Grand Unification Thresholds and the NMSGUT

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Plan

- SO(10) MSGUT Virtues and Development Review
- GUT Threshold effects on Fermion Yukawas : Fixing $d = 5$ rates
- RG Features of Distinctive Spectra
- New Horizons :Grand Yukawonifcation

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  (with Charanjit K. Khosa) :

- $\{Q_L, L_L, u^c_L, d^c_L, l^c_L \oplus \nu^{\mathbf{c}}_{\mathbf{L}}\} \equiv 16$: Tight and complete Q-L unification

- Simple Tri-band FM Higgs Channel Spectrum:$\Rightarrow(\mathbf{10} + \mathbf{120} + \overline{\mathbf{126}})$

$$\overline{126} \quad = \quad (15,2,2) + \Delta_R(10,1,3) + \Delta_L(\overline{10},3,1) + (6,1,1)$$

NATURAL HOME TO BOTH SEESAWS :

- Type I : $M_{B-L} \sim < \vec{\Delta}_R >_{SM=0} \Rightarrow M_{\nu^c} \Rightarrow M^I_\nu \sim \frac{y_\nu^2 v^2}{M_{B-L}}$

- Type II : $\frac{v_W^2}{M_{B-L}} \sim < \vec{\Delta}_L >_{Y=2,T_{3L}=-1} \Rightarrow \Rightarrow$
  $M^{II}_\nu \sim f < \vec{\Delta}_L >$

- $\vec{\Delta}_R(1,3,-2), \vec{\Delta}_L(3,1,2) \subset \overline{126}$ PRESERVE R_p :
  $M_p = (-)^{3(B-L)} \subset U(1)_{B-L} \subset SO(10) \quad \bigoplus < \Delta_{L,R} > \Rightarrow R_p, \quad \Rightarrow$
  Stable LSP , Ideal CDM !

- "New" Minimal Susy GUT (NMSGUT):
  Oldest complete Susy SO(10) GUT!

  (1982)CSA, Mohapatra, Clark Kuo, Nakagawa

- CTF/USP : Minimal parameters, fully realistic fermion spectra ⇒

- Distinctive MSSM spectra (2008) :
  - Mini Split
  - Large A-terms
  - Light smuons
  - Normal sHierarchy
    :Standout Falsifiable !

Futility vs Precision

- $\sim 10^3$ fields BUT controllable Threshold effects at GUT scales :

$$M_X \sim 10^{16.75} - 10^{18} \quad GeV$$

- Large SO(10) gauge beta functions due to $\mathbf{210}, \mathbf{126}, \overline{\mathbf{126}}, \mathbf{120} \;\Rightarrow$ Landau pole just above $M_X \Rightarrow$ Physical cutoff Λ_X : Gauge Gravity unification ?

- Induced Gravity ? :

$$M_{Planck} \sim \Lambda_X$$

Higgs Structure

- **AM Higgs** : $< \mathbf{210}(\Phi_{ijkl}), \ \overline{\mathbf{126}}(\overline{\mathbf{\Sigma}}_{ijklm}) \ , \mathbf{126} > \Rightarrow$

$$Susy \ SO(10) \longrightarrow MSSM$$

- **Superpotential**

$$\begin{aligned}
W &= W_{MSGUT} + W_{120} + W_{16} \\
W_{MSGUT} &= m \ 210^2 + \lambda \ 210^3 + M \ 126 \cdot \overline{126} + \eta \ 210 \cdot 126 \cdot \overline{126} \\
&+ 10 \cdot 210(\gamma \ 126 + \bar{\gamma} \ \overline{126}) + M_H \ 10^2 \\
W_{120} &= M_O \ 120 \cdot 120 + k \ 10 \cdot 120 \cdot 210 + \rho \ 120 \cdot 120 \cdot 210 \\
&+\zeta \ 120 \cdot 126 \cdot 210 + \bar{\zeta} \ 120 \cdot \overline{126} \cdot 210 \\
W_{16} &= h_{AB} \ 16_A \cdot 16_B + f'_{AB} \ 16_A 16_B + g_{[AB]} 16_A \cdot 16_B \cdot 120
\end{aligned}$$

- MSGUT Parameters (one fine tuned) : (**23**)

NMSGUT 15 more $\Rightarrow$ Total $= 38$ STILL MINIMAL !

