Solving the Strong CP Problem with Discrete Symmetries and the Right Unitarity Triangle

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SISSA
International School for Advanced Studies

Outline

- Motivation
- Strategy
- The Model
- Relation to Nelson Barr Models
- Summary and Conclusions

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The Strong CP Problem

- QCD violates CP via

$$\mathcal{L} \supset \frac{\bar{\theta}}{32\pi^2} \tilde{G}_{\mu\nu} G^{\mu\nu}$$

- This term has two contributions

$$\bar{\theta} = \theta + \arg\det(M_u M_d)$$

- Experiment tells us

$$\bar{\theta} \lesssim 10^{-11}$$

[PDG '13]

- Why is the cancellation so perfect?

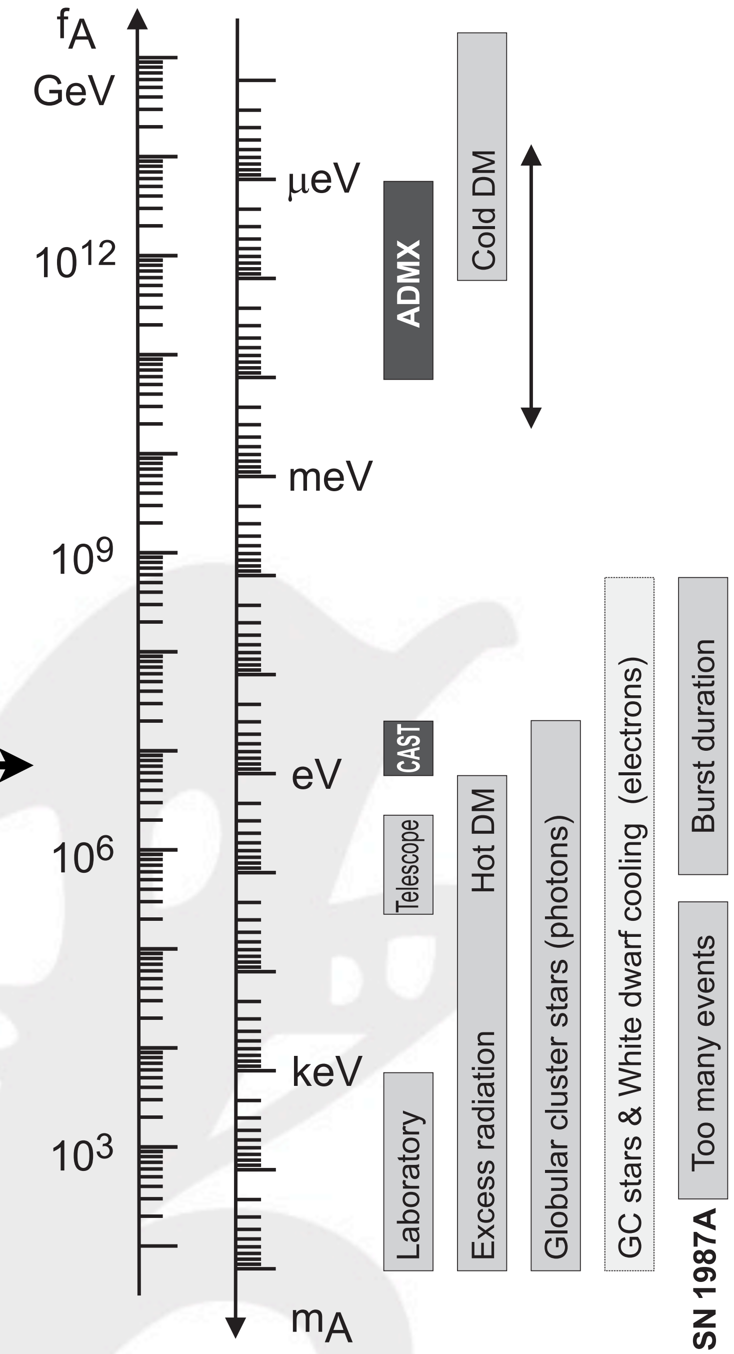
3 Popular Solutions

1. Massless up-quark, but:

$$m_u(2\ \mathrm{GeV}) = 2.3^{+0.7}_{-0.5}\ \mathrm{GeV}$$

[PDG '13]

