\sqrt{n})^k + (x_1 - y_1\sqrt{n})^k]$$ Bhaskara II (1150)
$$y_k = \frac{1}{2\sqrt{n}}[(x_1 + y_1\sqrt{n})^k - (x_1 - y_1\sqrt{n})^k]$$

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    $$y_k = \frac{1}{2\sqrt{n}}[(x_1 + y_1\sqrt{n})^k - (x_1 - y_1\sqrt{n})^k]$$

→ Next minimal sol.: (x_2,y_2)=(7,4) SU(2) septet w/ Y=2

Septet can have sizable VEV & give significant contributions to EWSB!!

Why Higgs septet?

- Models with $\rho_{tree}=1$

At least 1 Higgs doublet is required to have Yukawa interaction (mass generation for SM fermions)

  - SM [1 Higgs doublet]

Why Higgs septet?

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At least 1 Higgs doublet is required to have Yukawa interaction (mass generation for SM fermions)

  - SM [1 Higgs doublet]
  - 2HDM [2 Higgs doublet]
    - ✓ MSSM (Minimal Supersymmetric SM)

      an even number of Higgs doublets is required by the theory

      (holomorphy of superpotential, anomaly cancellation, mass generation of up & down)
    - ✓ Zee model (a radiative seesaw model for neutrino mass generation)

      at least 2 Higgs doublets is required to have lepton number violation
    - ✓ etc.

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- **New [1 Higgs doublet + 1 Higgs septet]**
- etc. (usually VEV alignment is required)

Physical Higgs bosons

- Models with $\rho_{tree}=1$

At least 1 Higgs doublet is required to have Yukawa interaction (mass generation for SM fermions)

- SM [1 Higgs doublet] h
- 2HDM [2 Higgs doublet] h, H, A, $H^{\pm}$
  - CP even Higgs bosons (h, H)
  - A CP odd Higgs boson (A)
  - A pair of charged Higgs bosons ($H^{\pm}$)
  - ✓ MSSM (Minimal Supersymmetric SM)
    - an even number of Higgs doublets is required by the theory
    - (holomorphy of superpotential, anomaly cancellation, mass generation of up & down)
  - ✓ Zee model (a radiative seesaw model for neutrino mass generation)
    - at least 2 Higgs doublets is required to have lepton number violation
  - ✓ etc.
- **New [1 Higgs doublet + 1 Higgs septet]** h, H, A, $H_1^{\pm}$, $H_2^{\pm}$, $H^{2\pm}$, $H^{3\pm}$, $H^{4\pm}$, $H^{5\pm}$
  - 2 pairs of charged Higgs bosons ($H_1^{\pm}$, $H_2^{\pm}$)
  - Multiply charged Higgs bosons ($H^{2\pm}$, $H^{3\pm}$, $H^{4\pm}$, $H^{5\pm}$)
- etc. (usually VEV alignment is required)

Higher dim. reps in the histroy

Before experimental confirmation of SM

"A Phenomenological Profile of the Higgs Boson "
J. Ellis, M. K. Gaillard, D. V. Nanopoulos, Nucl. Phys. B106, 292 (1976)

2.2. - Ambiguities

The model described above is the simplest version of the Weinberg-Salam model. As soon as we consider more complicated versions of this model, or other models of weak and electromagnetic interactions, then considerable ambiguities arise in the Higgs boson couplings. For example :

i) - Even in the context of the Weinberg-Salam [11] model we can choose to have several Higgs fields $\underset{\sim}{H}_i$ belonging to several multiplets i with weak isospins I_i. Then if the uncharged member H_i^0 of each multiplet has as its third component of isospin I_{3i} and acquires a vacuum expectation value $\langle 0|H_i^0|0\rangle = v_i$ we find

$$m_W^2 = \frac{g^2}{2}\sum_i v_i^2\left(I_i^2 + I_i - I_{3i}^2\right) \qquad (2.12)$$

and

$$m_Z^2 = \frac{g^2}{\cos^2\theta_W}\sum_i v_i^2 I_{3i}^2 \qquad (2.13)$$

Higher dim. reps in the histroy

There are basically two major constraints. First, it is an experimental fact [2,3] that $\rho = m_W^2/(m_Z^2 \cos^2\theta_W)$ is very close to 1. In the Standard Model, the ρ parameter is determined by the Higgs structure of the theory. It is well known [4] that in a model with only Higgs doublets (and singlets), the tree-level value of $\rho = 1$ is automatic, without adjustment of any parameters in the model. Although the minimal Higgs satisfies this property, so does any version of the Standard Model with any number of Higgs doublets (and singlets). In fact, there are other ways to satisfy the $\rho \approx 1$ constraint. First, there are an infinite number of more complicated Higgs representations which also satisfy $\rho = 1$ at tree level [5]. The general formula is

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum_{T,Y}[4T(T+1)-Y^2] \mid V_{T,Y} \mid^2 c_{T,Y}}{\sum_{T,Y} 2Y^2 \mid V_{T,Y} \mid^2}, \tag{4.1}$$

where $\langle \phi(T,Y) \rangle = V_{T,Y}$ defines the vacuum expectation values of each neutral Higgs field, and T and Y specify the total $SU(2)_L$ isospin and the hypercharge of the Higgs representation to which it belongs. In addition, we have introduced the notation:

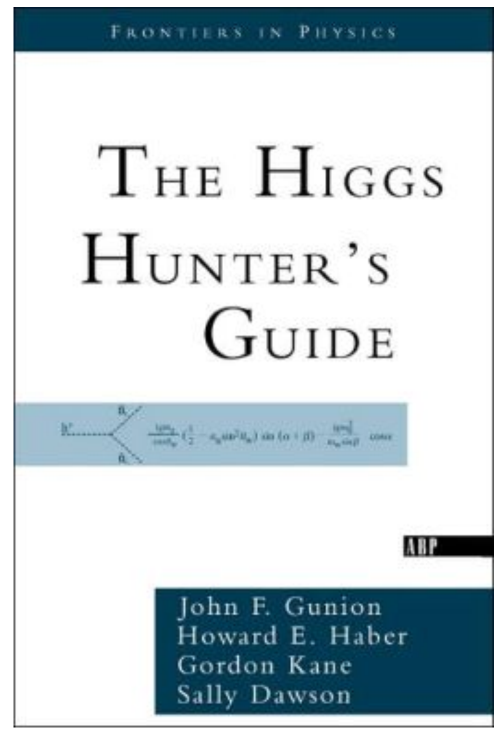
$$c_{T,Y} = \begin{cases} 1, & (T,Y) \in \text{complex representation}, \\ \frac{1}{2}, & (T, Y=0) \in \text{real representation}. \end{cases} \tag{4.2}$$

Here, we employ a rather narrow definition of a real representation as consisting of a real multiplet of fields with integer weak isospin and $Y = 0$. The requirement that $\rho = 1$ for arbitrary $V_{T,Y}$ values is

$$(2T+1)^2 - 3Y^2 = 1. \tag{4.3}$$

The possibilities beyond $T = 1/2$, $Y = \pm 1$ are usually discarded since the representations are rather complicated (the simplest example is a representation with weak isospin 3 and $Y = 4$). Second, one can take a model with multiple copies of "bad" Higgs representations, and arrange a "custodial" SU(2) symmetry among the copies, which then naturally imposes $\rho = 1$ at tree level. Examples of this type will be considered in §6.4. Finally, one can always choose arbitrary Higgs representations and fine tune the parameters of the Higgs potential to arrange $\rho \approx 1$. We will discard this latter "unnatural" possibility from further consideration.