SSB

- Calculable SSB at M_X : **GUT scale VEVS** : $SO(10) \to MSSM$

$$\begin{array}{lcl} \langle(15,1,1)\rangle_{210} & : & a \qquad \langle(15,1,3)\rangle_{210} : \omega \\ \langle(1,1,1)\rangle_{210} & : & p \qquad \langle(10,1,3)\rangle_{126,\overline{126}} : \sigma, \bar{\sigma} \end{array}$$

- Homogenous Degree 1 F-terms $\Sigma, \overline{\Sigma}$ equations $\Rightarrow$ extra $(D_{B-L} = 0)$ term condition : $|\sigma| = |\overline{\sigma}|$
- F Terms : SSB completely analyzable 4 eqns $\Rightarrow$ **Cubic in** $x = -\lambda\omega/m$

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1-x)^2$$

$$\xi = \frac{\lambda M}{\eta m}$$

Fields and Types

- MSGUT : 45+48+ 10+252+210=565 (super)Fields
- NMSGUT: 565+120= 685 fields
- 26 MSSM-irrep types
- GUT scale spectra and Threshold effects calculable

MSSM Higgs

- Multiple sources of Higgs Doublets only one pair light.

$$[1,2,-1](\bar{h}_1,\bar{h}_2,\bar{h}_3,\bar{h}_4)\oplus[1,2,1](h_1,h_2,h_3,h_4)$$
$$(H^{\alpha}_{\dot{2}},\overline{\Sigma}^{(15)\alpha}_{\dot{2}},\Sigma^{(15)\alpha}_{\dot{2}},\frac{\phi^{\dot{2}\alpha}_{44}}{\sqrt{2}})\oplus(H_{\alpha\dot{1}},\overline{\Sigma}^{(15)}_{\alpha\dot{1}},\Sigma^{(15)}_{\alpha\dot{1}},\frac{\phi^{44\dot{1}}_{\alpha}}{\sqrt{2}})$$

- Doublet Mass Matrix in MSGUT :

$$\mathcal{H}=\begin{pmatrix} -M_H & +\overline{\gamma}\sqrt{3}(\omega-a) & -\gamma\sqrt{3}(\omega+a) & -\bar{\gamma}\bar{\sigma} \\ -\overline{\gamma}\sqrt{3}(\omega+a) & 0 & -(2M+4\eta(a+\omega)) & 0 \\ \gamma\sqrt{3}(\omega-a) & -(2M+4\eta(a-\omega)) & 0 & -2\eta\overline{\sigma}\sqrt{3} \\ -\sigma\gamma & -2\eta\sigma\sqrt{3} & 0 & -2m+6\lambda(\omega-a) \end{pmatrix}$$

- Fine Tuning : $Det\mathcal{H}=0$
- Left and Right Null eigenvectors determine Higgs Fractions

Fermion data fits

- SO(10) $t-b-\tau$ unification as striking as gauge unification !
- Supports third generation as real core of fermion hierarchy.
- Convincing only if single 10-plet dominates Requires

$$\tan\beta \sim 45-60 \sim m_t/m_b$$

- MSGUT fails to produce adequate neutrino masses :
  - *Type I* $>>$ *Type II*
  - $m_\nu >>$ *Type I*
  - Because M_ν^c ($\overline{126}$ yukawa !) too large.
- NMSGUT Higgs Role reassignment :
  - $h \oplus g >> f \quad \Rightarrow$ Charged fermion data :$(m_{q,l}, \theta_q^i, \delta_c)$.
  - $f << h, g \Rightarrow$ Type I boosted

TENSIONS and Calmatives

- $10 \oplus 120$ only for Charged fermion fit $\Rightarrow$ TENSIONS :
  - $m_{d,s}^{MSSM}(M_Z) \sim m_{d,s}^{SM}(M_Z)/5$
  - Tree Level: $m_s - m_\mu = m_b - m_\tau$
- $\tan\beta$ driven threshold corrections to lower $y_{d,s}^{MSSM}$ while preserving or raising y_b Necessary for fermion fit !
- Lowering via gluino corrections. Preservation via $\ominus A_t vs \oplus \mu$ cancellation.
- Implies
  - $A_0 < 0, \mu \sim 10^2\, TeV$
  - $m_0 \sim 10\, TeV >> M_i \sim TeV$
  - Mini-Split necessary for fermion Fit !

