D-term Triggered Dynamical Supersymmetry Breaking

H. I. & N. Maru (Osaka City University)

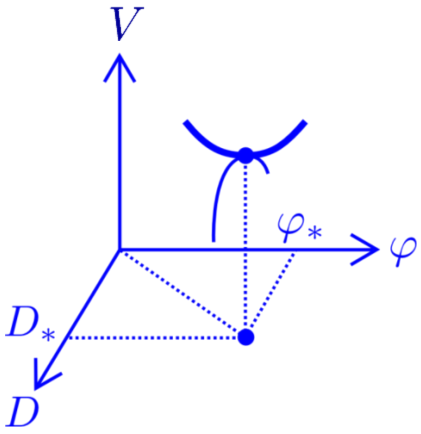
based on
- IJMPA 27 (2012) 1250159, arXiv:1109.2276
- PRD 88 (2013) 025012, arXiv:1301.7548

I)
- most desirable to break $\mathcal{N}=1$ SUSY dynamically (DSB)
- In the past, instanton generated superpotential e.t.c. $\langle F \rangle_{nonpt} \rightarrow \langle D \rangle \neq 0$

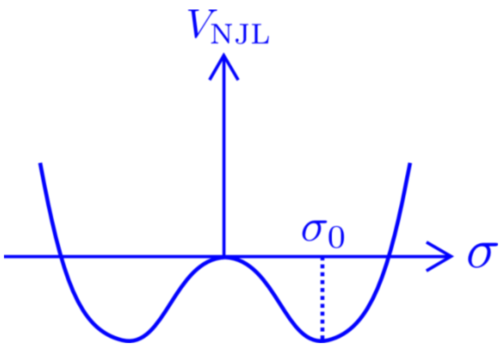
In this talk, we will accomplish

D-term DSB (DDSB)
in a self-consistent
Hartree-Fock approximation

- based on the nonrenormalizable D-gaugino-matter fermion which is present in generic $\mathcal{N}=1$ SUSY (effective) U(N) gauge action
- the vac. is metastable, can be made long lived
- requires the discovery of scalar gluons in nature, so that distinct from the previous proposals



compare this with
the NJL ptl.



Contents

II) ・action

$$\mathcal{L} = \int d^4\theta K(\Phi^a, \bar{\Phi}^a) + (gauging) + \int d^2\theta \mathrm{Im}\frac{1}{2}\tau_{ab}(\Phi^a)\mathcal{W}^{\alpha a}\mathcal{W}^b_\alpha + \left(\int d^2\theta W(\Phi^a) + c.c.\right)$$

$$= \mathcal{L}_{\text{Kähler}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{sup}}$$

K; Kähler potential

τ_{ab}; gauge kinetic superfield from the second derivatives of a generic holomorphic function $\mathcal{F}(\Phi^a)$

W; superpotential

We look at its component expansion

⋮

・special cases

- demand the Kähler function K to be special Kähler,

$$\Rightarrow \quad K = \mathrm{Im}\mathrm{Tr}\, \bar{\Phi}\frac{\partial \mathcal{F}(\Phi)}{\partial \Phi}, \quad \text{and} \quad g_{ab} = \mathrm{Im}\mathcal{F}_{ab} \quad \text{etc.}$$

- further, choose W such that the action possess the rigid $\mathcal{N}=2$ supersymmetry

$\Rightarrow$ tree vacua $\mathcal{N}=2 \rightarrow \mathcal{N}=1$ spontaneously (APT, FIS)

▪ assumptions made

1) general $\mathcal{N}=1$ action with adjoint Φ^a & V^a
with three input functions K, $\mathcal{F}_{ab}$, W