"The Higgs Hunter's Guide" (1990)
F. Gunion, H. Haber, G. Kane, S. Dawson



Difficulty of the model

- An accidental global U(1) symmetry in the Higgs potential

$$\mathcal{V} = -\mu_2^2 \Phi^\dagger \Phi + M_7^2 \chi^\dagger \chi + \lambda(\Phi^\dagger \Phi)^2 + \sum_{A=1}^{4} \lambda_A (\chi^\dagger \chi \chi^\dagger \chi)_A + \sum_{B=1}^{2} \kappa_B (\Phi^\dagger \Phi \chi^\dagger \chi)_B$$

$$\Phi \to e^{i\theta_\Phi} \Phi$$
$$\chi \to e^{i\theta_\chi} \chi$$

Φ(doublet) and χ(septet) are invariant under the separate U(1)

→ Exact Massless NG boson (experimentally disfavored)

$$(\Phi^\dagger \Phi \chi^\dagger \chi)_1 = \phi^{*i} \phi_i \chi^{*abcdef} \chi_{abcdef}$$
$$(\Phi^\dagger \Phi \chi^\dagger \chi)_2 = \phi^{*i} \phi_j \chi^{*jabcde} \chi_{iabcde}$$
$$(\chi^\dagger \chi \chi^\dagger \chi)_1 = \chi^{*ijklmn} \chi_{ijklmn} \chi^{*abcdef} \chi_{abcdef}$$
$$(\chi^\dagger \chi \chi^\dagger \chi)_2 = \chi^{*ijklmn} \chi_{ijklmf} \chi^{*abcdef} \chi_{abcden}$$
$$(\chi^\dagger \chi \chi^\dagger \chi)_3 = \chi^{*ijklmn} \chi_{ijklef} \chi^{*abcdef} \chi_{abcdmn}$$
$$(\chi^\dagger \chi \chi^\dagger \chi)_4 = \chi^{*ijklmn} \chi_{ijkdef} \chi^{*abcdef} \chi_{abclmn}$$

$$\begin{cases} \Phi_1 = \omega_2^+ \\ \Phi_2 = (v_2 + h_2 + i\, z_2)/\sqrt{2} \end{cases}$$

$$\begin{cases} \chi_{111111} = \chi_3 \\ \chi_{111112} = \chi_2/\sqrt{6} \\ \chi_{111122} = \chi_1/\sqrt{15} \\ \chi_{111222} = \chi_0/\sqrt{20} \\ \chi_{112222} = \chi_{-1}/\sqrt{15} \\ \chi_{122222} = \chi_{-2}/\sqrt{6} \\ \chi_{222222} = \chi_{-3} \end{cases}$$

with

$$\chi_{-2} = (v_7 + h_7 + i\, z_7)/\sqrt{2}$$
$$\chi_3 = H^{5+}, \chi_2 = H^{4+}, \chi_1 = H^{3+}, \chi_0 = H^{2+}$$

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$$\mathcal{V} = -\mu_2^2\Phi^\dagger\Phi + M_7^2\chi^\dagger\chi + \lambda(\Phi^\dagger\Phi)^2 - \frac{1}{\Lambda^3}\{(\chi^*\Phi^5\Phi^*) + \mathrm{H.c.}\}$$
$$+\sum_{A=1}^{4}\lambda_A(\chi^\dagger\chi\chi^\dagger\chi)_A + \sum_{B=1}^{2}\kappa_B(\Phi^\dagger\Phi\chi^\dagger\chi)_B$$

U(1) breaking term

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~~→ Exact Massless NG boson (experimentally disfavored)~~

$$(\chi^*\Phi^5\Phi^*) = \chi^{*abcdef}\Phi_a\Phi_b\Phi_c\Phi_d\Phi_e\Phi^{*g}\epsilon_{fg}$$

$$(\Phi^\dagger\Phi\chi^\dagger\chi)_1 = \phi^{*i}\phi_i\chi^{*abcdef}\chi_{abcdef}$$
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Physical basis

- An accidental global U(1) symmetry in the Higgs potential
For simplicity

$$\mathcal{V} = -\mu_2^2 \Phi^\dagger \Phi + M_7^2 \chi^\dagger \chi + \lambda(\Phi^\dagger \Phi)^2 - \frac{1}{\Lambda^3}\{(\chi^* \Phi^5 \Phi^*) + \text{H.c.}\}$$

~~$$+\sum_{A=1}^{4} \lambda_A (\chi^\dagger \chi \chi^\dagger \chi)_A + \sum_{B=1}^{2} \kappa_B (\Phi^\dagger \Phi \chi^\dagger \chi)_B$$~~

$$\begin{pmatrix} h_7 \\ h_2 \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

$$\begin{pmatrix} z_7 \\ z_2 \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} z \\ A \end{pmatrix}$$

$$\begin{pmatrix} \chi_{-1} \\ \chi_{-3}^* \\ \omega_2^+ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{10}}{4} & \frac{\sqrt{6}}{4} & 0 \\ -\frac{\sqrt{6}}{4} & \frac{\sqrt{10}}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\beta & 0 & -s_\beta \\ 0 & 1 & 0 \\ s_\beta & 0 & c_\beta \end{pmatrix} \begin{pmatrix} \omega^+ \\ H_2^+ \\ H_1^+ \end{pmatrix}$$

$z, \omega^\pm$:EW NG bosons which are absorbed by Z, $W^\pm$ bosons

$$\begin{cases} \chi_{111111} = \chi_3 \\ \chi_{111112} = \chi_2/\sqrt{6} \\ \chi_{111122} = \chi_1/\sqrt{15} \\ \chi_{111222} = \chi_0/\sqrt{20} \\ \chi_{112222} = \chi_{-1}/\sqrt{15} \\ \chi_{122222} = \chi_{-2}/\sqrt{6} \\ \chi_{222222} = \chi_{-3} \end{cases}$$

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Parameters

- An accidental global U(1) symmetry in the Higgs potential

For simplicity

Mass of Septet

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$$+\sum_{A=1}^{4}\lambda_A(\chi^\dagger\chi\chi^\dagger\chi)_A + \sum_{B=1}^{2}\kappa_B(\Phi^\dagger\Phi\chi^\dagger\chi)_B$$

CP even Higgs mixing

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$$\begin{pmatrix} \chi_{-1} \\ \chi^*_{-3} \\ \omega_2^+ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{10}}{4} & \frac{\sqrt{6}}{4} & 0 \\ -\frac{\sqrt{6}}{4} & \frac{\sqrt{10}}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_\beta & 0 & -s_\beta \\ 0 & 1 & 0 \\ s_\beta & 0 & c_\beta \end{pmatrix}\begin{pmatrix} \omega^+ \\ H_2^+ \\ H_1^+ \end{pmatrix}$$

