Four dimensional supersymmetric Yang-Mills quantum mechanics with SU(3) gauge group.

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INNOVATIVE ECONOMY
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The model

The hamiltonian is given by

$$H = \frac{1}{2} p_a^i p_a^i + \frac{g^2}{4} f_{abc} f_{ade} x_b^i x_c^j x_d^i x_e^j + \frac{ig}{2} f_{abc} \psi_a^T \Gamma^i \psi_b x_c^i$$

$i, j = 1, \ldots, D-1$ - spatial indices

$a, b, c, d, e = 1, \ldots, N^2 - 1$ - color indices

$\psi_{a,\alpha}$ - Majorana spinor

H is supersymmetric. Supersymmetry generators are:

$$Q_\alpha = (\Gamma^i \psi_a)_\alpha p_a^i + igf_{abc}(\Sigma^{ij}\psi_a)_\alpha x_b^i x_c^j, \qquad (1)$$

where Γ^k - alpha matrices, $\Sigma^{ij} = -\frac{i}{4}[\Gamma^i, \Gamma^j]$.

$$\{Q_\alpha, Q_\beta^\dagger\} = 4H\delta_{\alpha\beta} \qquad (2)$$

In our case $D = 4, \; N = 3$.

Motivations

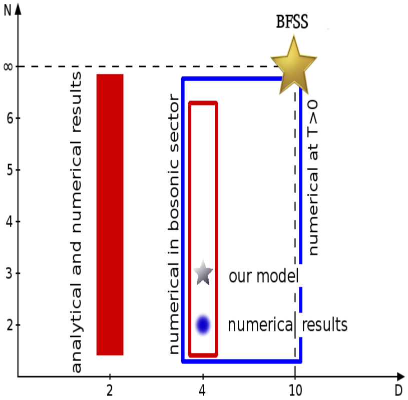
BFSS conjecture

uncompactified 11 dimensional M-theory $\Leftrightarrow$ large N limit of supersymmetric quantum mechanics in 10 dimensions [BFSS]

small volume approximation to QCD

bosonic sector of considered model is 0 - order approximation of QCD in small volume approach (i.e.: dynamics of homogeneous fields)

Earlier results - overview



- analytical and numerical results for $D = 2$ and arbitrary N [Trzetrzelewski; Korcyl]
- numerical results for $D = 4$, $N \leq 6$ in bosonic sector only as 0-order approximation to Yang-Mills QFTs [Lüscher; Ziemann]
- numerical results for $D = 4$, $N = 2$ [Wosiek, Campostrini]
- numerical results at finite tempetature [Catterall, Wiseman; Anagnostopoulos et al.]

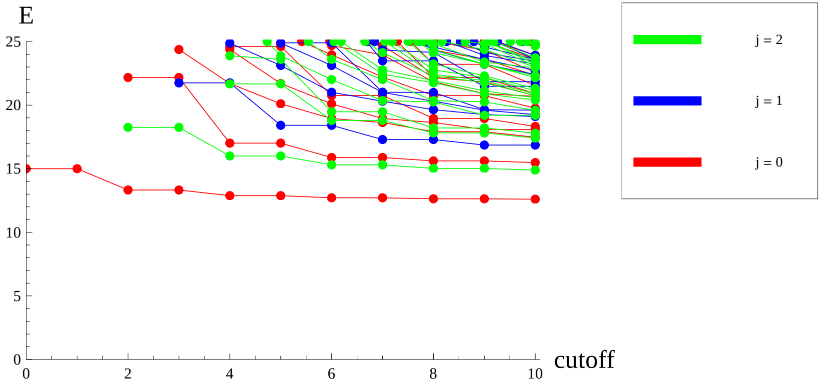
Method

- construct Fock space of gauge invariant states with cutoff on number of bosonic excitations
- construct matrices of hamiltonian, angular momentum and supersymmetry generators
- for energies - diagonalize the hamiltonian
- fermionic number is conserved (for $D = 2, 4$) - consider each fermionic sector separately
- rotation symmetry - use sectors of definite angular momentum