2) third derivatives of $\mathcal{F}$ at the scalar vev's nonvanishing

3) W at tree level preserves $\mathcal{N}=1$ susy

4) the gauge group $U(N)$, the vac. being in its unbroken phase

▪ supercurrent & D^a, F^a eqs

off-shell form of the $\mathcal{N}=1$ supercurrent

$$\eta_1 \mathcal{S}^{(1)\mu} = \sqrt{2} g_{ab} \eta_1 \sigma^\nu \bar{\sigma}^\mu \psi^a \mathcal{D}_\nu \bar{\phi}^b + \sqrt{2} i g_{ab} \eta_1 \sigma^\mu \bar{\psi}^a F^b - i \mathcal{F}_{ab} \eta_1 \sigma_\nu \bar{\lambda}^a F^{\mu\nu b} + \frac{1}{2} \mathcal{F}_{ab} \epsilon^{\mu\nu\rho\delta} \eta_1 \sigma_\nu \bar{\lambda}^a F^{\rho\delta b} - \frac{i}{2} \bar{\mathcal{F}}_{ab} \eta_1 \sigma^\mu \bar{\lambda}^a D^b + \frac{\sqrt{2}}{4} \left(\mathcal{F}_{abc} \psi^c \sigma^\nu \bar{\sigma}^\mu \lambda^b - \bar{\mathcal{F}}_{abc} \bar{\lambda}^c \bar{\sigma}^\mu \sigma^\nu \bar{\psi}^b \right) \eta_1 \sigma_\nu \bar{\lambda}^a$$

$\Rightarrow$ NGF will be l.c. of λ^0 & ψ^0

・ reasoning to DDSB

$$\langle D^0 \rangle = -\frac{1}{2\sqrt{2}} \langle g^{00} \left(\mathcal{F}_{0cd} \psi^d \lambda^c + \bar{\mathcal{F}}_{0cd} \bar{\psi}^d \bar{\lambda}^c \right) \rangle,$$

- holomorphic part of the mass matrix:

$$M_{Fa} \equiv \begin{pmatrix} 0 & -\frac{\sqrt{2}}{4} \langle \mathcal{F}_{0aa} D^0 \rangle \\ -\frac{\sqrt{2}}{4} \langle \mathcal{F}_{0aa} D^0 \rangle & \langle \partial_a \partial_a W \rangle \end{pmatrix}.$$

mixed Majorana-Dirac type

eigenvalues: $$\Lambda^{(\pm)}_{a\mathbf{11}} = \frac{1}{2} \langle \partial_a \partial_a W \rangle \left(1 \pm \sqrt{1 + \frac{\langle \mathcal{F}_{0aa} D^0 \rangle^2}{2 \langle \partial_a \partial_a W \rangle^2}} \right).$$

- however, the non-vanishing F^0 term induced as well, as the stationary value of the scalar fields gets shifted.

the holo. part of the complete mass matrix

$$\mathcal{M}_a = \begin{pmatrix} -\frac{i}{2} g^{aa} \mathcal{F}_{0aa} F^0, & -\frac{\sqrt{2}}{4} \sqrt{g^{aa} (\mathrm{Im}\mathcal{F})^{aa}} \mathcal{F}_{0aa} D^0 \\ -\frac{\sqrt{2}}{4} \sqrt{g^{aa} (\mathrm{Im}\mathcal{F})^{aa}} \mathcal{F}_{0aa} D^0, & g^{aa} \partial_a \partial_a W + g^{aa} g_{0a,a} \bar{F}^0 \end{pmatrix} = \begin{pmatrix} m^a_{\lambda\lambda} & m^a_{\lambda\psi} \\ m^a_{\psi\lambda} & m^a_{\psi\psi} \end{pmatrix}.$$

- suppress the indices as we work with the unbroken phase U(N) phase

$$\Delta \equiv -\frac{2 m_{\lambda\psi}}{m_{\psi\psi}}, \qquad f \equiv \frac{2 i m_{\lambda\lambda}}{\mathrm{tr}\mathcal{M}}.$$

The two eigenvalues $\Lambda^{(\pm)} \equiv (\mathrm{tr}\mathcal{M}) \lambda^{(\pm)}$,
where

$$\lambda^{(\pm)} = \frac{1}{2} \left(1 \pm \sqrt{(1 + if)^2 + \left(1 + \frac{i}{2} f \right)^2 \Delta^2} \right).$$

$\Rightarrow$ The masses for the two species of SU(N) fermions

III)

change the notation for expectation values from $\langle \ldots \rangle$ to $\ldots_*$
(∵ vev.= the stationary value in the variational analysis.)