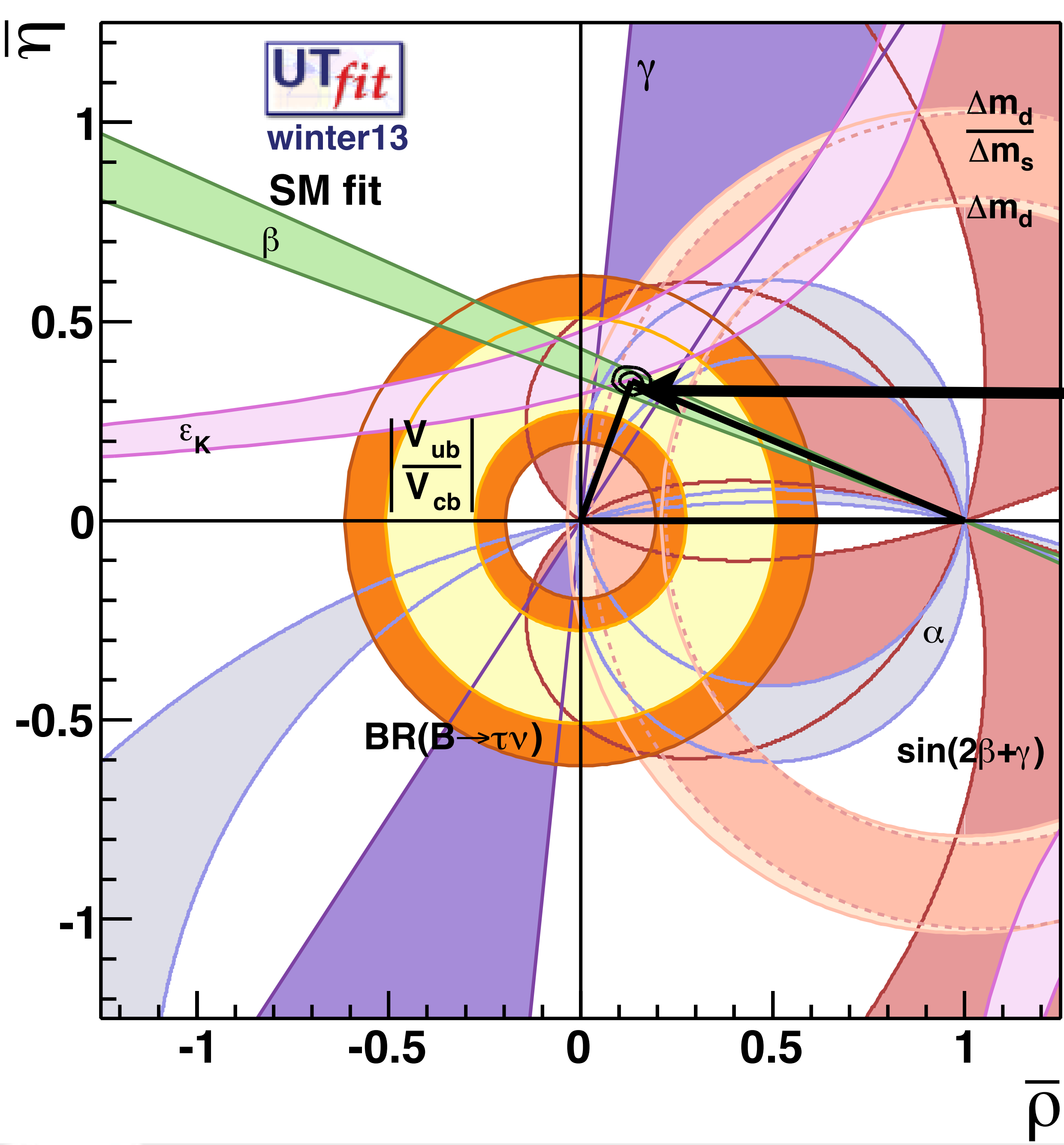
2. Axion solution →



[Plot taken from the PDG Axion Review '13]

3. Spontaneous CP violation:
   Topic of this talk

The CKM Unitarity Triangle



$$(V_{\mathrm{CKM}}^\dagger V_{\mathrm{CKM}})_{bd} = 0$$

Fit result:

$$\alpha = (88.7 \pm 3.1)^\circ$$

[UTFit Winter '13 SM Fit]

Accident or sign of spontaneous CP violation?