Fits Achieved

- Normal SHierarchy :
  $m_{\tilde{q}_3,\tilde{l}_3} >> m_{\tilde{q}_{1,2},\tilde{l}_{1,2}}$
- Light smuons ($m_{\tilde{\mu}} \sim m_{\chi^0}$)$\Rightarrow$: Favorable
- DM Co-annihilation $\surd$
- $a_\mu \surd$.
- $B \to \mu\mu, b \to s\gamma, \Delta\rho, \epsilon_{Lept}$ all in right ballpark.
- Excellent Fits achieved (2008) : compatible with

$$M_H \sim 126 GeV <<< m_A \sim \mu \sim 10^2 TeV$$

Transitory Glee

- Short lived (satisfaction) : Generic Catastrophic proton decay

$$\tau_p^{d=5} \sim 10^{28} \quad years$$

Fit	$\tau_p(M^+\nu)$	$\Gamma(p \to \pi^+\nu)$	$BR(p \to \pi^+\nu_{e,\mu,\tau})$
$Ex.1$	8.1×10^{28}	3.1×10^{-30}	$\{2.6 \times 10^{-5}, 0.09, 0.91\}$
$Ex.2$	1.7×10^{28}	7.2×10^{-30}	$\{3.04 \times 10^{-5}, 0.014, 0.986\}$
		$\Gamma(p \to K^+\nu)$	$BR(p \to K^+\nu_{e,\mu,\tau})$
$Ex.1$		9.2×10^{-30}	$\{1.1 \times 10^{-4}, 0.27, 0.73\}$
$Ex.2$		5.2×10^{-29}	$\{5.45 \times 10^{-5}, 0.01, 0.99\}$

Table of $d = 5$ operator mediated proton lifetimes τ_p(yrs), decay rates $\Gamma(yr^{-1})$ and branching ratios in the dominant Meson$^+ + \nu$ channels.

GUT Threshold Effects on $Y_f^{MSSM} - Y_{SO(10)}$ matching

- Light matter and specially MSSM Higgs in Yukawa vertex suffer major finite wave function renormalization due to manifold circulating heavy fields

$$\begin{aligned}
\Delta Z_j^i &= \frac{g^2}{8\pi^2}\sum_{A,k} Q_k^{Ai} Q_j^{Ak} F(m_A, m_k) \\
&- \frac{1}{32\pi^2}\sum y_{ikl}^* y_{jkl} F(m_k, m_l) \\
F(m_A, m_B) &= \frac{1}{M_A^2 - M_B^2}(M_A^2 Ln\frac{m_A^2}{Q^2} - M_B^2 Ln\frac{m_B^2}{Q^2}) - 1 \\
F(m_A) &= Ln\frac{m_A^2}{Q^2} - 1 \\
Q &= M_X^0
\end{aligned}$$

Redefinition to Canonical Kinetic terms

- Diagonalize among multiple copies to get canonical kinetic terms

$$Y'_f = \frac{\tilde{U}^T_{f^c} Y_f \tilde{U}_f}{\sqrt{Z_{H^f}}}$$
$$\tilde{U}_\phi = U_\phi \Lambda_\phi^{-\frac{1}{2}}$$

- 26 MSSM multiplet types (and conjugates). PS decomposition of SO(10) gauge and Superpotential vertices allows calculation of MSSM decomposition.