・point of the H.F. approximation

spirit: tree $\sim$ 1-loop in the $\hbar$ expansion

$\Rightarrow$ optimal configuration, which is transcendental

- three const. bkgd fields,

  $\varphi \equiv \varphi^0$ (complex), U(N) invariant scalar,

  $D \equiv D^0$(real) ,

  $F \equiv F^0$ (complex).

- denote our effective potential by

$$V = V^{\rm tree} + V_{\rm c.t.} + V_{1-{\rm loop}}.$$

after the elimination of the auxiliary fields denote by $V_{\rm scalar}$

▪tree part & warm up

all config. U(N) inv. $\Rightarrow$ suppress indices

$$V^{\rm tree}(D,F,\bar F,\varphi,\bar\varphi) = -gF\bar F - \frac{1}{2}({\rm Im}\mathcal{F}'')D^2 - FW' - \bar F\bar W'.$$

all minus signs correct

- stationary conditions $\Rightarrow$

$$V^{\rm tree}_{\rm scalar}(\varphi,\bar\varphi) \equiv V^{\rm tree}(\varphi,\bar\varphi,D_*=0,F=F_*(\varphi,\bar\varphi),\bar F=\overline{F_*(\varphi,\bar\varphi)}) = g^{-1}(\varphi,\bar\varphi)|W'(\varphi)|^2.$$

$$\left.\frac{\partial^2 V^{\rm tree}_{\rm scalar}(\varphi,\bar\varphi)}{\partial\varphi\partial\bar\varphi}\right|_{\varphi_*,\bar\varphi_*} = g^{-1}(\varphi_*,\bar\varphi_*)\left|W''(\varphi_*)\right|^2,$$

$$m_s(\varphi,\bar\varphi) \equiv g^{-1}(\varphi,\bar\varphi)W''(\varphi),$$

$$m_{s*} = m_s(\varphi_*,\bar\varphi_*).$$

$$\Delta \equiv -2\frac{m_{\lambda\psi}}{m_{\psi\psi}} = \frac{\sqrt{2}}{2}\frac{\sqrt{g^{-1}({\rm Im}\mathcal{F}'')^{-1}}\mathcal{F}'''}{g^{-1}W'' + g^{-1}\partial g\bar F}D \equiv r(\varphi,\bar\varphi,F,\bar F)D.$$

$$f_3 \equiv \frac{g^{-1}\mathcal{F}'''F}{g^{-1}W'' + g^{-1}\partial g\bar F},$$

- the mass scales of the problem:
  m_{s*}, the scalar gluon mass and $g^{-1}\mathcal{F}'''_*$, the third prepotential derivative,
  (and $g^{-1}\partial g$), SUSY breaking scale being essentially the geometric mean.

▪ treatment of UV infinity

UV scale and infinity reside in $\mathcal{F}$. The supersymmetric counterterm:

$$V_{\mathrm{c.t.}} = -\frac{1}{2}\mathrm{Im}\int d^2\theta \Lambda \mathcal{W}^{0\alpha}\mathcal{W}_{0\alpha} = -\frac{1}{2}(\mathrm{Im}\Lambda)D^2.$$

It is a counterterm associated with $\mathrm{Im}\mathcal{F}''$.

A renormalization condition

$$\frac{1}{N^2}\frac{\partial^2 V}{(\partial D)^2}\bigg|_{D=0,\varphi=\varphi_*,\bar{\varphi}=\bar{\varphi}_*} = 2c,$$

relate (or transmute) the original infinity of the dimensional reduction scheme with that of $\mathrm{Im}\mathcal{F}''$.