What do we obtain

- energies (with distinction between continuous and discrete spectrum)
- energy eigenstates (with definite fermion number and angular momentum)
- supersymmetric fractions → identifying supermultiplets
- restricted Witten index (sum over sectors with n_F even)

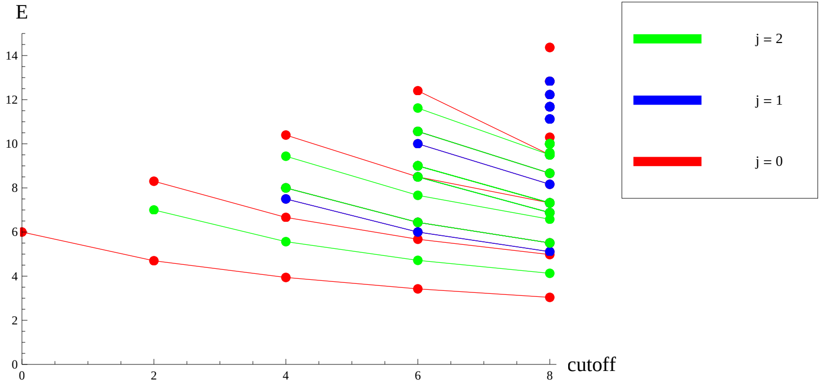
Energies in bosonic sector



Very fast convergence of lowest energies ⇒ spectrum is discrete.

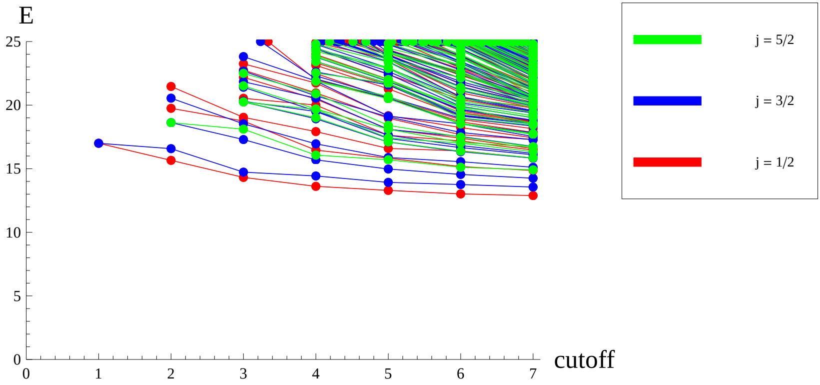
Digression: signature of continuous spectrum

Spectrum is continuous for kinetic energy only: $H = \frac{1}{2} p_a^i p_a^i$.



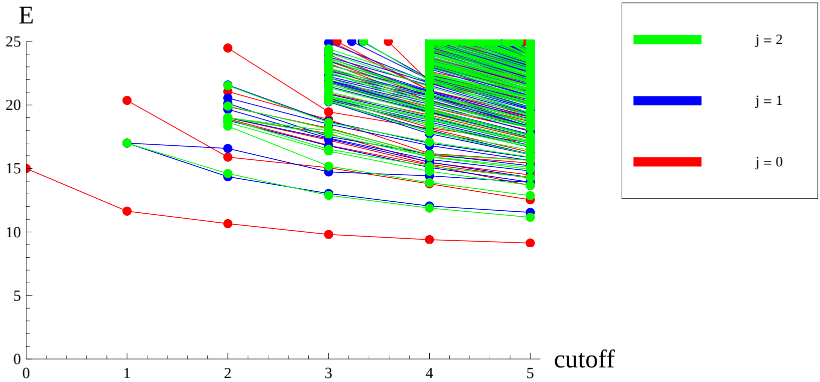
Energy behavior is $E \sim 1/N_{cut}$.