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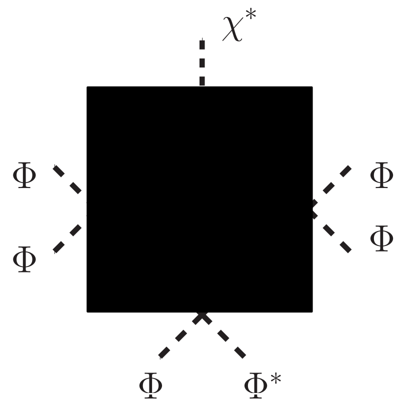
Ratio of VEV

$$\tan\beta = \frac{v_2}{4v_7}$$

A model

A renormalizable model with Higgs septet

Don't introduce VEV of exotic multiplets other than those of doublet and septet



$$-\frac{1}{\Lambda^3}\{(\chi^*\Phi^5\Phi^*) + \text{H.c.}\}$$

Non-renormalizable term

External fields are fixed by dim.7 operator

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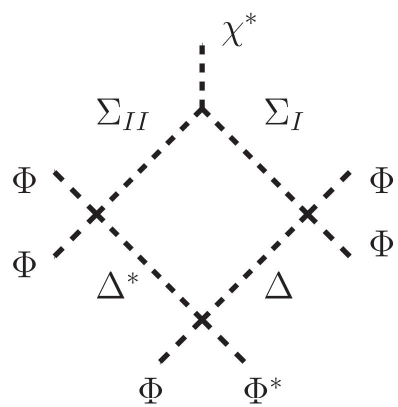


$$-\frac{1}{\Lambda^3}\{(\chi^*\Phi^5\Phi^*) + \text{H.c.}\}$$

~~Non-renormalizable term~~

External fields are fixed by dim.7 operator

Decompose the diagram

- 2 quintuplets ($\Sigma_{I,II}$) and 1 triplet (Δ) with exact Z_2 parity

Forbid VEV of extra multiplet
(Bonus: Dark Matter candidate)

$$\mathcal{L}_{\cancel{U(1)}} = \mu\,\chi_{abcdef}\Sigma_I^{*abci}\Sigma_{II}^{*defj}\epsilon_{ij} + \Phi_i\Phi_j(c_I\,\Sigma_I^{*ijkl} + c_{II}\,\Sigma_{II}^{*ijkl})\Delta_{kl} + f\,\Phi_a\Phi^{*b}\Delta^{*ac}\Delta_{bc} + \text{H.c.}$$

Soft breaking term of the global U(1)

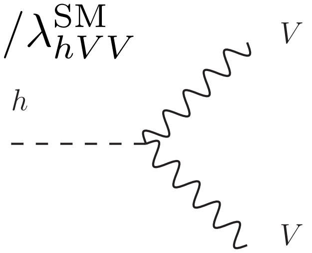
We obtain correct dim.7 operator from a renormalizable theory!!

Model predictions

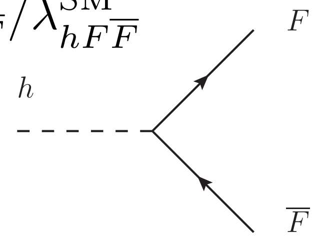
Model predictions

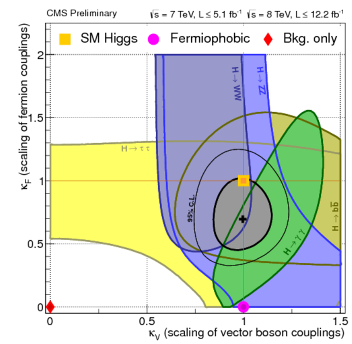
Modified Gauge/Yukawa coupling

$$\kappa_V = \lambda_{hVV}/\lambda_{hVV}^{\rm SM}$$



$$\kappa_F = \lambda_{hF\overline{F}}/\lambda_{hF\overline{F}}^{\rm SM}$$



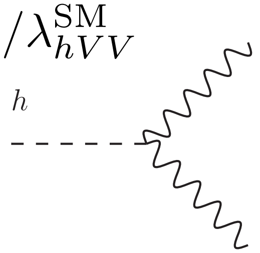


- ✧ SM: $\kappa_V^{\rm SM} = 1, \kappa_F^{\rm SM} = 1$

Model predictions

Modified Gauge/Yukawa coupling

$\kappa_V = \lambda_{hVV}/\lambda_{hVV}^{\rm SM}$



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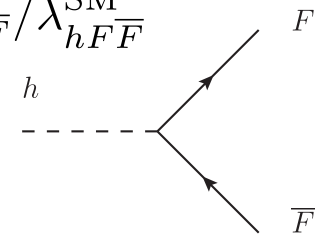
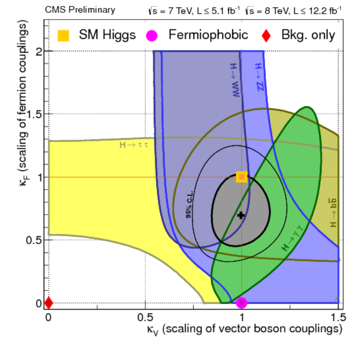




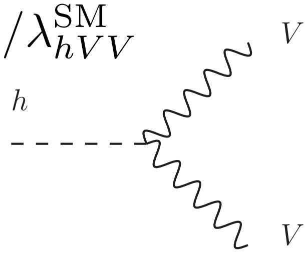
- ✧ SM: $\kappa_V^{\rm SM} = 1, \kappa_F^{\rm SM} = 1$
- ✧ 2HDM: $\kappa_V^{\rm 2HDM} = \sin(\beta - \alpha), \kappa_F^{\rm 2HDM(\text{-}I)} = \cos\alpha / \sin\beta$

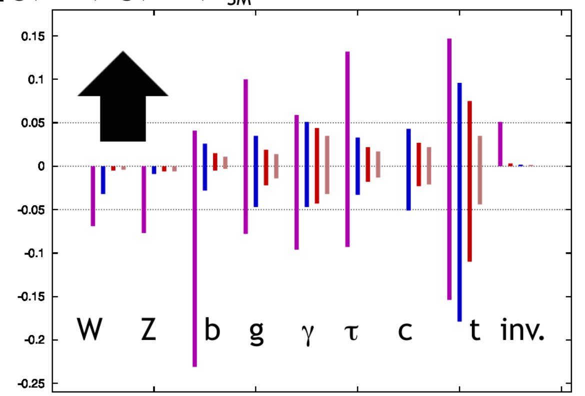
$$\kappa_V^{\rm 2HDM} \leq 1$$

Model predictions

Modified Gauge/Yukawa coupling

$\kappa_V = \lambda_{hVV}/\lambda_{hVV}^{\rm SM}$





- SM: $\kappa_V^{\rm SM} = 1, \kappa_F^{\rm SM} = 1$
- 2HDM: $\kappa_V^{\rm 2HDM} = \sin(\beta - \alpha), \kappa_F^{\rm 2HDM(\text{-}I)} = \cos\alpha/\sin\beta$