▪ the one-loop part

$$V_{1-\mathrm{loop}} = \frac{N^2|\mathrm{tr}\mathcal{M}|^4}{32\pi^2}\left[A(\varepsilon,\gamma)\left(|\lambda^{(+)}|^4 + |\lambda^{(-)}|^4 - \left|\frac{m_s}{\mathrm{tr}\mathcal{M}}\right|^4\right)\right.$$
$$\left. -|\lambda^{(+)}|^4 \log|\lambda^{(+)}|^2 - |\lambda^{(-)}|^4\log|\lambda^{(-)}|^2 + \left|\frac{m_s}{\mathrm{tr}\mathcal{M}}\right|^4 \log\left|\frac{m_s}{\mathrm{tr}\mathcal{M}}\right|^4\right].$$

$$A(\varepsilon,\gamma) = \frac{1}{2} - \gamma + \frac{1}{\varepsilon}, \qquad \varepsilon = 2 - \frac{d}{2}.$$

IV) ・variational analysis

$$\begin{cases} \dfrac{\partial V}{\partial D} = 0 \\ \dfrac{\partial V}{\partial F} = 0 \text{ and its complex conjugate} \\ \dfrac{\partial V}{\partial \varphi} = 0 \text{ and its complex conjugate} \end{cases}$$

- work in the region where the strength $||F_*||$ small and can be treated perturbatively.

gap eq.

$$\frac{\partial V(D, \varphi, \bar{\varphi}, F=0, \bar{F}=0)}{\partial D} = 0,$$

stationary cond. for scalars

$$\frac{\partial V(D, \varphi, \bar{\varphi}, F=0, \bar{F}=0)}{\partial \varphi} = 0$$

$\Rightarrow$ stationary values $(D_*, \varphi_*, \bar{\varphi}_*)$

- $$\left.\frac{\partial V(D = D_*(0,0), \varphi = \varphi_*(0,0), \bar{\varphi} = \bar{\varphi}_*(0,0), F, \bar{F})}{\partial F}\right|_{D,\varphi,\bar{\varphi},\bar{F} \text{ fixed}} = 0$$

$\Rightarrow$ $\bar{F} = \bar{F}_*$ perturbatively

▪ the analysis in the region $F_* \approx 0$

- explicit determination of $V(D,\phi,\bar{\phi},F=0,\bar{F}=0)$:

  first solve the normalization condition
$$2cN^2 = \left.\frac{\partial^2 V}{(\partial D)^2}\right|_{D=0,*}$$
  to obtain
$$A = \frac{1}{2} + \frac{32\pi^2}{|m_{s*}|^4(r_{0*}^2+\bar{r}_{0*}^2)}\left(2c + \frac{\mathrm{Im}\mathcal{F}''_*}{N^2} + \frac{\mathrm{Im}\Lambda}{N^2}\right) \equiv \tilde{A}(c,\Lambda,\varphi_*,\bar{\varphi}_*).$$

- r, Δ , complex in general, put sub. 0, as $F,\ \bar{F} \to 0$
$$V_0 = V(D,\varphi,\bar{\varphi},F=0,\bar{F}=0)$$

- If Δ_0 real,
$$\frac{V_0}{N^2|m_s|^4} = \left(\left(c' + \frac{1}{64\pi^2}\right) - \delta\right)\Delta_0^2 + \frac{1}{32\pi^2}\left[\frac{\tilde{A}}{8}\Delta_0^4 - \lambda_0^{(+)^4}\log\lambda_0^{(+)^2} - \lambda_0^{(-)^4}\log\lambda_0^{(-)^2}\right]$$

- gap equation
$$\left.\frac{\partial V_0}{\partial D}\right|_{\varphi,\bar{\varphi}} = 0.$$
$$0 = \Delta_0\left[2\left(\left(c' + \frac{1}{64\pi^2}\right) - \delta\right) + \frac{1}{32\pi^2}\left\{\frac{\tilde{A}}{2}\Delta_0^2 - \frac{1}{\sqrt{1+\Delta_0^2}}\left(\lambda_0^{(+)3}\left(2\log\lambda_0^{(+)2}+1\right) - \lambda_0^{(-)3}\left(2\log\lambda_0^{(-)2}+1\right)\right)\right\}\right],$$
$$\delta_* = 0$$