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How to suppress the strong CP phase?

[Antusch, Holthausen, Schmidt, MS '13]

- Step 1: Promote CP to be fundamental

$$\bar{\theta} = 0$$

- Step 2: Break CP such that arg det (M_u M_d) = 0
  - Step 2a: Make M_u real with vanishing 1-3 element
  - Step 2b: For M_d use (* real elements)

$$M_d = \begin{pmatrix} 0 & * & 0 \\ * & \mathrm{i} * & * \\ 0 & 0 & * \end{pmatrix}$$

The Phase Sum Rule

[Antusch, King, Malinsky, MS '10]

- We parameterise the mixing matrices as, e.g.

$$U_d^\dagger = U_{23}^d U_{13}^d U_{12}^d \text{ with } U_{12}^d = \begin{pmatrix} c_{12}^d & s_{12}^d \mathrm{e}^{-\mathrm{i}\,\delta_{12}^d} & 0 \\ -s_{12}^d \mathrm{e}^{\mathrm{i}\,\delta_{12}^d} & c_{12}^d & 0 \\ 0 & 0 & 1 \end{pmatrix} (U_d M_d M_d^\dagger U_d^\dagger = \text{diag.})$$

- For mass matrices with vanishing 1-3 element:

$$\alpha \approx \delta_{12}^d - \delta_{12}^u \overset{!}{\approx} 90^\circ$$

[Antusch, King, Malinsky, MS '09] see also [Fritzsch and Xing; Masina and Savoy '06; Harrison, Dallison, Roythorne, Scott '09]

- Simple solution (ignoring signs), e.g.

$$M_d = \begin{pmatrix} 0 & * & 0 \\ * & \mathrm{i}\,* & * \\ 0 & 0 & * \end{pmatrix} ,\ M_u \text{ real } \Rightarrow\ \delta_{12}^d = 90^\circ ,\ \delta_{12}^u = 0$$

Discrete Vacuum Alignment

[Antusch, King, Luhn, MS '11]

- Yukawas proportional to flavon vevs, e.g.

$$\langle\phi\rangle \propto (0,0,x)^T \quad \text{or} \quad \langle\phi\rangle \propto (x,x,x)^T$$

- Add term to W compatible with Z_n symmetry

$$\mathcal{W} \supset P\left(\kappa\frac{\phi^n}{\Lambda^{n-2}} \mp \lambda M^2\right)$$

- Solve F-term conditions ($|F_P|$=0) + CP symmetry*

$$\arg(\langle\phi\rangle) = \arg(x) = \begin{cases} \frac{2\pi}{n}q\ , \quad q = 1,\ldots,n & \text{for “}-\text{”} \\ \frac{2\pi}{n}q + \frac{\pi}{n}\ , \quad q = 1,\ldots,n & \text{for “}+\text{”} \end{cases}$$

* We assume here that CP enforces κ and λ to be real.

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Overview

[Antusch, Holthausen, Schmidt, MS '13]

- Symmetry of the Model

$$\underbrace{SU(3)_C \times SU(2)_L \times U(1)_Y}_{\text{gauge sym.}} \times \underbrace{A_4 \times Z_2 \times Z_4^5}_{\text{discrete family/shaping sym.}} \times \underbrace{U(1)_R}_{R \text{ sym.}}$$

