

# Interpreting the “Higgs-mass oracle”

Gino Isidori

[ *INFN, Frascati & CERN* ]



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- ▶ Introduction
- ▶ Consistency with the e.w. precision tests
- ▶ Stability and metastability bounds
- ▶ The running of  $\lambda$
- ▶ High-scale matching & SUSY
- ▶ Conclusions

► Introduction [ *Ibis redibis...* ]

Before the Higgs discovery there were strong hopes that knowing  $m_h$  we would have gained a clear clues about the nature of physics beyond the SM, especially in the context of SUSY

These hopes were justified, and indeed  $m_h$  *could have* given us unambiguous answers...



► Introduction [ *Ibis redibis...* ]

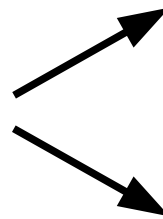
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These hopes were justified, and indeed  $m_h$  *could have* given us unambiguous answers...

...however, the measured value resembles very much the typical “sibylline answer” of the ancient oracles.

Such as

*Ibis redibis non morieris in bello*



*You will go, be back,  
and not die in the battle*

*You will go, not be back,  
and die in the battle*



► Introduction [ *Ibis redibis...* ]

Before the Higgs discovery there were strong hopes that knowing  $m_h$  we would have gained a clear clues about the nature of physics beyond the SM, especially in the context of SUSY

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As I will illustrate in this talk, it is hard to conceive a more intricate/ambiguous situation concerning possible UV extensions of the SM than the one we have after the LHC8 results:

- $m_h = 125 - 126 \text{ GeV}$

- no evidences of physics beyond the SM @ LHC8



► Consistency with the e.w. precision tests

The Higgs mechanism is the most economical & simple choice to achieve the spontaneous symmetry breaking of both gauge and flavor symmetries that we observe in nature.

$$\mathcal{L}_{\text{higgs}} = D\phi^+ D\phi - V(\phi) - Y^{ij} \psi_L^i \psi_R^j \phi$$

$$V(\phi) = -\mu^2 \phi^+ \phi + \lambda(\phi^+ \phi)^2$$

Before the start of the LHC only the ground state determined by  $V(\phi)$  (*and the corresponding Goldstone boson structure*) was tested with good accuracy:

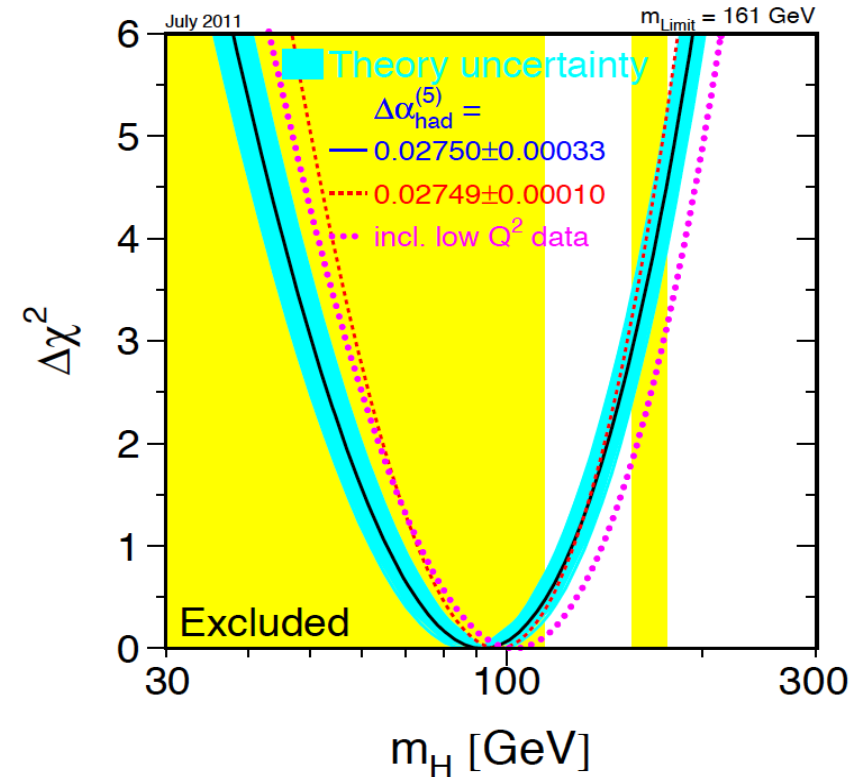
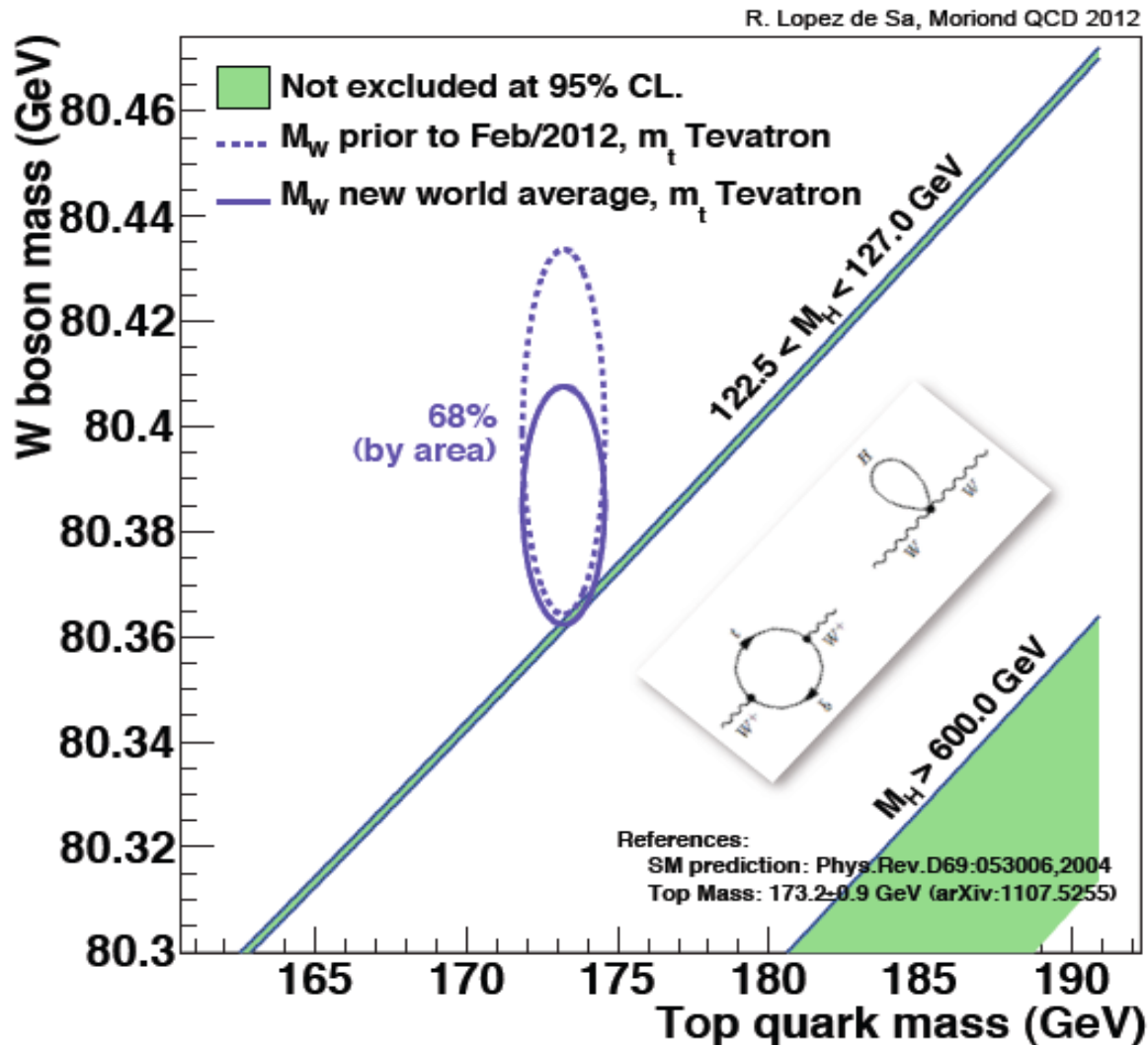
$$v = \langle \phi^+ \phi \rangle^{1/2} \sim 246 \text{ GeV} \quad [ m_W = \frac{1}{2} g v ]$$