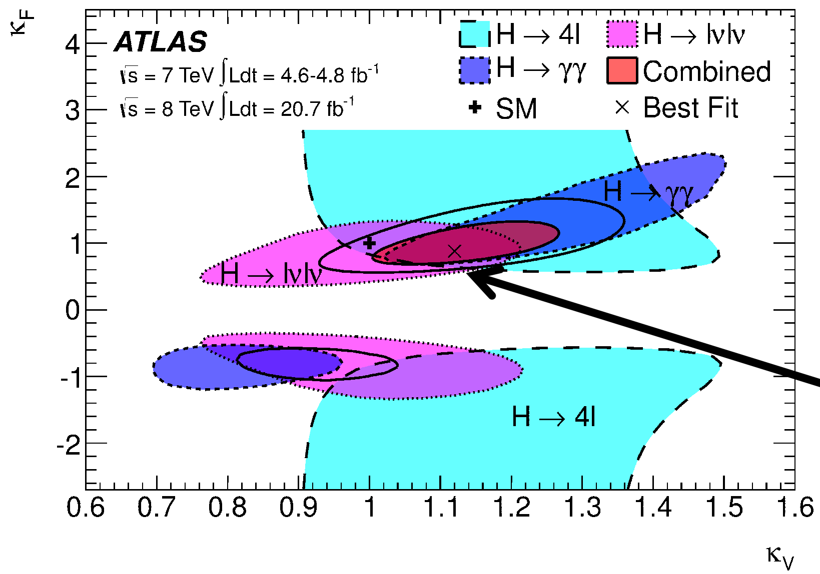
$$\kappa_V^{\rm 2HDM} \leq 1$$

$$\kappa_{\mathbf{V}}^{\rm septet} \geq 1$$

- **Septet:** $\kappa_V^{\rm septet} = \sin\beta\cos\alpha - 4\cos\beta\sin\alpha, \kappa_F^{\rm septet} = \cos\alpha/\sin\beta$

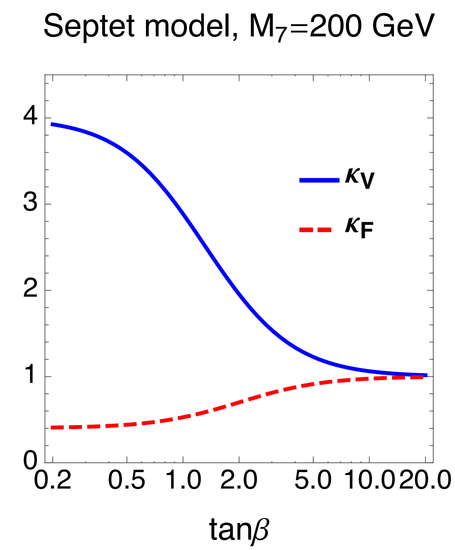
κ_V can be larger than one!!

Model predictions



ATLAS (EPS-HEP 2013)

$\kappa_V > 1$ is favored



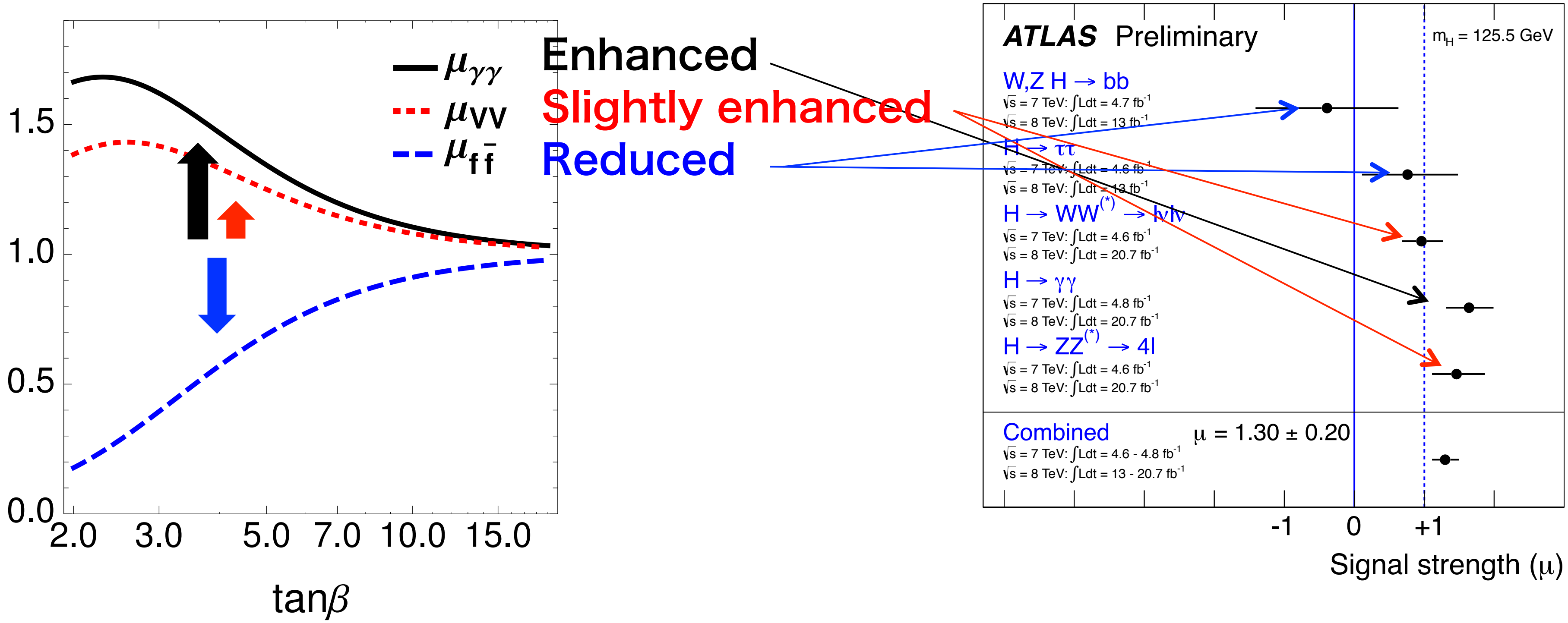
✧ **Septet:** $\kappa_V^{\text{septet}} = \sin\beta\cos\alpha - 4\cos\beta\sin\alpha$, $\kappa_F^{\text{septet}} = \cos\alpha/\sin\beta$

κ_V can be larger than one!!

Distinctive feature of the septet model

Signal strength

M_7=200 GeV



Seems to be compatible with "ATLAS" results

$$\mu_x = \frac{\sigma \cdot \mathcal{B}_x}{\sigma^{\rm SM} \cdot \mathcal{B}_x^{\rm SM}}$$

σ : Production cross section of the Higgs boson

B_{xx}: Decay Branching ratio of the Higgs boson into xx

Normalized by SM

More phenomenology

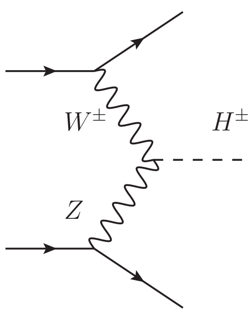
$W^{\pm}ZH^{\pm}$ vertex

- Anomalous $W^{\pm}ZH^{\mp}$ coupling?
  - No $H^{\pm}$ in the SM
  - Forbidden also in the MSSM (2HDM)
  - Case with septet

    → Septet naturally induces $W^{\pm}ZH^{\mp}$ vertex

$$\chi = \begin{pmatrix} H^{+++++} \\ H^{++++} \\ H^{+++} \\ H^{++} \\ H_1^{+} \\ (v_7 + h_7 + i\, z_7)/\sqrt{2} \\ H_2^{-} \end{pmatrix}$$

- WZ fusion @ LHC



  - Simulation studies have been done

Asakawa, Kanemura, Kanzaki, PRD75, 075022 (2007)

v_7 ~ O(10GeV) can be tested!!