Energies in sector with one fermion



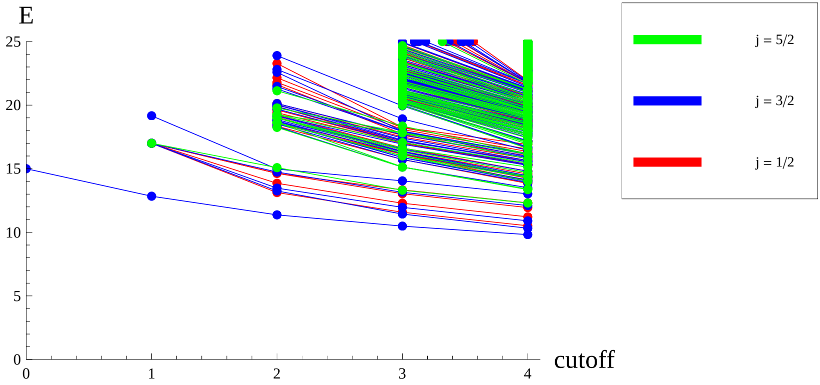
Still only discrete spectrum.

Energies in sector with two fermions



Spectrum is rather discrete.

Energies in sector with three fermions



Candidate for continuous spectrum.

Supersymmetric fractions

$$\mathcal{Q}^\dagger_{1/2} = \frac{1}{2}(Q_1 - iQ_2 + Q_3 + iQ_4)$$

$$\mathcal{Q}^\dagger_{-1/2} = \frac{1}{2}(iQ_1 + Q_2 - iQ_3 + Q_4)$$

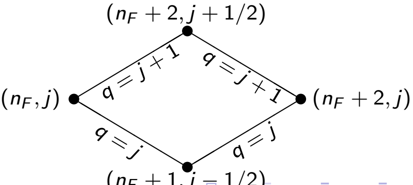
$$H = \tfrac{1}{4}\{\mathcal{Q}, \mathcal{Q}^\dagger\}$$

$|n_F jmE\rangle$ - eigenstate of hamiltonian

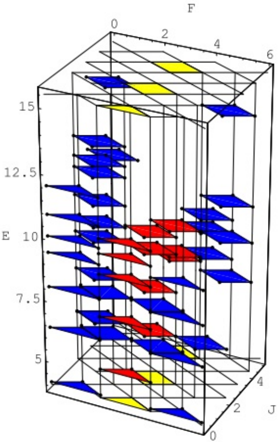
$$(2j+1) = \frac{1}{E}\langle n_F jmE|H|n_F jmE\rangle$$

$$= \sum_{j'E'} \Bigg(\underbrace{\frac{1}{4E}\sum_{mm'} \left|\langle (n_F-1)j'm'E'|\mathcal{Q}|n_F jmE\rangle\right|^2}_{\text{supersymmetric fraction } q_{n_F}(j'E'|jE)} + q_{n_F+1}(jE|j'E') \Bigg)$$

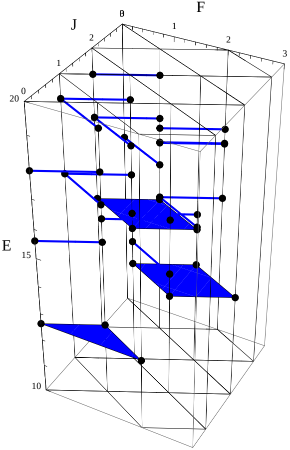
supermultiplets form diamonds like that on the right



Overall picture - identification of supermultiplets



SU(2)



SU(3)

Our results vs earlier results of [Campostrini, Wosiek] for $SU(2)$. Single lines mean that the whole supermultiplet was not identified.

Conclusions

- our method gives us a qualitative picture of the spectrum
- the cutoff is still too low (for larger number of fermions) to determine continuous spectrum and to calculate Witten index

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Main challenge

size of basis

- at present we have matrices of sizes up to 4k x 4k
- getting nb=7 in each fermion sector would require matrices 1M x 1M
- possible solution for higher D or N: take only most significant basis states?

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