The situation has substantially changed with the measurement of the Higgs boson mass, that has allowed us to precisely (and separately) fix both  $\mu^2$  &  $\lambda$ :

$$\lambda_{(\text{tree})} = \frac{1}{2} m_h^2 / v^2 \approx 0.13 \qquad \mu_{(\text{tree})}^2 = \frac{1}{2} m_h^2$$

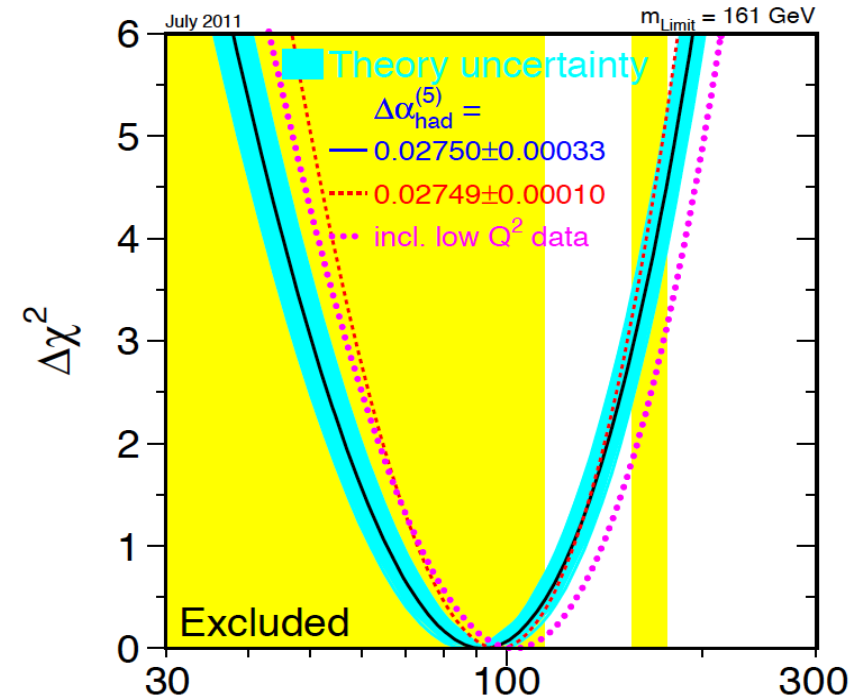
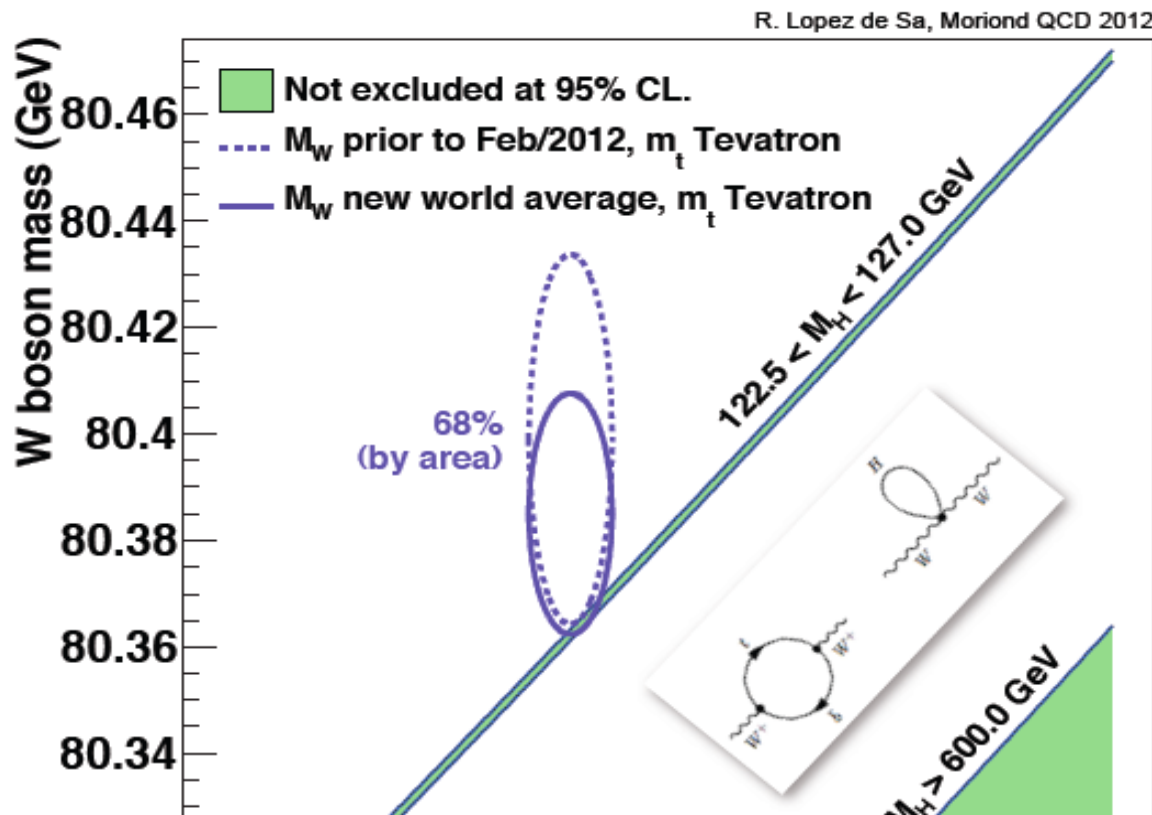
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Actually some information about the Higgs mass was already present in the e.w. precision tests (assuming the validity of the SM up to high scales):