$W^{\pm}ZH^{\pm}$ vertex

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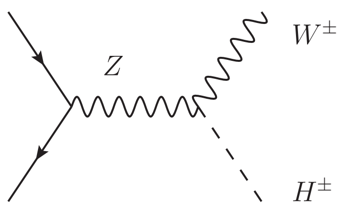
Charged Higgs strahlung @ ILC



- Counter measurement of Higgs strahlung ($e^+e^- \to Zh$)

  Most important measurement of hVV coupling @ ILC
- Recoil method can be applied

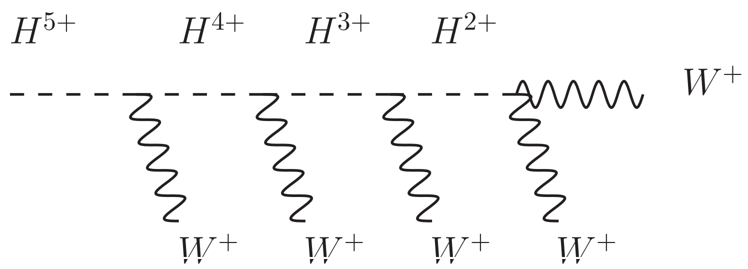
  $W^{\pm}ZH^{\pm}$ vertex is tested without measuring $H^{\pm}$

  Kanemura, Yagyu, Yanase, PRD83, 075018 (2011)

v_7 ~ O(GeV) can be tested!!

Multiply charged Higgs bosons

- Multiple W bosons



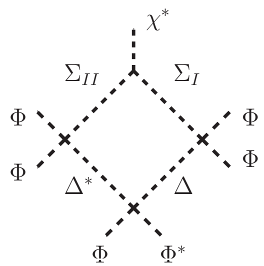
- ✓ Long decay chain (Maybe long-lived)
- ✓ Large cross section (Q=5)

Summary

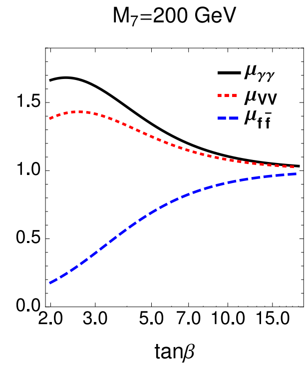
- **Beyond the Higgs**
  - ρ parameter and Beyond the SM → Septet (next minimal)

$$\rho_{\text{tree}} = \frac{\sum_\alpha [I_\alpha(I_\alpha+1) - Y_\alpha^2] v_\alpha^2}{\sum_\beta 2Y_\beta^2 v_\beta}$$

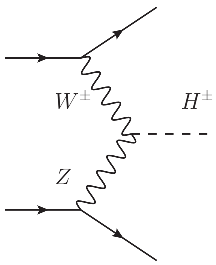
  - Model with septet

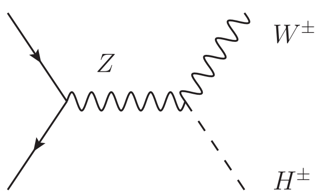


  - LHC Higgs signal vs Septet



  - Smoking gun of Septet @ colliders





Thank you very much for your attention

Back up

SU(2)

$$[J^a, J^b] = i\,\epsilon^{abc}\,J^c$$

$$\begin{cases} \mathbf{J}^2|j,m\rangle = j(j+1)|j,m\rangle \\ J^3|j,m\rangle = m|j,m\rangle \end{cases}$$

$$j = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \cdots$$

$$m = -j, -j+1, \cdots, j-1, j$$

Lowering/Raising operators $J^{\pm} \equiv J_1 \pm i\,J_2$

$$[J^3, J^{\pm}] = \pm\,J^{\pm}$$

$$[J^+, J^-] = 2\,J^3$$

$$J^{\pm}|j,m\rangle = \sqrt{(j \mp m)(j+1 \pm m)}|j,m\pm 1\rangle$$

(2j+1) representation in j_3 space

$$|j,m\rangle = \phi_{j,m} = \begin{pmatrix} \varphi_{j,j} \\ \varphi_{j,j-1} \\ \cdots \\ \varphi_{j,-j+1} \\ \varphi_{j,-j} \end{pmatrix}$$

SU(2)

With fundamental rep. indices $$\phi^i (i = 1, 2) = \mathbf{2} = \square$$

$$\phi = \mathbf{1}, \quad \phi_i = \epsilon_{ij}\phi^j = \mathbf{2}^*$$

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3} \qquad \square \otimes \square = \mathbf{1} \oplus \square\square$$

$$\mathbf{3} \otimes \mathbf{2} = \mathbf{2} \oplus \mathbf{4} \qquad \square\square \otimes \square = \square \oplus \square\square\square$$

$$\cdots \qquad \cdots$$

$$\mathbf{6} \otimes \mathbf{2} = \mathbf{5} \oplus \mathbf{7} \qquad \square\square\square\square\square \otimes \square = \square\square\square\square \oplus \square\square\square\square\square\square$$

Higher dim. rep.

$$\phi^{\overbrace{ij\cdots mn}^{(N-1)}} = \mathbf{N} = \overbrace{\square\square\cdots\square\square}^{(N-1)}$$

Mass spectrum

- For simplicity
- An accidental global U(1) symmetry in the Higgs potential

$$\mathcal{V} = -\mu_2^2\Phi^\dagger\Phi + M_7^2\chi^\dagger\chi + \lambda(\Phi^\dagger\Phi)^2 - \frac{1}{\Lambda^3}\{(\chi^*\Phi^5\Phi^*) + \mathrm{H.c.}\}$$

$$~~+\sum_{A=1}^{4}\lambda_A(\chi^\dagger\chi\chi^\dagger\chi)_A + \sum_{B=1}^{2}\kappa_B(\Phi^\dagger\Phi\chi^\dagger\chi)_B~~$$