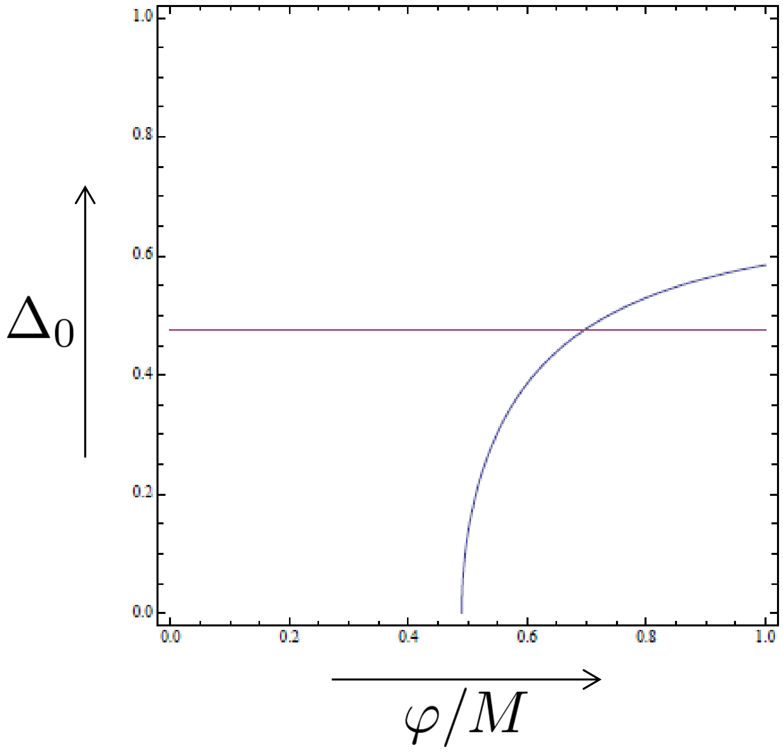
H. I. & N. M. IJMPA (2012)

- stationary cond.
$$\left.\frac{\partial V_0}{\partial \varphi}\right|_{D,\bar{\varphi}} = 0$$
$$2\partial(\ln|m_s|^2)\frac{V_0}{N^2|m_s|^4} = \left(\frac{\partial\delta}{\partial\varphi}\right)\Delta_0^2 - \frac{\partial\Delta_0}{\partial\varphi}\frac{\partial}{\partial\Delta_0}\left(\frac{V_0}{N^2|m_s|^4}\right)$$

  Using the gap eq.
$$\frac{V_0}{N^2|m_s|^4} = \frac{\frac{\partial\delta}{\partial\varphi}}{2\partial(\ln|m_s|^2)}\Delta_0^2$$

$(\Delta_{0*}, \varphi_* = \bar{\varphi}_*)$ determined as the point of intersection of the two real curves in the $(\Delta_0, \varphi = \bar{\varphi})$ plane

Schematically,



DSB HAS BEEN REALIZED

▪numerical study

- the minimal choice for DDSB:

$$\mathcal{F} = \frac{c}{2N}\mathrm{tr}\varphi^2 + \frac{1}{3!MN}\mathrm{tr}\varphi^3 \equiv \frac{1}{2}c\varphi^2 + \frac{1}{3!M}\varphi^3,$$

$$W = \frac{m^2}{N}\mathrm{tr}\varphi + \frac{d}{3!N}\mathrm{tr}\varphi^3 \equiv m^2\varphi + \frac{d}{3!}\varphi^3,$$

- consistency check: $\left|\frac{F_*}{D_*}\right| \ll 1, \; |f_{3*}| \ll 1$

- samples:

$c' + \frac{1}{64\pi^2}$	$\tilde{A}/(4\cdot 32\pi^2)$	Δ_{0*}	φ_*/M $(-\frac{N^2}{\mathrm{Im}(i+\Lambda)})$	$\lvert F_*/D_*\rvert$	$\lvert f_{3*}\rvert$
0.002	0.0001	0.477	0.707 (10000)	2.621 ($m = M$)	1.77
0.002	0.0001	0.477	0.707 (10000)	0.524 ($m \ll M$)	0.35
0.002	0.0001	0.477	0.707 (10000)	0.860 ($m = 0.4M$)	0.58
0.003	0.001	1.3623	0.8639 (2000)	0.825 ($m = M$)	>1
0.003	0.001	1.3623	0.8639 (2000)	0.224 ($m \ll M$)	0.43
0.003	0.001	1.3623	0.5464 (5000)	1.092 ($m = M$)	>1
0.003	0.001	1.3623	0.5464 (5000)	0.142 ($m \ll M$)	0.27
0.003	0.001	1.3623	0.5464 (5000)	0.911 ($m = 0.9M$)	1.76
0.003	0.001	1.3623	0.3863 (10000)	1.444 ($m = M$)	>1
0.003	0.001	1.3623	0.3863 (10000)	0.100 ($m \ll M$)	0.19
0.003	0.001	1.3623	0.3863 (10000)	0.960 ($m = 0.8M$)	1.85