- Flavour model for quarks only (d_R is A_4 triplet)!
- 5 singlet flavons ξ with real vevs, 4 triplets:

$$\langle\phi_1\rangle \sim \begin{pmatrix}1\\0\\0\end{pmatrix} \;,\quad \langle\phi_2\rangle \sim \begin{pmatrix}0\\1\\0\end{pmatrix} \;,\quad \langle\phi_3\rangle \sim \begin{pmatrix}0\\0\\1\end{pmatrix} \;,\quad \langle\tilde{\phi}_2\rangle \sim \mathrm{i}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

Coupling to Matter

[Antusch, Holthausen, Schmidt, MS '13]

- The effective superpotential reads

$$\mathcal{W}_d = Q_1 \bar{d} H_d \frac{\phi_2 \xi_d}{\Lambda^2} + Q_2 \bar{d} H_d \frac{\phi_1 \xi_d + \tilde{\phi}_2 \xi_s + \phi_3 \xi_t}{\Lambda^2} + Q_3 \bar{d} H_d \frac{\phi_3}{\Lambda}$$

$$\mathcal{W}_u = Q_1 \bar{u}_1 H_u \frac{\xi_u^2}{\Lambda^2} + Q_1 \bar{u}_2 H_u \frac{\xi_u \xi_c}{\Lambda^2} + Q_2 \bar{u}_2 H_u \left(\frac{\xi_c}{\Lambda} + \frac{\xi_t^2}{\Lambda^2} \right)$$

$$+ (Q_2 \bar{u}_3 + Q_3 \bar{u}_2) H_u \frac{\xi_t}{\Lambda} + Q_3 \bar{u}_3 H_u$$