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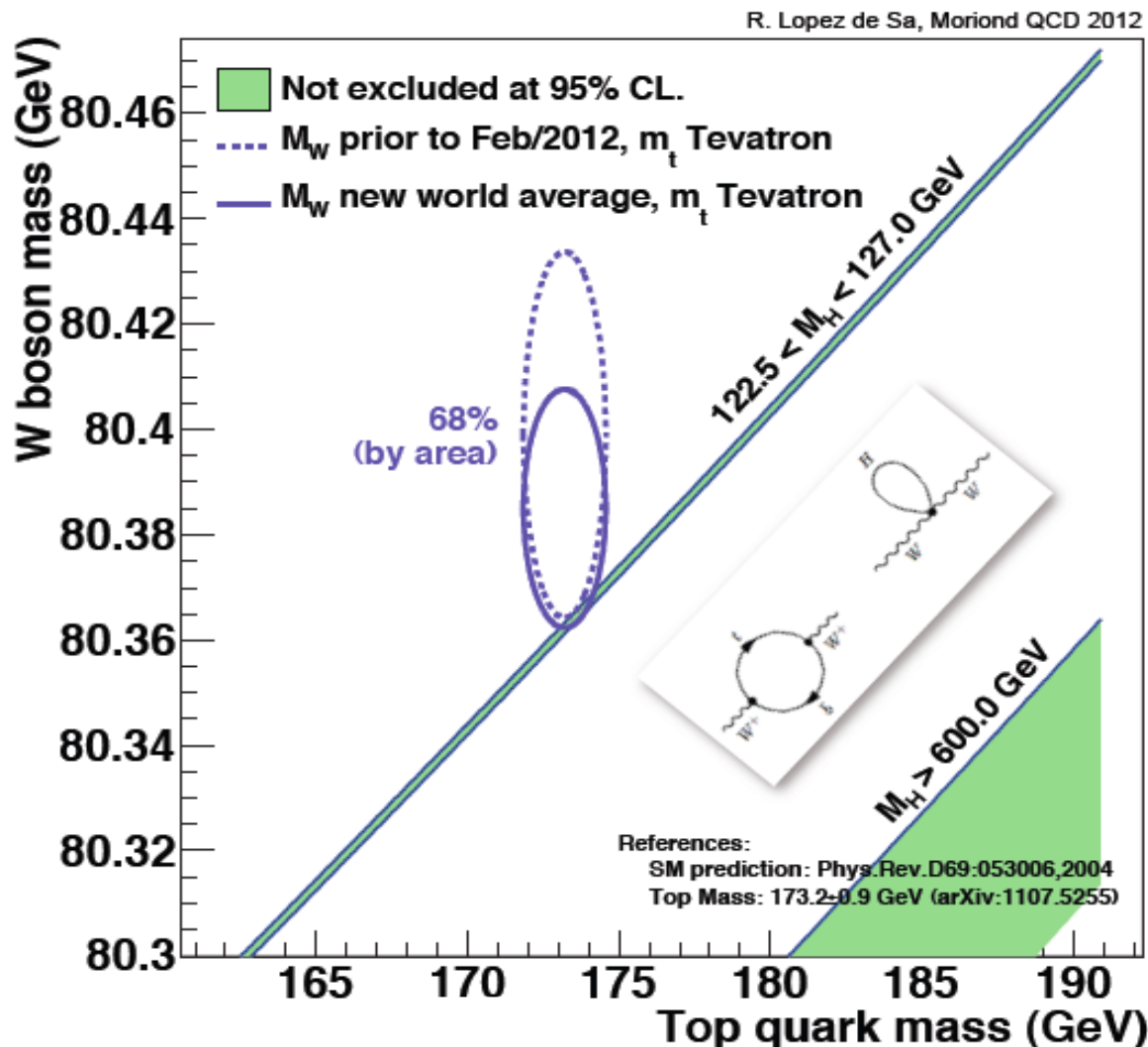


Message n.1: The observation of the physical Higgs boson with  $m_h$  well consistent with the (indirect) prediction of the e.w. precision tests is a great success of the SM.



## ► Consistency with the e.w. precision tests

Actually some information about the Higgs mass was already present in the e.w. precision tests (assuming the validity of the SM up to high scales):



direct  $m_h \neq$  indirect  $m_h$



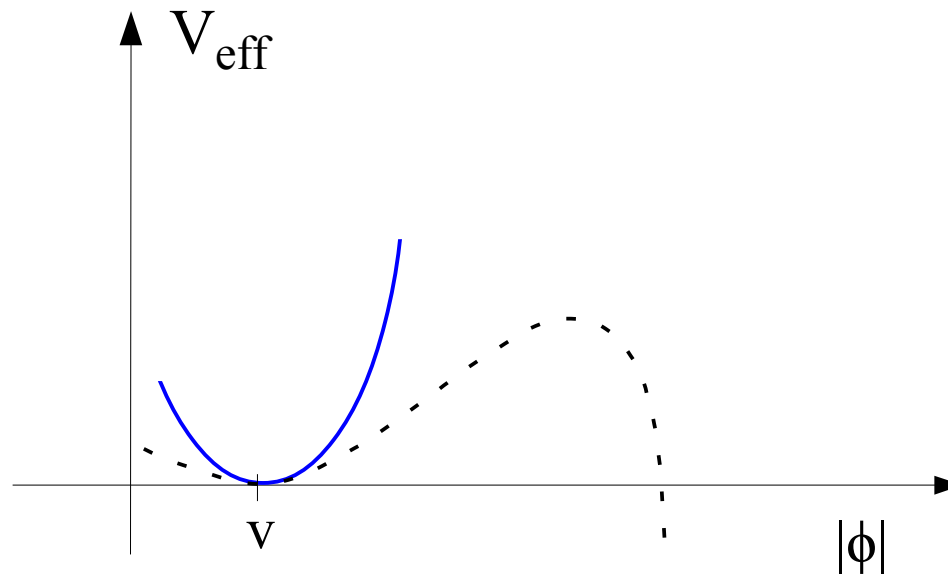
*unambiguous indication*

NP close to the TeV scale

The opposite is not necessarily true, but it indicates that (up to conspiracies) the theory has a *minimal & weakly coupled* structure around the TeV scale, as in the SM.

► Stability and metastability bounds

A completely independent (and unambiguous) indication for NP could have been obtained by the high-energy behavior of the Higgs potential

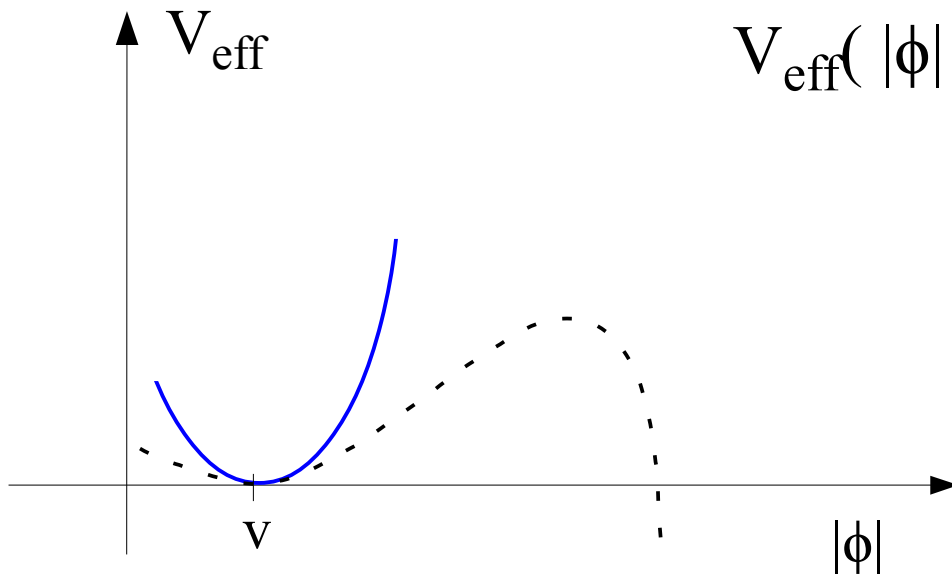


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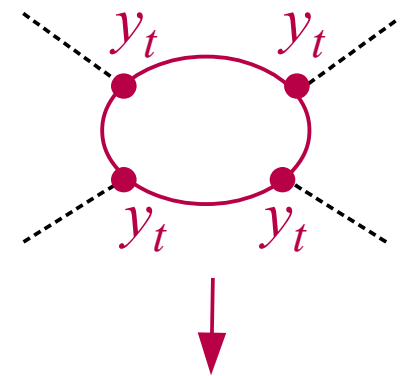
At large field values:

$$V_{\text{eff}}(|\phi| \gg v) \approx \lambda(|\phi|) \times |\phi|^4 + O(v^2|\phi|^2)$$



$$\lambda(v) \propto \frac{m_h^2}{v^2}$$

$$y_t(v) \propto \frac{m_t}{v}$$



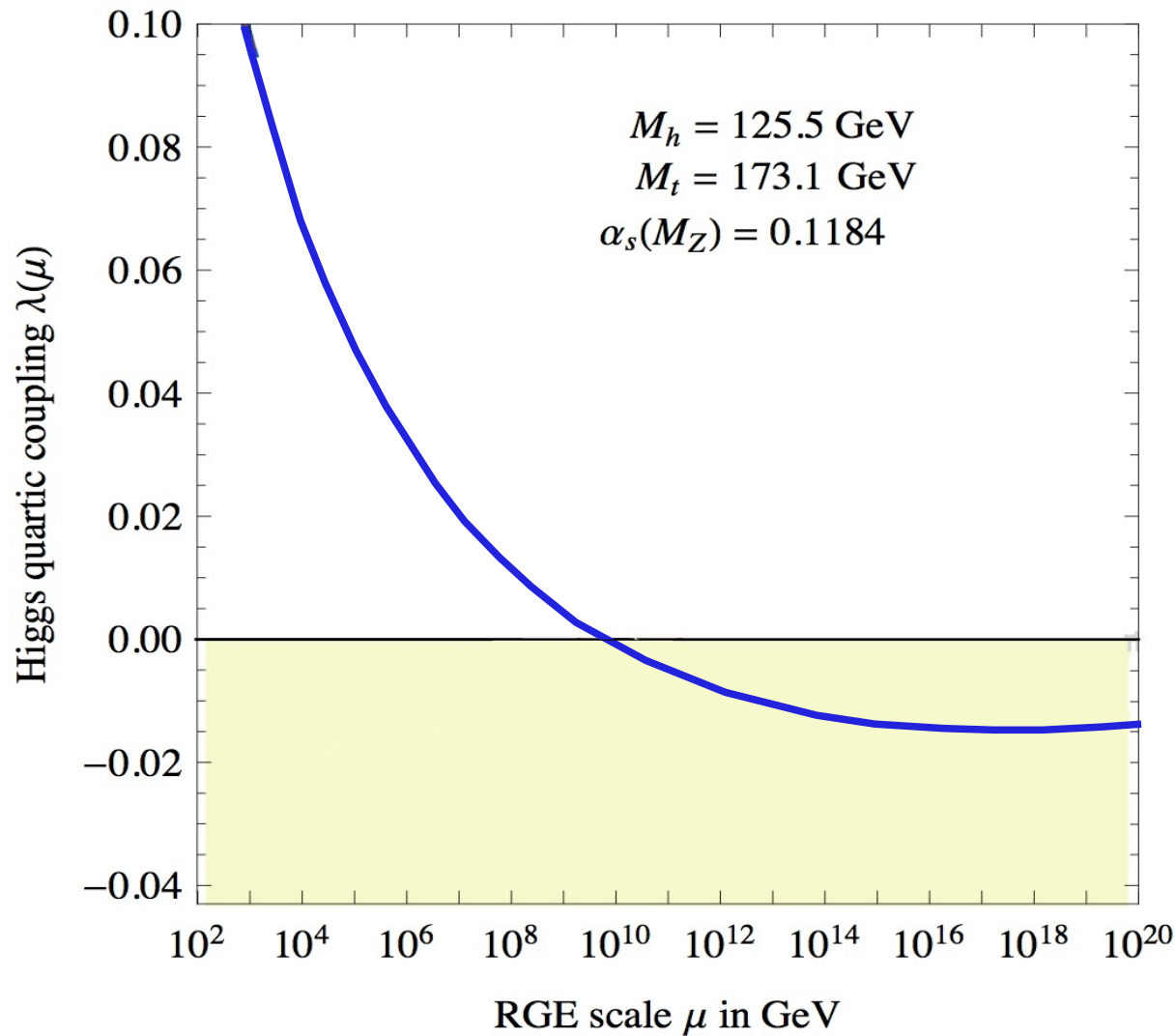
decreasing  $\lambda$   
at large energies

A too-light  $m_h$  could imply an  
unstable Higgs potential → need for NP

Cabibbo, Maiani, Parisi, Petronzio, '79;  
Hung '79; Lindner 86; Sher '89; ....

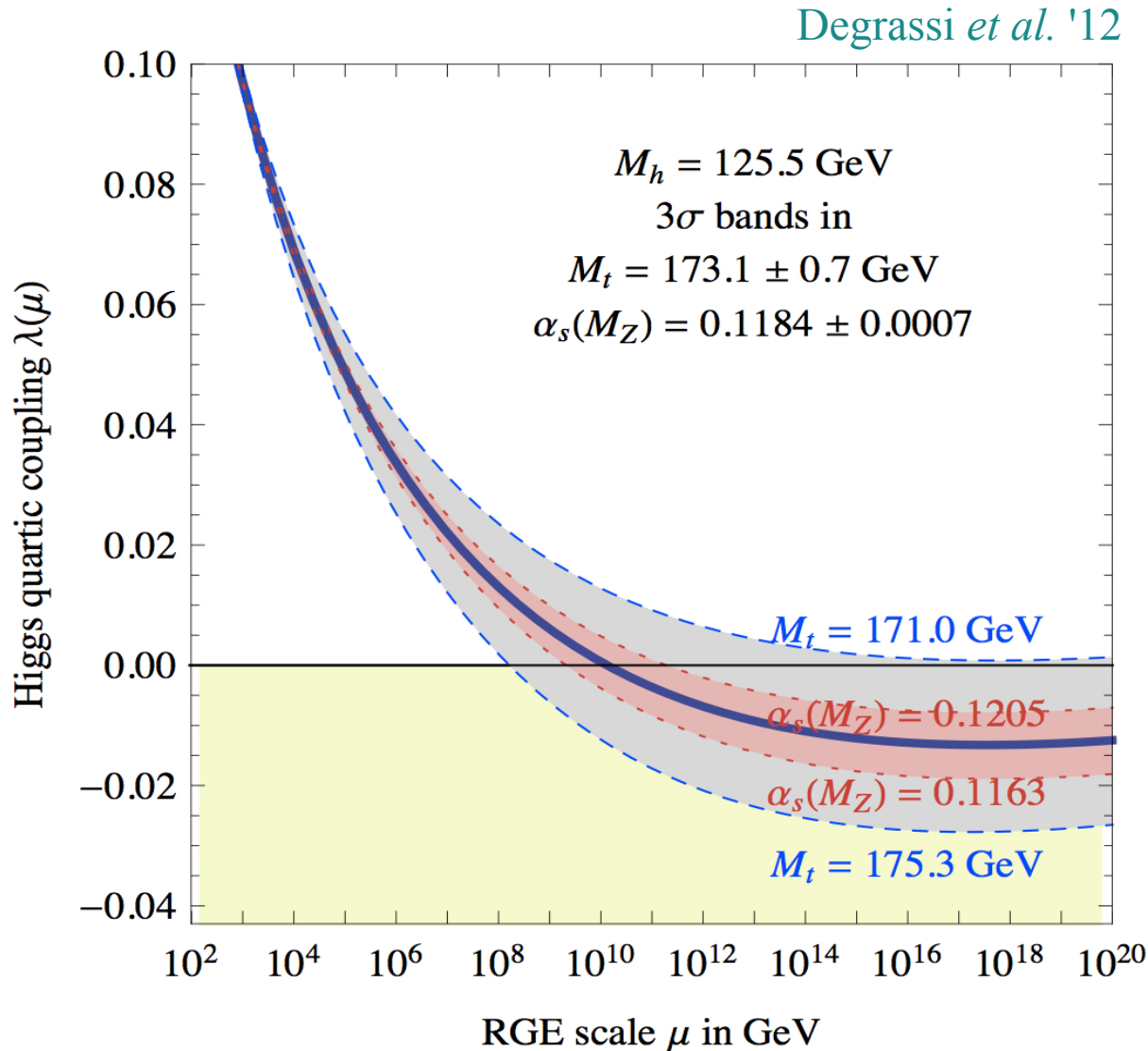
► Stability and metastability bounds

This is indeed what happens for  $m_h \approx 125$  GeV and  $m_t \approx 173$  GeV !