Redefinition

- Multiple Higgs doublets in GUT descend to single Light pair.
- Each couples to 26 pairwise combinations with MSSM conjugate quantum numbers.
- E.g two fold $K[3,1,-\frac{8}{3}]$ and 3-fold $\bar{X}[\bar{3},2,\frac{5}{3}] \sim [1,2,-1]$ couple to $H[1,2,1]$ and circulate in loop with multiplicity 3 ($Z_H = 1+3\Delta_{K\bar{X}}+..$)
- 1300 vertices : evaluate with correct relative signs to get $Z_{f,f^c,H}$

Home Safe

- Matter line corrections are generally small(max 30%).
- (Specially gauge) corrections on Higgs lines add up.
- Search for fermion fits at M_X with threshold corrected Yukawas and penalties on size of $d = 5$ B violation operator

$$C_{d=5} \sim \frac{Y^2_{SO(10)}}{M_X}$$

- Parameter sets where $Z_H \to 0$ are selected for !
- SO(10) yukawas required to match MSSM are much smaller
- Same yukawas enter the LLLL and RRRR effective $d = 5$ B violation operators without Higgs external line.
- Lifetimes $\tau_p^{d=5} > 10^{34}$ years easily achieved !

$\chi_X = 0.4234$ $h_{11} 3.3612 = \times 10^{-6}$ $h_{33} = 0.0241$ $g_{13} = (-9.26 + 6.87i)10^{-5}$ $\Delta_X = 0.74$ $\Delta\alpha_3(M_Z) = .0024$ $M^{\nu^c}(GeV) = \{7.79E8$	$\chi_Z = .0754$ $h_{22} = 3.0910^{-4}$ $g_{12} = (0.13 + 0.13i)10^{-4}$ $g_{23} = -(3.2 + 1.4i)10^{-4}$ $\Delta_G = -23.5$ $, 1.02E12, 3.79E13\}$
$\tan\beta = 51.0$ $\mathrm{m}_{\frac{1}{2}} = -180.6$ $\mu = 1.09 \times 10^5$ $\mathrm{A}_0 = -1.36 \times 10^5$ $\mathrm{M}^2_{\bar{\mathrm{H}}} = -8.61 \times 10^9$ $Max(\|L^{d=5}\|, \|R^{d=5}\|)$	$\mathrm{R}_{\frac{b\tau}{s\mu}} = 0.0485$ $\mathrm{m}_0 = 6463.0$ $\mathrm{B} = -8.59 \times 10^9$ $\mathrm{M}^2_{\mathrm{H}} = -7.72 \times 10^9$ $8.28 \times 10^{-22}\,\mathrm{GeV}^{-1}$

Table : Fit : Values of the NMSGUT-SUGRY-NUHM parameters at M_X from an accurate fit to 18 fermion data.

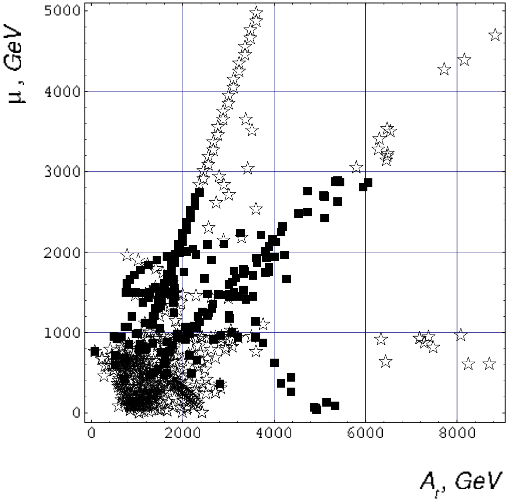
Soft	Susy	parameters	at $\mathbf{M_Z}$
M_1	120.22	$M_{\tilde{\bar{u}}_{1,2}}$	8190.37
M_2	346.13	$M_{\tilde{\bar{u}}_3}$	15617.44
M_3	500.05	$A^{0(l)}_{11,22}$	-80607.35
$M_{\tilde{\bar{l}}_1}$	1314.07	$A^{0(l)}_{33}$	-48613.40
$M_{\tilde{\bar{l}}_2}$	200.77	$A^{0(u)}_{11,22}$	-99127.34
$M_{\tilde{\bar{l}}_3}$	12750.91	$A^{0(u)}_{33}$	-49080.19
$M_{\tilde{L}_{1,2}}$	8918.04	$A^{0(d)}_{11,22}$	-80820.34
$M_{\tilde{L}_3}$	12784.02	$A^{0(d)}_{33}$	-25953.43
$M_{\tilde{\bar{d}}_{1,2}}$	4701.17	$M_{\tilde{Q}_{1,2}}$	6635.99
$M_{\tilde{\bar{d}}_3}$	34511.30	$\mu(M_Z)$	82076.07
$M_{\tilde{Q}_3}$	26677.45	$M^2_{\bar{H}}$	-6.7041×10^9
$B(M_Z)$	1.0886×10^9	M^2_H	-7.4518×10^9