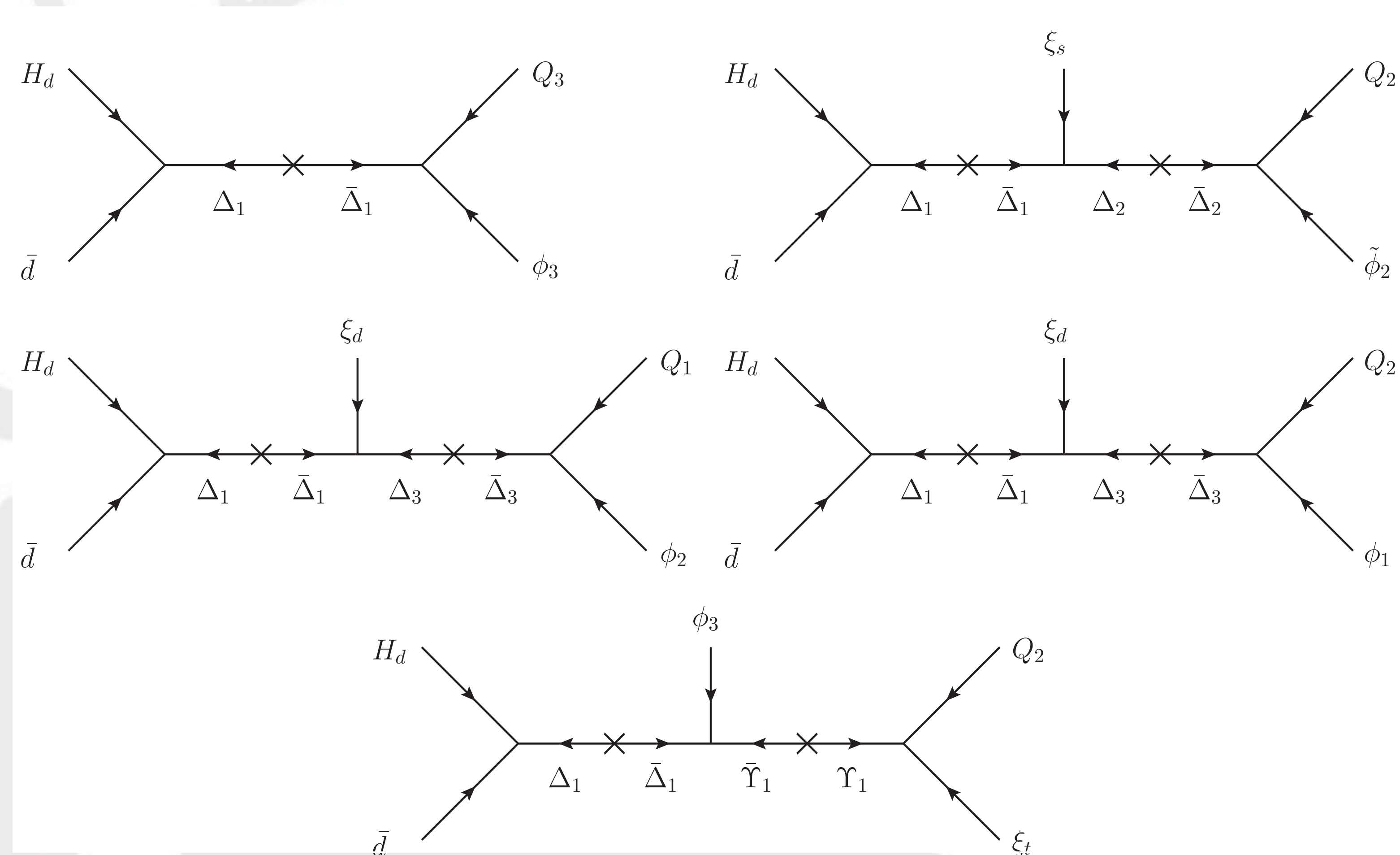
- Giving the mass matrix structure

$$M_d = \begin{pmatrix} 0 & b_d & 0 \\ b_d' & \mathrm{i} c_d & d_d \\ 0 & 0 & e_d \end{pmatrix} \text{ and } M_u = \begin{pmatrix} a_u & b_u & 0 \\ 0 & c_u & d_u \\ 0 & d_u' & e_u \end{pmatrix}$$

Higher Dimensional Operators I

[Antusch, Holthausen, Schmidt, MS '13]

We give a "UV completion" of the model giving full control over the effective operators!



Higher Dimensional Operators II

[Antusch, Holthausen, Schmidt, MS '13]

1. Corrections to the flavon sector:
   At least dimension seven, do not change the flavon directions and phases
2. Corrections to the up-quark sector:
   Suppressed (real) corrections to the 1-1, 1-2, 2-2 elements of M_{u}
3. Corrections to the down-quark sector:
   No corrections from higher-dim. operators!

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Nelson-Barr Models

[Nelson '84; Barr '84]

Nelson-Barr models defined by two conditions

1. At the tree level there are no Yukawa or mass terms coupling *F* fermions to C^* fermions, or *C* fermions to *C* fermions.

2. The CP-violating phases appear at the tree level only in those Yukawa terms that couple *F* fermions to $R = (C + C^*)$ fermions.

In terms of a mass matrix including heavy vector-like fields:

$$M_D \sim \begin{pmatrix} Y v_d & 0 \\ \langle \phi \rangle & M_\Upsilon \end{pmatrix}$$

Yv_d and M_Υ real, $\langle\phi\rangle$ complex

Relation to our Model Class

[Antusch, Holthausen, Schmidt, MS '13]

In our model we have:

$$M_D \sim \begin{pmatrix} 0 & 0 & 0 & \langle \phi_2^T \rangle & 0 & 0 \\ 0 & 0 & \langle \tilde{\phi}_2^T \rangle & \langle \phi_1^T \rangle & \langle \xi_t \rangle & \langle \xi_c \rangle \\ 0 & \langle \phi_3^T \rangle & 0 & 0 & 0 & \langle \xi_t \rangle \\ v_d & M_{\Delta_1} & 0 & 0 & 0 & 0 \\ 0 & \langle \xi_s \rangle & M_{\Delta_2} & 0 & 0 & 0 \\ 0 & \langle \xi_d \rangle & 0 & M_{\Delta_3} & 0 & 0 \\ 0 & \langle \phi_3^T \rangle & 0 & 0 & M_{\Upsilon_1} & \langle \xi_t \rangle \\ 0 & 0 & 0 & 0 & 0 & M_{\Upsilon_2} \end{pmatrix}$$

with a real determinant as well:

$$\det M \sim v_d^3 M_{\Delta_2}^3 M_{\Delta_3}^3 M_{\Upsilon_1} M_{\Upsilon_2} \langle \xi_d^2 \rangle \langle \phi_1 \rangle \langle \phi_2 \rangle \langle \phi_3 \rangle$$