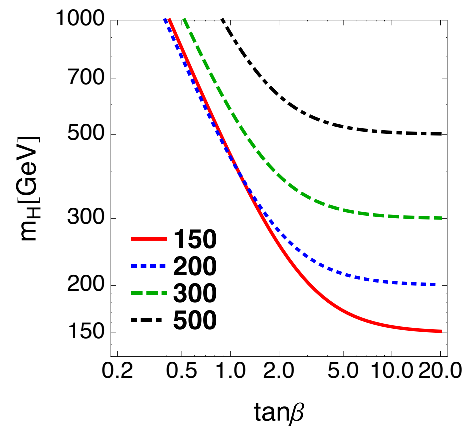
- Mass eigenvalues

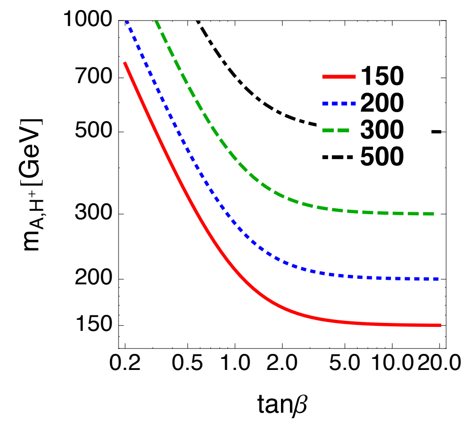
$$m_h^2 = \left(1 + \frac{3}{2}\,\frac{1}{t_\beta\, t_\alpha}\right) M_7^2$$

$$m_H^2 = \left(1 - \frac{3}{2}\,\frac{t_\alpha}{t_\beta}\right) M_7^2$$

$$m_A^2 = m_{H_1^\pm}^2 = M_7^2/s_\beta^2$$

$$m_{H_2^\pm}^2 = m_{H^{2\pm}}^2 = m_{H^{3\pm}}^2 = m_{H^{4\pm}}^2 = m_{H^{5\pm}}^2 = M_7^2$$





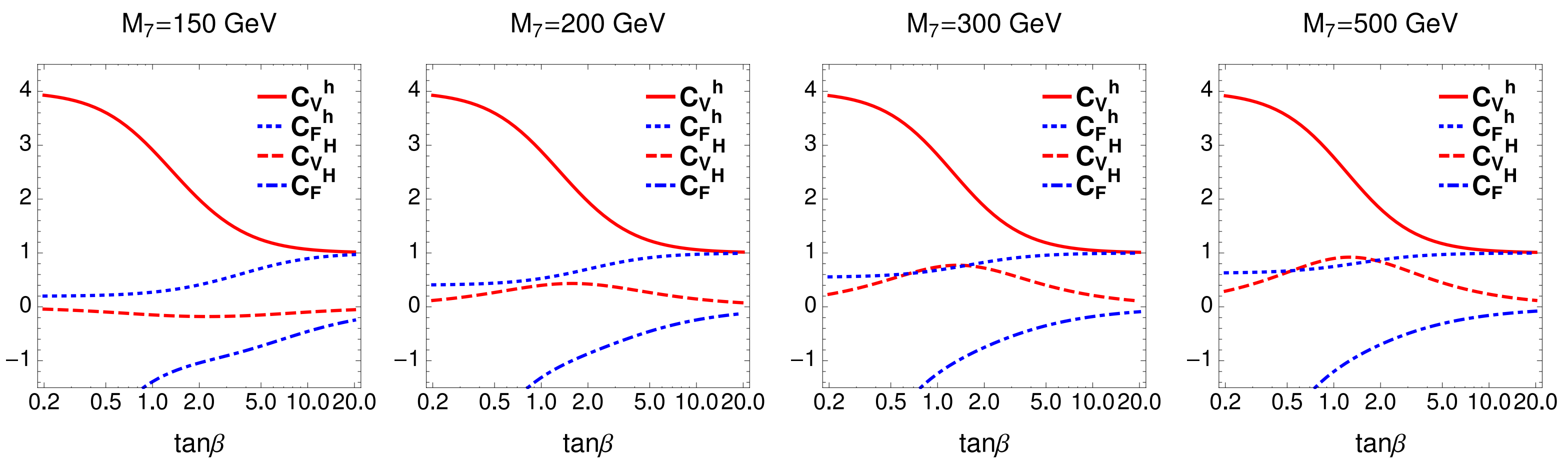
$$\tan\beta = \frac{v_2}{4v_7}$$

More κ_V and κ_F

- 2 CP even Higgs bosons (h, H)

$$\kappa_V^h = \sin\beta\cos\alpha - 4\cos\beta\sin\alpha, \kappa_F^h = \cos\alpha/\sin\beta$$

$$\kappa_V^H = \sin\beta\sin\alpha + 4\cos\beta\cos\alpha, \kappa_F^H = \sin\alpha/\sin\beta$$



Electroweak precision data

- For simplicity An accidental global U(1) symmetry in the Higgs potential

$$\mathcal{V} = -\mu_2^2\Phi^\dagger\Phi + M_7^2\chi^\dagger\chi + \lambda(\Phi^\dagger\Phi)^2 - \frac{1}{\Lambda^3}\{(\chi^*\Phi^5\Phi^*) + \text{H.c.}\}$$

$$\cancel{+\sum_{A=1}^{4}\lambda_A(\chi^\dagger\chi\chi^\dagger\chi)_A + \sum_{B=1}^{2}\kappa_B(\Phi^\dagger\Phi\chi^\dagger\chi)_B}$$

- Oblique parameters

$$S = \frac{1}{4\pi}\Big[(C_V^h)^2\,G^{hZ\prime} + (C_V^H)^2\,G^{HZ\prime} + 30\,c_\beta^2\,G^{H_2^\pm W\prime} + 30\,s_\beta^2\,F^{H_1^\pm H_2^\pm\prime} - \tfrac{1}{3}\,\ln m_{H_1^\pm}^2 - 15\,\ln m_{H_2^\pm}^2 + (4\,s_\alpha s_\beta - c_\alpha c_\beta)F^{hA\prime} + (4\,c_\alpha s_\beta + s_\alpha c_\beta)F^{HA\prime}\Big]$$

$$T = \frac{\sqrt{2}G_F}{\alpha_{\text{EM}}(4\pi)^2}\Big[(C_V^h)^2\,\Delta G^h + (C_V^H)^2\,\Delta G^H - 15\,c_\beta^2\,\Delta G^{H_2^\pm}\Big]$$

$$\begin{cases} F^{xy} = \dfrac{m_x^2+m_y^2}{2} - \dfrac{m_x^2 m_y^2}{m_x^2-m_y^2}\ln\dfrac{m_x^2}{m_y^2} \\ G^{xV} = F^{xV} + 4\,m_V^2\Big(-1 + \dfrac{m_x^2\ln m_x^2 - m_V^2\ln m_V^2}{m_x^2-m_V^2}\Big) \\ \Delta G^x = G^{xW} - G^{xZ} \\ F^{xy\prime} = -\dfrac{1}{3}\Big(+\dfrac{4}{3} - \dfrac{m_x^2\ln m_x^2 - m_y^2\ln m_y^2}{m_x^2-m_y^2} - \dfrac{m_x^2+m_y^2}{(m_x^2-m_y^2)^2}\,F^{xy}\Big) \\ G^{xV\prime} = F^{xV\prime} + 4\,m_V^2\Big(-\dfrac{1}{(m_x^2-m_V^2)^2}\,F^{xV}\Big) \end{cases}$$

$$\tan\beta = \frac{v_2}{4v_7}$$

Electroweak precision data

- Best fit values

$$\Delta S = 0.04 \pm 0.09$$

$$\Delta T = 0.07 \pm 0.08$$

$$(\sigma_{ST} = 0.88)$$

- Oblique parameters

$$S = \frac{1}{4\pi}\Big[(C_V^h)^2\, G^{hZ\prime} + (C_V^H)^2\, G^{HZ\prime} + 30\, c_\beta^2\, G^{H_2^\pm W\prime} + 30\, s_\beta^2\, F^{H_1^\pm H_2^\pm\prime} - \tfrac{1}{3}\ln m_{H_1^\pm}^2 - 15\,\ln m_{H_2^\pm}^2 + (4\, s_\alpha s_\beta - c_\alpha c_\beta) F^{hA\prime} + (4\, c_\alpha s_\beta + s_\alpha c_\beta) F^{HA\prime}\Big]$$