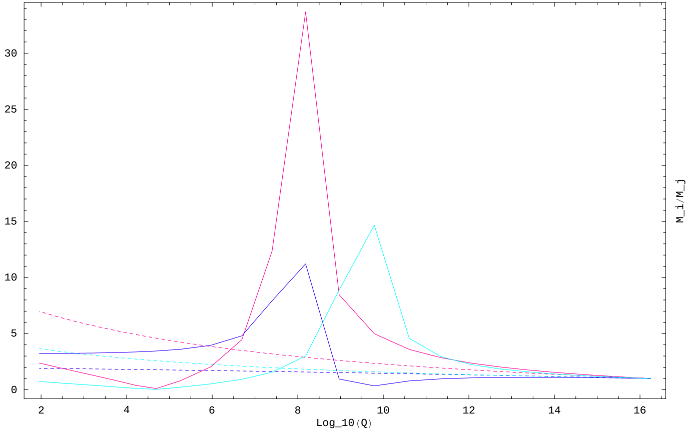
Field	$Mass(GeV)$
$M_{\tilde{G}}$	500.05
$M_{\chi^\pm}$	346.13, 82076.16
M_{χ^0}	120.22, 346.13, 82076.13, 82076.13
$M_{\tilde{\nu}}$	8917.793, 8870.645, 12783.853
$M_{\tilde{e}}$	1314.85, 8918.17, 200.57, 8871.15, 12483.32, 13045.62
$M_{\tilde{u}}$	6635.76, 8190.29, 6634.61, 8189.44, 15614.25, 26680.06
$M_{\tilde{d}}$	4701.24, 6636.27, 4699.43, 6635.15, 26670.33, 34516.86
M_A	235669.27
$M_{H^\pm}$	235669.28
M_{H^0}	235669.25
M_{h^0}	124.00

Table : Susy spectra ignoring generation mixing. Mini-split with large μ, A_0 parameters avoids FCNC and CCB/UFB. Masses ordered by generation not magnitude.

Parameter	*Value*
ϵ_{Lepto}	0.35×10^{-7}
a_μ	0.698×10^{-9}
δ_{PMNS}(in degrees)	6.2370
$\tau(p \rightarrow Meson + \nu)$	$5.233 * 10^{34} yr$
$\Gamma(\pi\nu)$	$6.77 * 10^{-37} yr^{-1}$
$\{BR(\pi\nu)\}$	$\{0.0045, 0.839, 0.1569\}$
$\Gamma(K + \nu)$	$\{1.84 * 10^{-35} yr^{-1}$
$BR(K + \nu)$	$\{0.00139, 0.335, 0.664\}$