▪ second variation and mass of scalar gluons

$$V_{\rm scalar} = V(D = D_*(\varphi, \bar{\varphi}), F = F_*(\varphi, \bar{\varphi}) \approx 0, \bar{F} = \bar{F}_*(\varphi, \bar{\varphi}) \approx 0, \varphi, \bar{\varphi})$$

at the stationary point $(D_*(\varphi_*, \bar{\varphi}_*), 0, 0, \varphi_*, \bar{\varphi}_*)$.

⋮

- scalar gluon mass: $\frac{1}{g}|W'' - (\partial\partial_F V_{1-{\rm loop}})|_*^2$

in the region $|(\partial_F \partial_{\bar{F}} V)_0|_*, |(\partial_F^2 V)_0|_*, \ll g_*,$

consistency checked

- samples:

$c' + \frac{1}{64\pi^2}$	$\tilde{A}/(4 \cdot 32\pi^2)$	Δ_{0*}	φ_*/M $(-\frac{N^2}{{\rm Im}(i+\Lambda)})$	scalar gluon mass
0.002	0.0001	0.477	0.707 (10000)	$0.4998 + 0.0056\, N^2 + 8.607 \times 10^{-7} N^4$
0.003	0.001	1.3623	0.8639 (2000)	$0.7463 + 0.0106\, N^2 + 2.653 \times 10^{-4} N^4$
0.003	0.001	1.3623	0.5464 (5000)	$0.2986 + 0.0008\, N^2 + 4.694 \times 10^{-5} N^4$
0.003	0.001	1.3623	0.3863 (10000)	$0.1492 - 0.0024\, N^2 + 7.235 \times 10^{-5} N^4$

D-term Triggered Dynamical Supersymmetry Breaking

H. I. & N. Maru (Osaka City University)

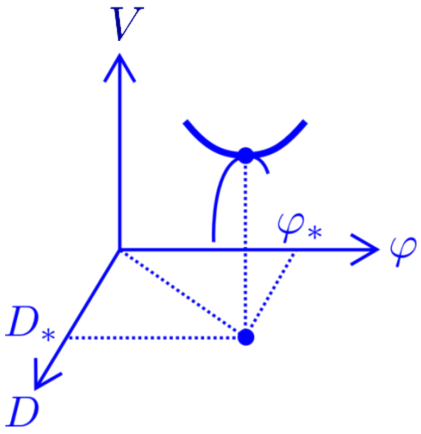
based on
- IJMPA 27 (2012) 1250159, arXiv:1109.2276
- PRD 88 (2013) 025012, arXiv:1301.7548

I)
- most desirable to break $\mathcal{N}=1$ SUSY dynamically (DSB)
- In the past, instanton generated superpotential e.t.c. $\langle F \rangle_{nonpt} \to \langle D \rangle \neq 0$

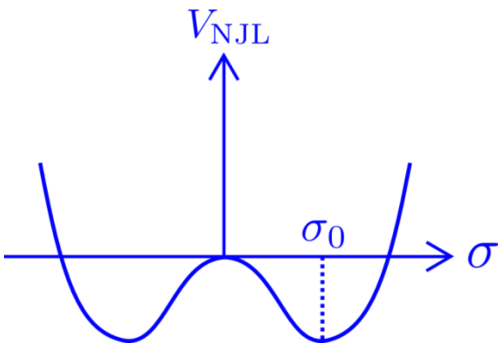
In this talk, we will accomplish

D-term DSB (DDSB)
in a self-consistent
Hartree-Fock approximation

- based on the nonrenormalizable D-gaugino-matter fermion which is present in generic $\mathcal{N}=1$ SUSY (effective) U(N) gauge action
- the vac. is metastable, can be made long lived
- requires the discovery of scalar gluons in nature, so that distinct from the previous proposals



compare this with
the NJL ptl.