In Words

Nelson-Barr	Our Class of Models
Real Yukawa Couplings	Effective Yukawa Couplings (apart from top)
Real "Messenger" Masses	Real Messenger Masses
Complex Couplings between light & heavy Fields	Real couplings but complex Flavon vevs
Structure of Couplings	Alignment of Flavon vevs

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Summary and Conclusions

- Solution to Strong CP problem with discrete Symmetries but without Axions
- Ingredients:
  - Fundamental CP symmetry
  - Phase sum rule (right CKM unitarity triangle)
  - Discrete symmetries to fix flavour directions and phases
  - Messenger sector to control higher-dimensional operators

Thank you for your attention!

Flavon Alignment

[Antusch, Holthausen, Schmidt, MS '13]

Study only one example for simplicity:

$$\mathcal{W} = \tilde{A}_2 \cdot (\tilde{\phi}_2 \star \tilde{\phi}_2) + \tilde{O}_{1;2}(\phi_1 \cdot \tilde{\phi}_2) + \tilde{O}_{2;3}(\tilde{\phi}_2 \cdot \phi_3) + \frac{P}{\Lambda^2}\left(\tilde{\phi}_2^4 \pm M_F^4\right)$$

1. Fix the possible vev direction in flavour space

$$F_{\tilde{A}_i} = 2(\tilde{\phi}_2)_j(\tilde{\phi}_2)_k = 0, \text{ where } i \neq j \neq k \neq i \Rightarrow \langle \tilde{\phi}_2 \rangle \sim (0,1,0)$$

2. Fix the direction in relation to other vevs

$$F_{\tilde{O}_{1;2}} = 0 \Rightarrow \langle \phi_1 \rangle \perp \langle \tilde{\phi}_2 \rangle \text{ and } F_{\tilde{O}_{2;3}} = 0 \Rightarrow \langle \phi_3 \rangle \perp \langle \tilde{\phi}_2 \rangle$$

3. Fix the phase

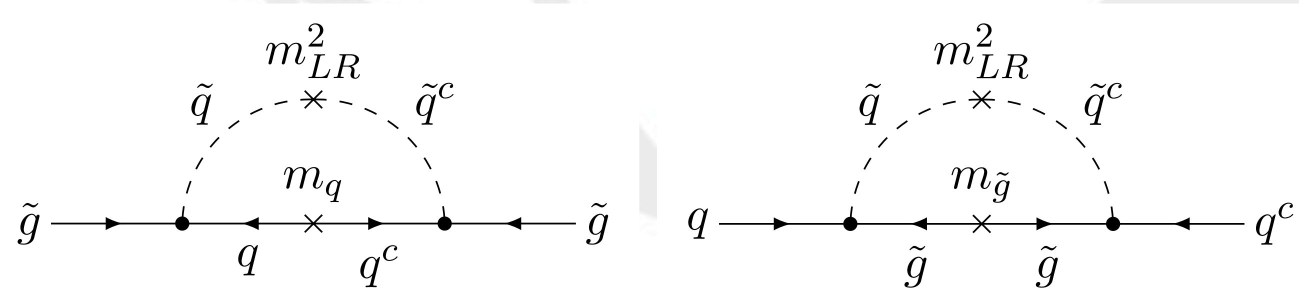
$$F_P = 0 \Rightarrow \arg\langle \tilde{\phi}_2 \rangle = 0, \frac{\pi}{2}, \pi, \frac{\pi}{2}$$

Backup: Corrections from SUSY breaking

- SUSY breaking might give corrections, e.g.

$$\delta\bar{\theta} = 3 \arg(m_{\tilde{g}})$$

- Also LR sfermion mixing gives corrections



- If SUSY breaking conserves CP and is close to MFV, corrections are potentially small enough.