Table : B and Lepton violation parameters


5000
4000
3000
2000
1000
0
0
2000
4000
6000
8000
μ, GeV
A_t, GeV


30
25
20
15
10
5
0
2
4
6
8
10
12
14
16
Log_10(Q)
M_i/M_j

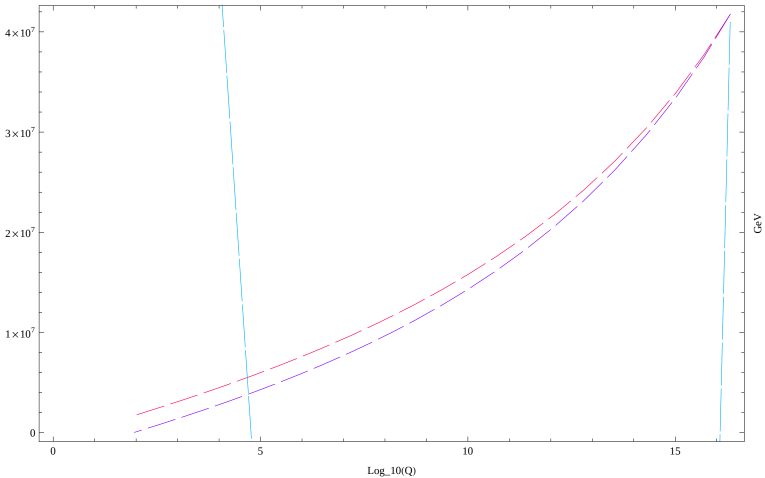


Figure : Right slepton masses

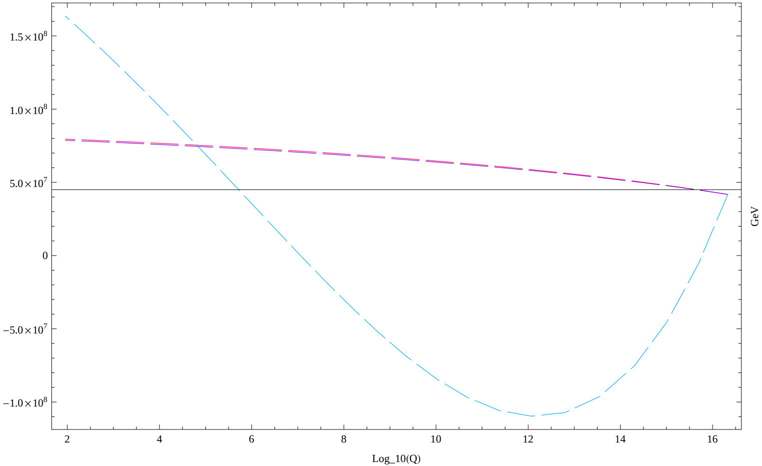


Figure : Left slepton masses

- Yukawonification : Gauge symmetry $SO(10) \times O(N_g)_F$

$$
\begin{aligned}
Matter &: \quad 16 - plets \quad \psi_A, A = 1..N_g \\
Higgs &: \quad \{\Phi, \overline{\Sigma}, \Sigma, H\}_{AB} = \{\Phi, \overline{\Sigma}, \Sigma, H\}_{BA} \\
NMSGUT &: \quad \Theta_{AB} = -\Theta_{BA}
\end{aligned}
$$

- Superpotential

$$
\begin{aligned}
W_{GUT} &= \mathrm{Tr}(m\Phi^2 + \lambda\Phi^3 + M\overline{\Sigma}.\Sigma + \eta\Phi.\overline{\Sigma}.\Sigma) \\
&+ \Phi.H.(\gamma\Sigma + \bar{\gamma}.\overline{\Sigma}) + M_H H.H) \\
W_F &= \Psi_A.((hH) + (f\Sigma) + (g\Theta))_{AB}\Psi_B
\end{aligned}
$$

- SSB

$$
\begin{aligned}
W &= \mathrm{Tr}[m(p^2 + 3a^2 + 6\omega^2) + 2\lambda(a^3 + 3p\omega^2 + 6a\omega^2)] \\
&+\mathrm{Tr}[M\bar{\sigma}\sigma + \eta(p + 3a - 6\omega)\frac{(\bar{\sigma}\sigma + \sigma\bar{\sigma})}{2}]
\end{aligned}
$$