Contents

I) Introduction and punch lines

II) action, assumptions and some properties

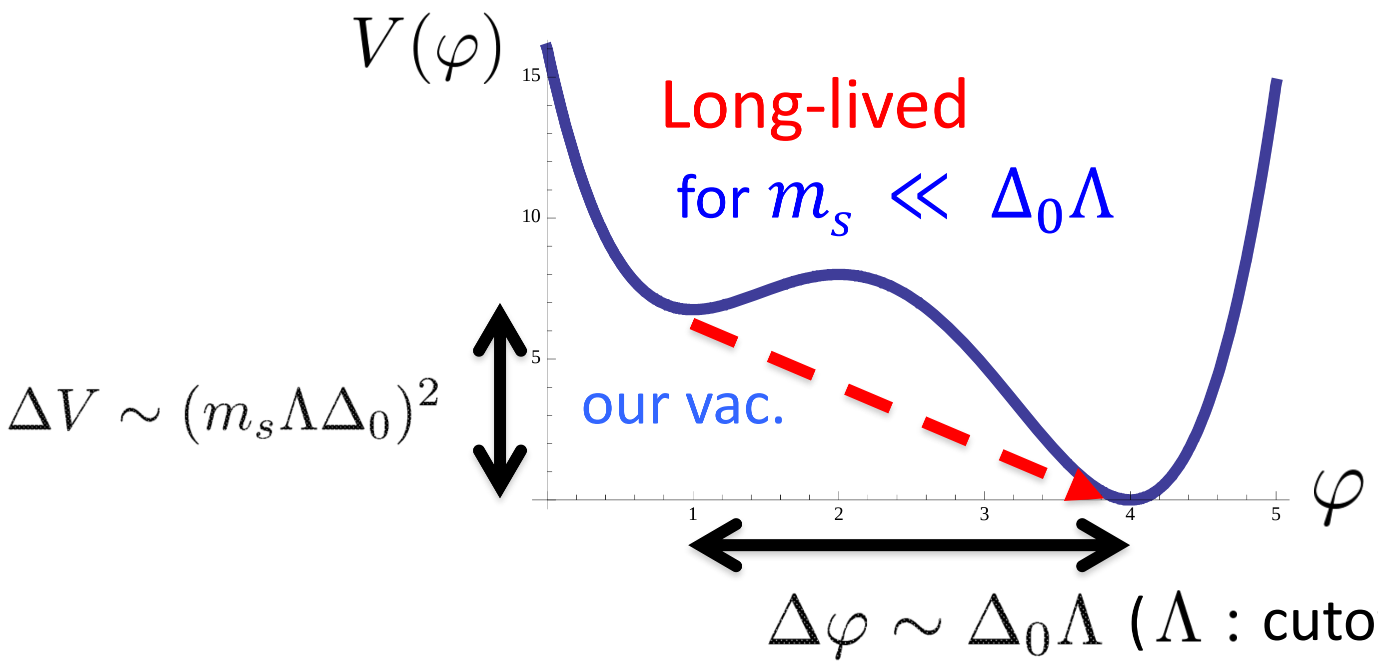
III) effective potential in the Hartree-Fock approximation

IV) stationary conditions and gap equation

・Metastability of our false vacuum

$\langle D \rangle = 0$ tree vacuum is not lifted

$\Rightarrow$ check if our vacuum $\langle D \rangle \neq 0$ is sufficiently long-lived



Coleman & De Luccia (1980)

Decay rate of our vacuum $\propto \exp\left[-\frac{\langle \Delta\phi \rangle^4}{\langle \Delta V \rangle}\right] = \exp\left[-\frac{(\Delta_0 \Lambda)^2}{m_s^2}\right] \ll 1 \qquad \Delta_0 \Lambda \gg m_s$