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$$\left\{\begin{aligned}
F^{xy} &= \frac{m_x^2 + m_y^2}{2} - \frac{m_x^2 m_y^2}{m_x^2 - m_y^2} \ln\frac{m_x^2}{m_y^2} \\
G^{xV} &= F^{xV} + 4\, m_V^2 \Big(-1 + \frac{m_x^2 \ln m_x^2 - m_V^2 \ln m_V^2}{m_x^2 - m_V^2}\Big) \\
\Delta G^x &= G^{xW} - G^{xZ} \\
F^{xy\prime} &= -\frac{1}{3}\Big(+\frac{4}{3} - \frac{m_x^2 \ln m_x^2 - m_y^2 \ln m_y^2}{m_x^2 - m_y^2} - \frac{m_x^2 + m_y^2}{(m_x^2 - m_y^2)^2}\, F^{xy}\Big) \\
G^{xV\prime} &= F^{xV\prime} + 4\, m_V^2 \Big(-\frac{1}{(m_x^2 - m_V^2)^2}\, F^{xV}\Big)
\end{aligned}\right.$$

$$\tan\beta = \frac{v_2}{4 v_7}$$

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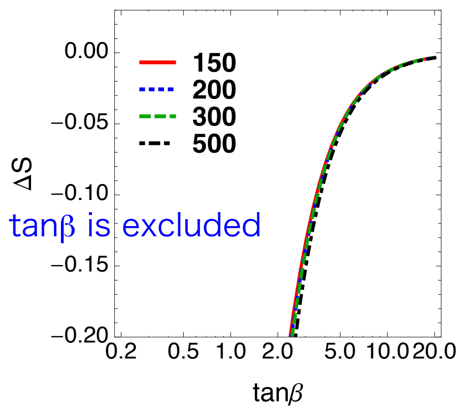
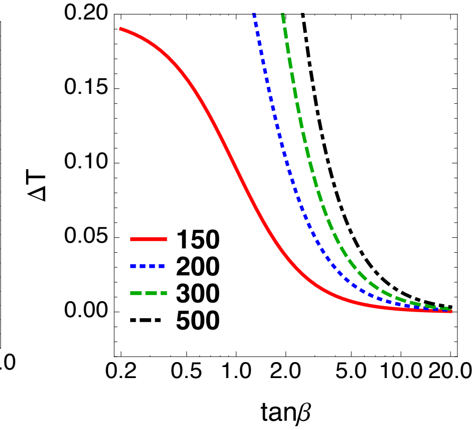


Small tanβ is excluded



- Oblique parameters

$$S = \frac{1}{4\pi}\Big[(C_V^h)^2\, G^{hZ'} + (C_V^H)^2\, G^{HZ'} + 30\, c_\beta^2\, G^{H_2^\pm W'} + 30\, s_\beta^2\, F^{H_1^\pm H_2^\pm{}'}$$

$$- \tfrac{1}{3}\ \ln m_{H_1^\pm}^2 - 15\ \ln m_{H_2^\pm}^2 + (4\, s_\alpha s_\beta - c_\alpha c_\beta) F^{hA'} + (4\, c_\alpha s_\beta + s_\alpha c_\beta) F^{HA'}\Big]$$

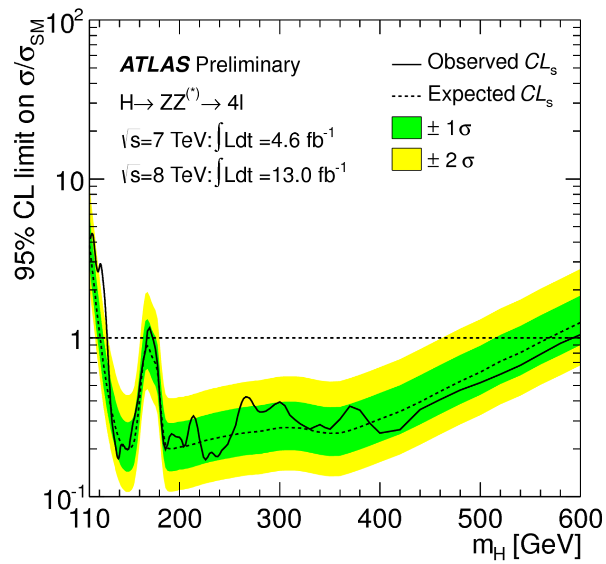
$$T = \frac{\sqrt{2} G_F}{\alpha_{\rm EM} (4\pi)^2}\Big[(C_V^h)^2\, \Delta G^h + (C_V^H)^2\, \Delta G^H - 15\, c_\beta^2\, \Delta G^{H_2^\pm}\Big]$$

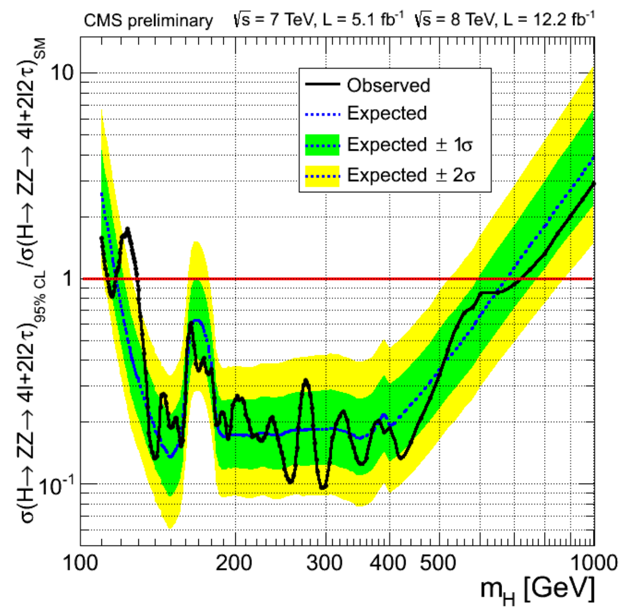
$(\Delta S, \Delta T)$	$\tan\beta = 3$	$\tan\beta = 5$	$\tan\beta = 10$	$\tan\beta = 20$
$M_7 = 150$ GeV	(-0.13, 0.019)	(-0.05, 0.007)	(-0.013, 0.002)	(-0.003, 0.)
$M_7 = 200$ GeV	(-0.14, 0.050)	(-0.05, 0.019)	(-0.014, 0.005)	(-0.003, 0.001)
$M_7 = 300$ GeV	(-0.14, 0.088)	(-0.05, 0.033)	(-0.013, 0.008)	(-0.003, 0.002)
$M_7 = 500$ GeV	(-0.15, 0.14)	(-0.06, 0.053)	(-0.014, 0.013)	(-0.004, 0.003)

$$\tan\beta = \frac{v_2}{4 v_7}$$

Signal strength of the extra Higgs boson

- Search for the SM Higgs boson can be interpret as a constraint on extra Higgs bosn (H)
- VV decay channel [μ^H_{VV} (gg→H→VV)] gives stronger limits for heavier mass region