- Doublet Higgs $2N_g(N_g+1)$ dim mass matrix

$$\mathcal{H} \;=\; \begin{pmatrix} -M_H & \bar\gamma\sqrt{3}\Omega(\omega-a) & -\gamma\sqrt{3}\Omega(\omega+a) & -\bar\gamma\Omega(\bar\sigma) \\ \gamma\sqrt{3}\Omega(\omega-a) & -(2M+4\eta\Omega(a-\omega)) & \varnothing_d & -2\eta\sqrt{3}\Omega(\bar\sigma) \\ -\bar\gamma\sqrt{3}\Omega(\omega+a) & \varnothing_d & -(2M+4\eta\Omega(\omega+a)) & \varnothing_d \\ -\gamma\Omega(\sigma) & -2\eta\sqrt{3}\Omega(\sigma) & \varnothing_d & (-2m+6\lambda\Omega(\omega-a)) \end{pmatrix}$$

$$\Omega[V] = \begin{pmatrix} V_{11} & 0 & V_{12}/\sqrt{2} \\ 0 & V_{22} & V_{12}/\sqrt{2} \\ V_{12}/\sqrt{2} & V_{12}/\sqrt{2} & (V_{11}+V_{22})/2 \end{pmatrix}$$

- Yukawa couplings :

$$
\begin{aligned}
Y_u &= \begin{pmatrix} \hat{h}\hat{V}_1 + \hat{f}\hat{V}_4 & \hat{h}\hat{V}_3 + \hat{f}\hat{V}_6 \\ \hat{h}\hat{V}_3 + \hat{f}\hat{V}_6 & \hat{h}\hat{V}_2 + \hat{f}\hat{V}_5 \end{pmatrix} \\
Y_d &= \begin{pmatrix} \hat{h}\hat{W}_1 + \hat{f}\hat{W}_7 & \hat{h}\hat{W}_3 + \hat{f}\hat{W}_9 \\ \hat{h}\hat{W}_3 + \hat{f}\hat{W}_9 & \hat{h}\hat{W}_2 + \hat{f}\hat{W}_8 \end{pmatrix} \\
Y_\nu &= Y_u|_{f\to -3f} \quad ; \quad Y_l = Y_d|_{f\to -3f} \\
\hat{h} &= 2\sqrt{2}h \qquad ; \qquad \hat{f} = -4i\sqrt{\frac{2}{3}}f
\end{aligned}
$$

- $\hat{V}, \hat{W}$ are the normalized right and left null eigenvectors of the mass matrix

- GUT F terms

$$2m(p-a)-2\lambda a^2+2\lambda\omega^2=0$$
$$2m(p+\omega)+\lambda(p+2a+3\omega)\omega$$
$$+\lambda\omega(p+2a+3\omega)=0$$
$$M\sigma+\eta(\chi\sigma+\sigma\chi)/2=0$$
$$M\bar\sigma+\eta(\chi\bar\sigma+\bar\sigma\chi)/2=0$$
$$\bar\sigma\sigma+\sigma\bar\sigma=-\frac{4}{\eta}(mp+3\lambda\omega^2)\equiv F$$

where $\chi\equiv(p+3a-6\omega)$

- D-terms for $N_g=2$

$$|\sigma_{11}|^2+|\sigma_{22}|^2+|\sigma_{12}|^2=|\bar\sigma_{11}|^2+|\bar\sigma_{22}|^2+|\bar\sigma_{12}|^2$$
$$Re[\sigma_{12}^*(\sigma_{11}-\sigma_{22})+\bar\sigma_{12}^*(\bar\sigma_{11}-\bar\sigma_{22})+p_{12}^*(p_{11}-p_{22})$$
$$+3a_{12}^*(a_{11}-a_{22})+6\omega_{12}^*(\omega_{11}-\omega_{22})]=0$$

Conclusions

- Susy SO(10) GUTs fit fermion data, give LSP CDM, Leptogenesis, calculable LFV, Inflation...
- Complete GUT-SSB specific calculations carried out
- Spectra specific NMSGUT fits of complete fermion data, predicted(2008) Mini-Split large $A_0, \mu, m_A, m_{\tilde{f}}, m_h$ with controlled exotics.
- Require $A_0 < 0, \mu > 0$ driven modification of fermion yukawas at M_S
- $d = 5$ proton decay operators controllable by novel mechanism: Threshold lowering of SO(10) Yukawas required to match MSSM yukawas.
- Dynamical Yukawonification ! reduces number of parameters by over 50%
- Novel scenarios ? Re-evaluation of (N)MSGUT constraints and no go's.