## ► Stability and metastability bounds

This is indeed what happens for  $m_h \approx 125$  GeV and  $m_t \approx 173$  GeV ...



...however, unfortunately this is not enough to unambiguously claim the need of NP:

- The present error on  $m_t$  does not allow us to exclude at more than  $3\sigma$  that  $\lambda > 0$  up to  $M_{\text{Planck}}$

- Even for the central values of  $m_h$  and  $m_t$ , the Higgs potential remains sufficiently metastable

► Stability and metastability bounds

The metastability condition: even if the potential has a second deeper minimum, the model is consistent with observations (= no need for NP) if the lifetime of the (unstable) e.w. minimum is longer than the age of the Universe

The e.w. minimum is destabilized by:

Quantum fluctuations (at  $T=0$ )

computable in a  
model-independent way

Thermal fluctuations

the probability depends on the thermal  
history of the universe & competes with  
the quantum tunneling only for very high  $T$

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Most conservative bound

The quantum tunneling occurs via bubble formation in the homogeneous background of the false (e.w.) minimum [**“bounce” field configurations**]

$$\text{Probability} \sim e^{-S_0[\phi_{\text{bounce}}]} \sim e^{-\frac{8\pi^2}{3|\lambda|}}$$

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Most conservative bound

result from full one-loop action,  
assuming no further destabilization from  
Planck-scale dynamics [ → talk by Branchina ]

$$p = \max_R \frac{V_U}{R^4} \exp \left[ - \frac{8 \pi^2}{3 |\lambda(1/R)|} + \begin{array}{l} \text{tiny} \\ \text{higher-order} \\ \text{terms} \end{array} \right]$$

G.I., Ridolfi,  
Strumia '01

The tunneling is dominated by “bounces” of size  $R$ , such that  $\lambda(1/R)$  reaches its minimum value:  $\lambda$  can become negative, provided it remains small in magnitude.



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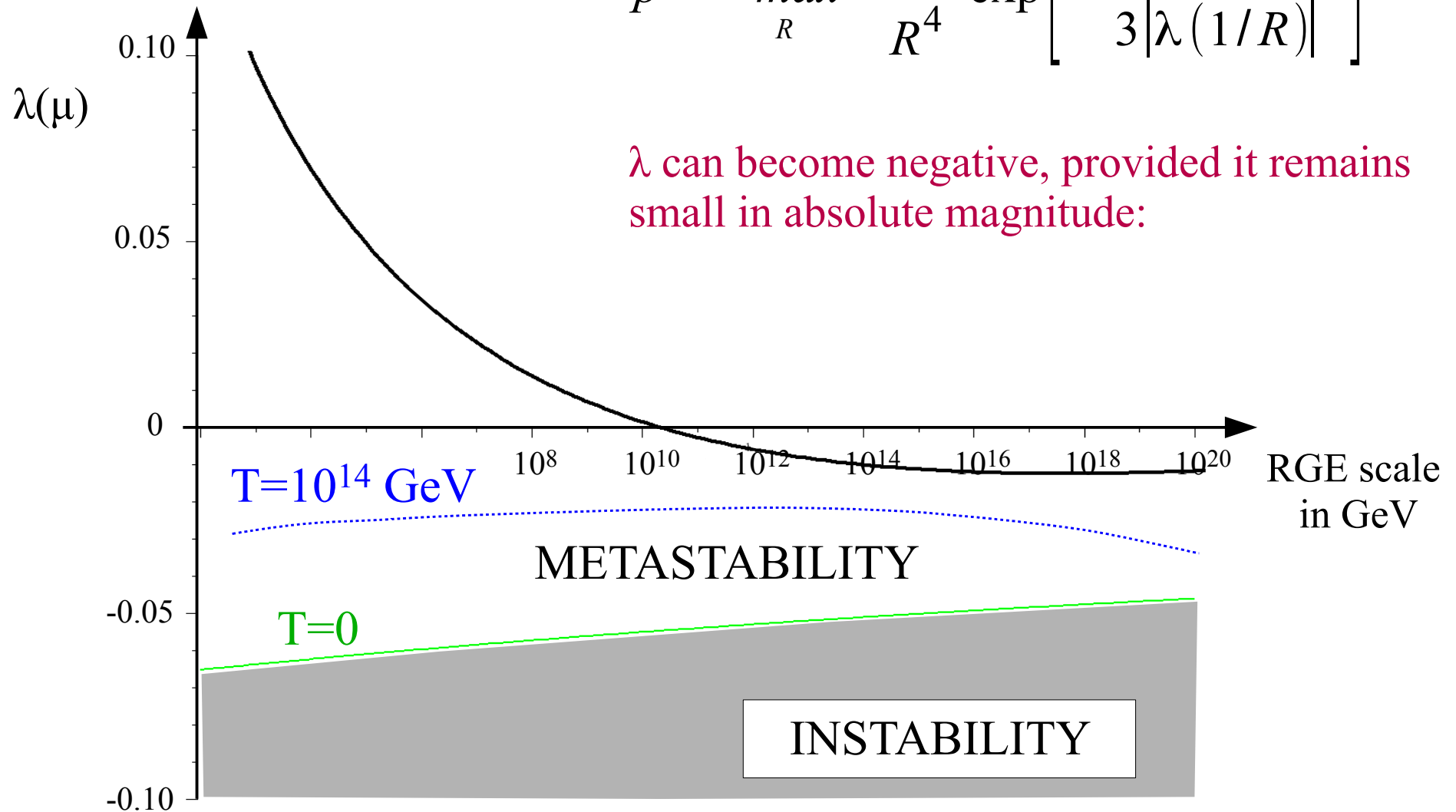
The tunneling is favored by the kinetic energy, but this effect is compensated by the appearance of an effective  $T$ -dependent positive mass:  $m_{\text{eff}}^2(T) \sim g^2(T) T^2$

- again tunneling needs non-trivial field configurations
- also in this case  $\lambda$  can become negative, provided it remains sufficiently small in magnitude.