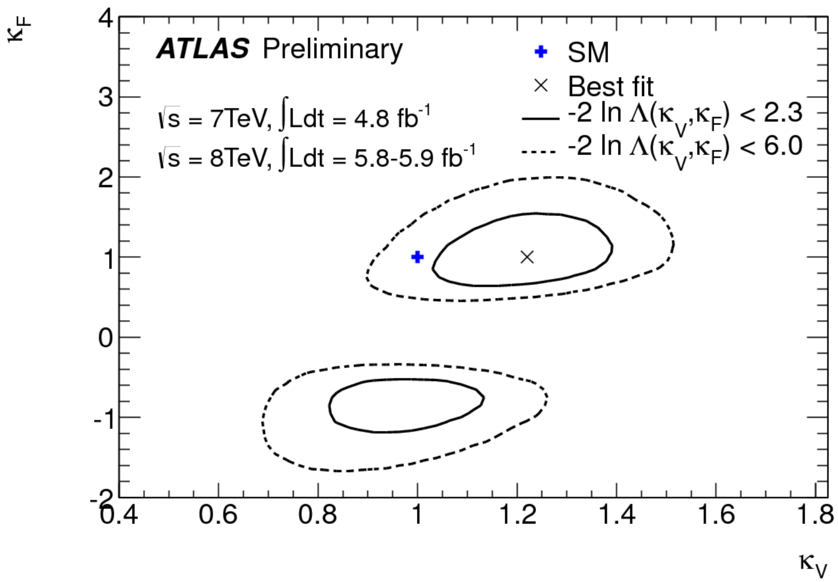
Results from the septet

$(m_H[\text{GeV}],\, \mu^H_{VV})$	$\tan\beta = 5$	$\tan\beta = 6$	$\tan\beta = 7$	$\tan\beta = 8$	$\tan\beta = 9$	$\tan\beta = 10$
$M_7 = 150$ GeV	(171., 0.44)	(165., 0.31)	(161., 0.20)	(159., 0.13)	(157., 0.081)	(156., 0.062)
$M_7 = 200$ GeV	(214., 0.21)	(210., 0.15)	(207., 0.11)	(206., 0.089)	(205., 0.071)	(204., 0.059)
$M_7 = 300$ GeV	(316., 0.12)	(311., 0.087)	(308., 0.065)	(306., 0.050)	(305., 0.040)	(304., 0.032)
$M_7 = 500$ GeV	(523., 0.12)	(516., 0.084)	(512., 0.063)	(509., 0.048)	(507., 0.038)	(503., 0.031)

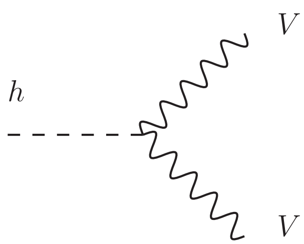
Small tanβ/m_H is excluded

Is it SM-like?

- κ_V vs κ_F



Production	Decay	LO SM	
VH	$H \to bb$	$\sim \frac{C_V^2 \times C_F^2}{C_F^2}$	$\sim C_V^2$
ttH	$H \to bb$	$\sim \frac{C_F^2 \times C_F^2}{C_F^2}$	$\sim C_F^2$
VBF/VH	$H \to \tau\tau$	$\sim \frac{C_V^2 \times C_F^2}{C_F^2}$	$\sim C_V^2$
ggH	$H \to \tau\tau$	$\sim \frac{C_F^2 \times C_F^2}{C_F^2}$	$\sim C_F^2$
ggH	$H \to ZZ$	$\sim \frac{C_F^2 \times C_V^2}{C_F^2}$	$\sim C_V^2$
ggH	$H \to WW$	$\sim \frac{C_F^2 \times C_V^2}{C_F^2}$	$\sim C_V^2$
VBF/VH	$H \to WW$	$\sim \frac{C_V^2 \times C_V^2}{C_F^2}$	$\sim C_V^4 / C_F^2$
ggH	$H \to \gamma\gamma$	$\sim \frac{C_F^2 \times (8.6 C_V - 1.8 C_F)^2}{C_F^2}$	$\sim C_V^2$
VBF	$H \to \gamma\gamma$	$\sim \frac{C_V^2 \times (8.6 C_V - 1.8 C_F)^2}{C_F^2}$	$\sim C_V^4 / C_F^2$



$$\kappa_V = \lambda_{hVV} / \lambda_{hVV}^{\rm SM}$$

κ_V can be different from unity

Signal strength of the Higgs boson

Signal strength for “xx” channel

$$\mu_{xx} = \frac{\sigma \cdot \mathcal{B}_{xx}}{\sigma^{\rm SM} \cdot \mathcal{B}_{xx}^{\rm SM}}$$

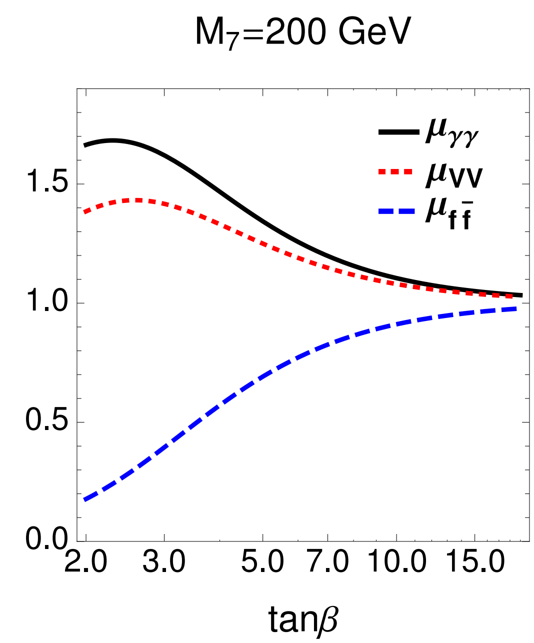
μ_{xx}=1 for SM

μ_{xx}≠1 for Beyond the SM

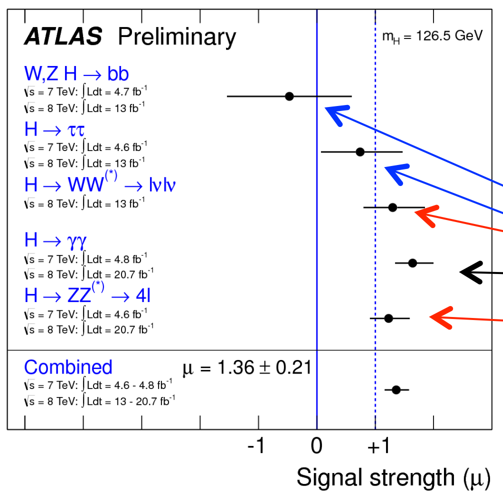
σ : Production cross section of the Higgs boson

B_{xx}: Decay Branching ratio of the Higgs boson into xx

Normalized by SM



Results from LHC



Seems to be compatible with “ATLAS” results

γγ: Enhanced

VV: Slightly Enhanced

FF: Reduced