► Stability and metastability bounds

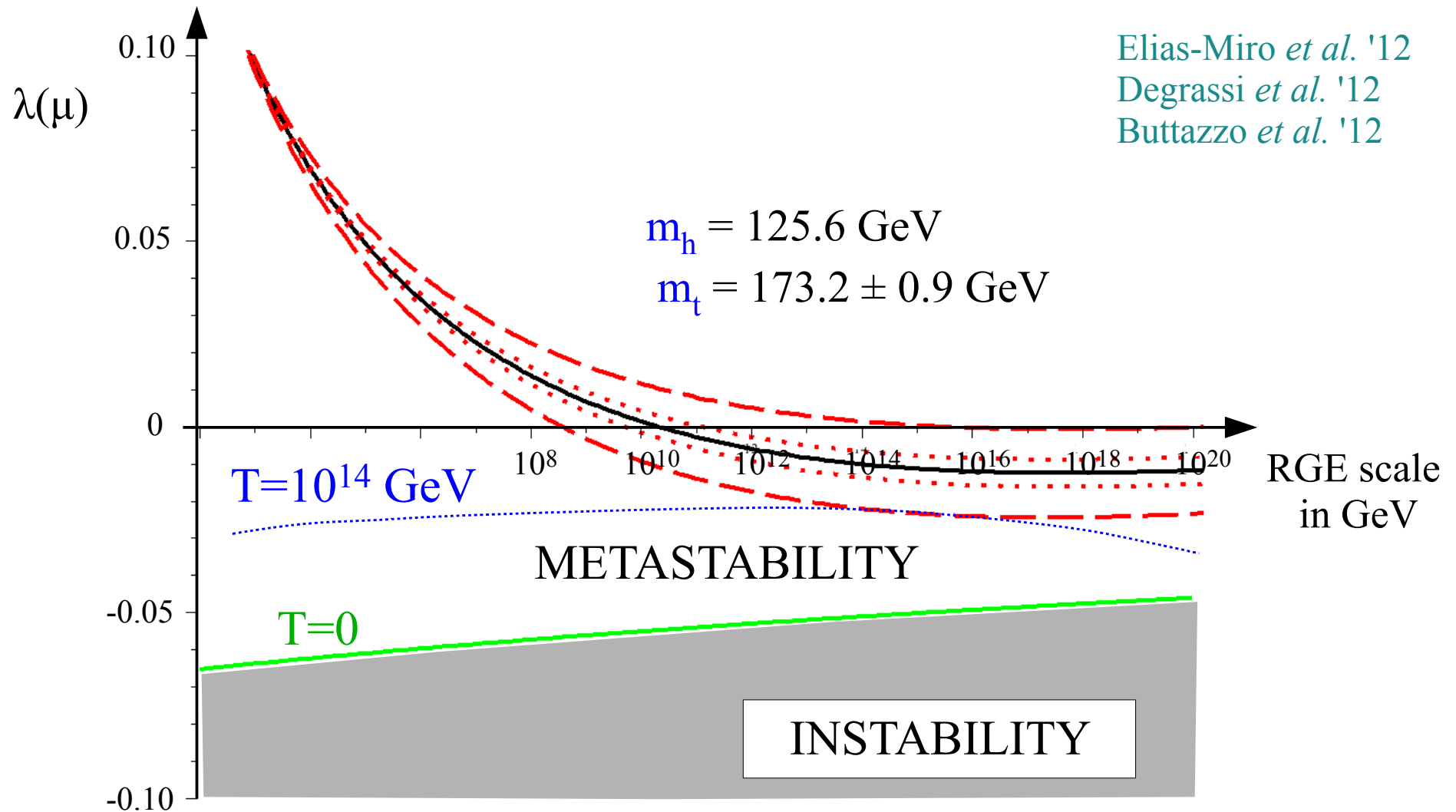
The metastability condition

$$p \approx \max_R \frac{V_U}{R^4} \exp \left[ -\frac{8\pi^2}{3|\lambda(1/R)|} \right]$$



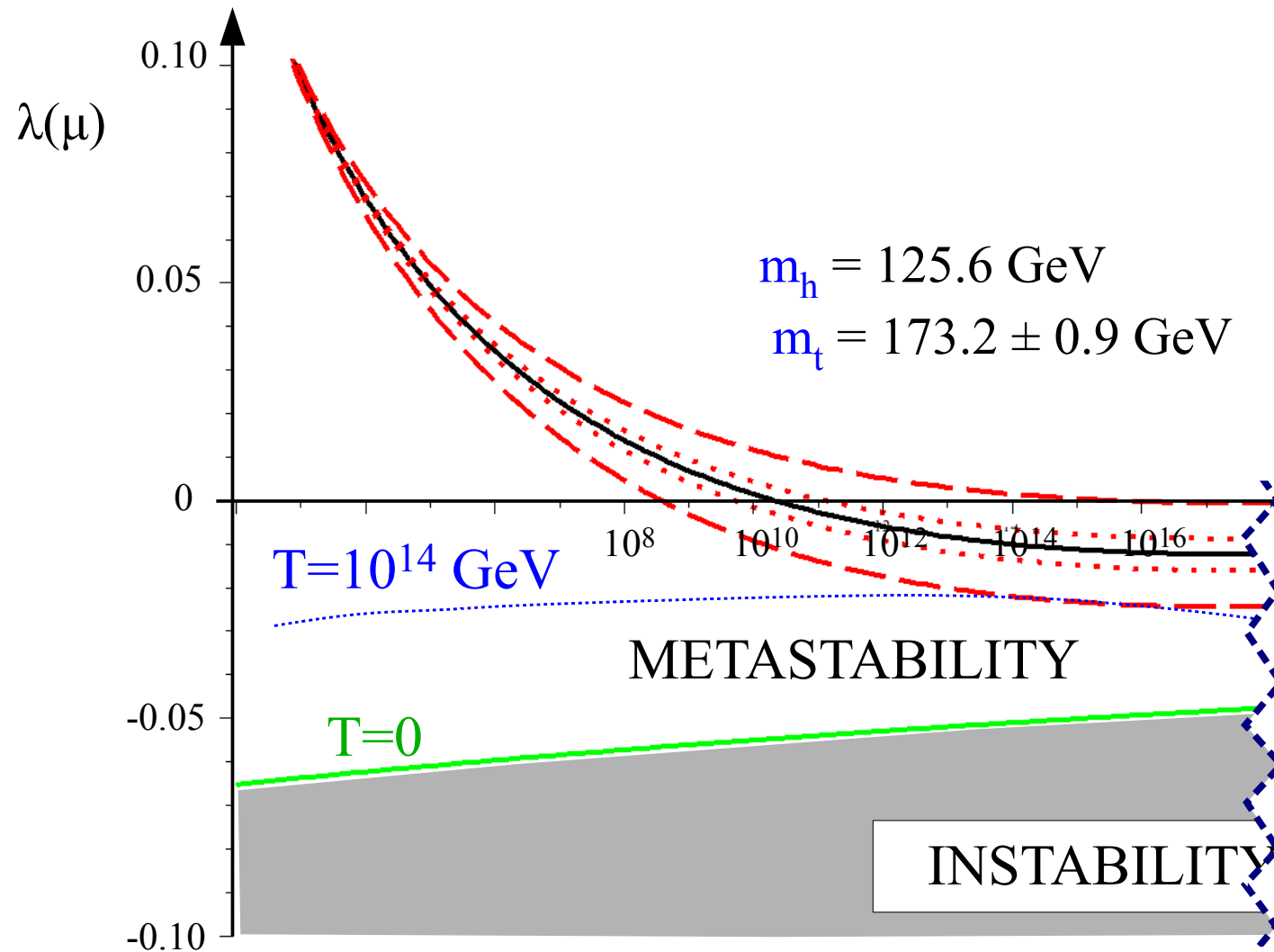
► Stability and metastability bounds

Message n.2: For  $m_h \approx 125\text{-}126$  GeV and the present central value of  $m_{\text{top}}$ , the SM vacuum is unstable but sufficiently long-lived, compared to the age of the Universe



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Elias-Miro *et al.* '12  
 Degraasi *et al.* '12  
 Buttazzo *et al.* '12

**N.B.:** we cannot trust the estimate of the tunneling rate too close to  $M_{\text{Pl}}$

► Stability and metastability bounds

Message n.2: The SM vacuum is unstable but sufficiently long-lived, compared to the age of the Universe → **No need of NP below  $M_{\text{Pl}}$  to stabilize the SM vacuum**

$\lambda(\mu)$  cross the metastability limit  
at a scale  $\mu_0 \ll M_{\text{Planck}}$

↓ *unambiguous  
indication*

NP around/below  $\mu_0$

N.B.: Also in this case the opposite is NOT true.

Moreover, we cannot say anything about possible destabilization of the e.w. vacuum by Planck-scale dynamics [New “vacuum decay channels” could easily “open-up” at  $M_{\text{Planck}}$  → talk by Branchina].

► The running of  $\lambda$



*Welcome to the...*

**SLOW RUNNERS CLUB**

**for those without a need for speed**

► The running of  $\lambda$

How “precise” is the estimate of the evolution of  $\lambda$   
(and the corresponding conclusion that the SM potential is unstable)?

A complete NNLO analysis has recently become possible:

- Two-loop potential Ford, Jack, Jones '92, '01
- Three-loop beta functions Mihaila, Salomon, Steinhauser '12  
Chetyrkin, Zoller, '12
- Two-loop threshold corrections in relating  $\lambda(v)$  to the Higgs mass:

$$\lambda(\mu) = \frac{G_F m_h^2}{\sqrt{2}} + \Delta\lambda(\mu)$$

*(dominant uncertainty)*

Yukawa×QCD Bezrukov *et al.* '12

Yukawa×QCD Degrassi *et al.* '12

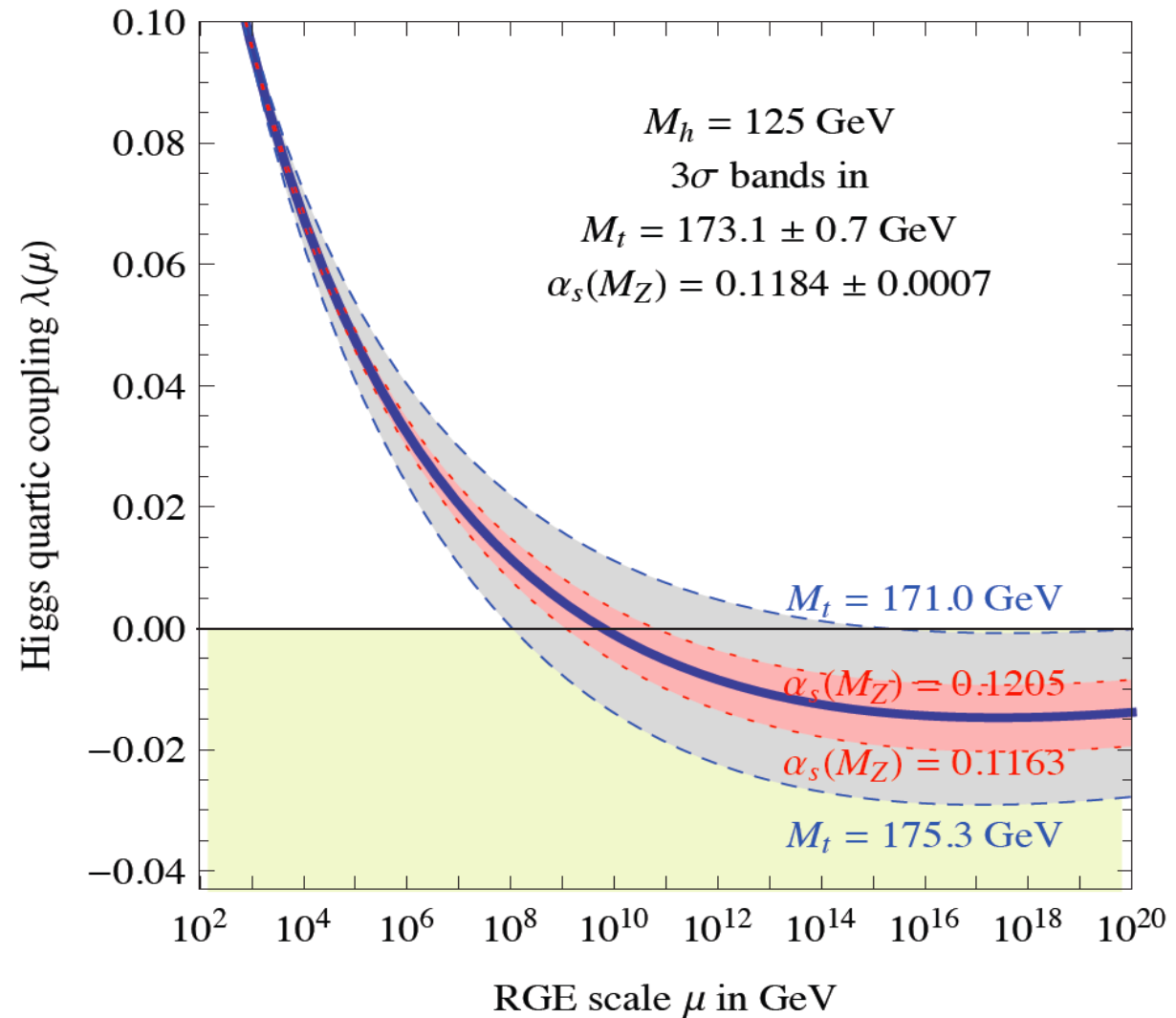
Yuk.×Yuk.

Full 2-loop Buttazzo *et al.* '13

## ► The running of $\lambda$

Given the fast running of  $\lambda$  close to the e.w. scale, the dominant uncertainty comes from threshold (non-log enhanced) corrections at the electroweak scale (or in the precise evaluation of the initial condition).

While the smallness of  $\lambda$  (and the other couplings) at high energies imply that the 3-loop terms in the beta functions play a very minor role (useful to control the error).





► The running of  $\lambda$

With the NNLO calculation we are able to derive a very precise relation between Higgs and top masses from vacuum stability:

*Absolute stability:*

Degrassi *et al.* '12

$$M_h \text{ [GeV]} > 129.4 + 2.0 \left( \frac{M_t \text{ [GeV]} - 173.1}{1.0} \right) - 0.5 \left( \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0_{\text{th}}$$

Conservative th. error given the size of the shifts from NLO to NNLO:

- + 0.6 GeV due to the QCD threshold corrections to  $\lambda$
- + 0.2 GeV due to the Yukawa threshold corrections to  $\lambda$
- 0.2 GeV from RG equation at 3 loops
- 0.1 GeV from the effective potential at 2 loops.

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Reliability of this th. error estimate fully confirmed by complete 2-loop threshold corr.

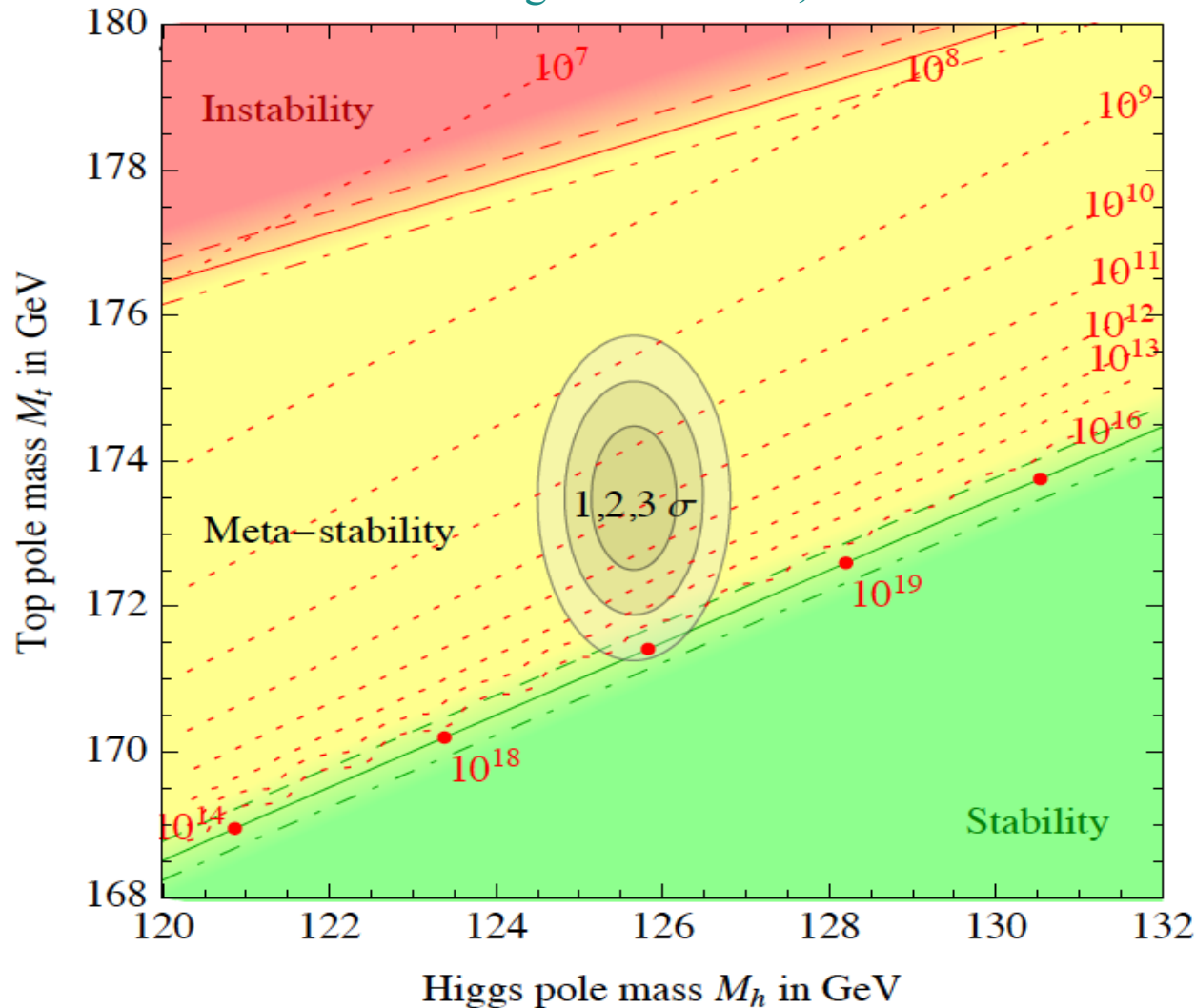
Buttazzo *et al.* '13

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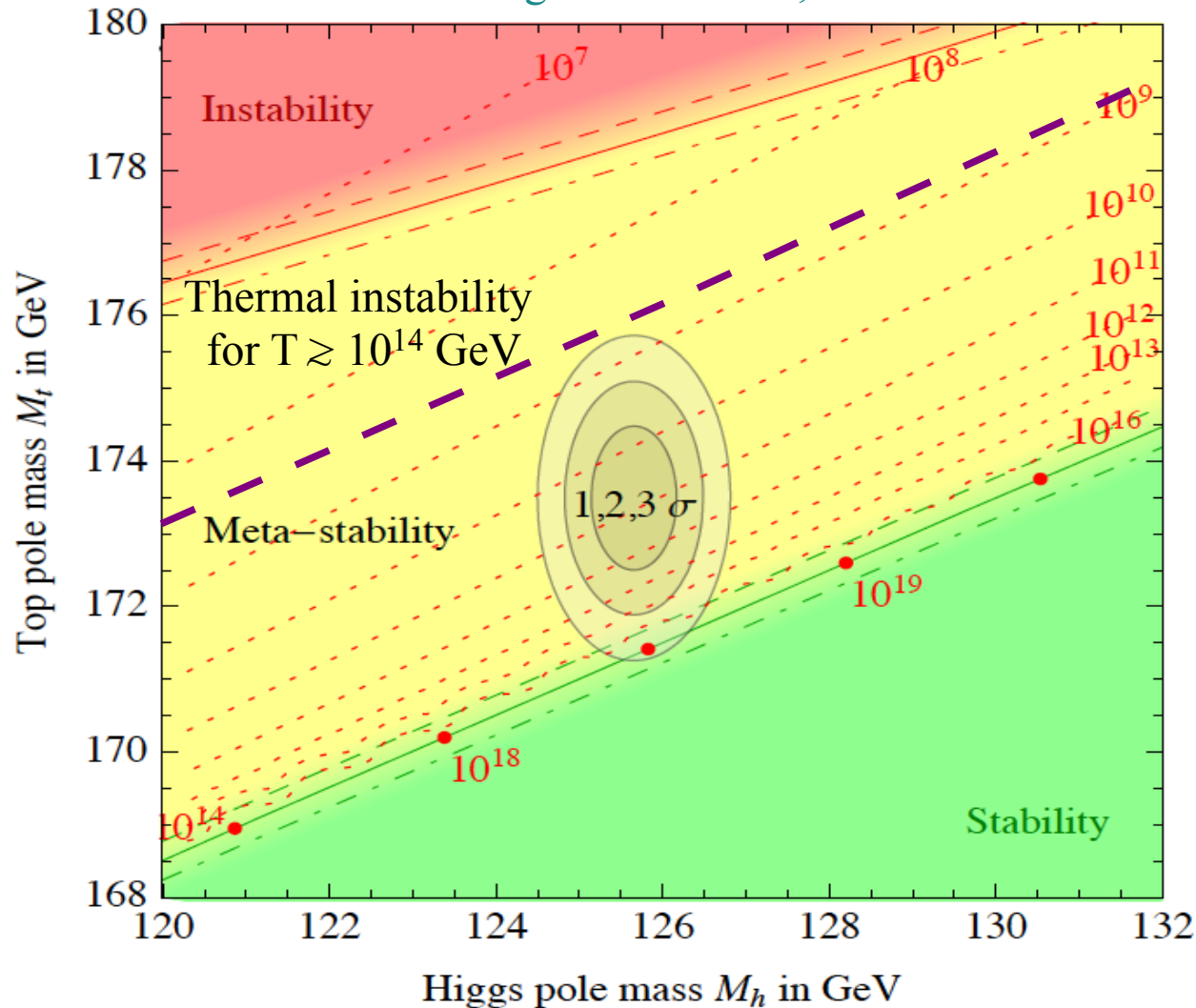
**N.B.:** This is not a standard “phase-diagram plot” [*we have not solved the model...*]

It denotes the stability/instability regions assuming no further destabilization from Planck-scale dynamics

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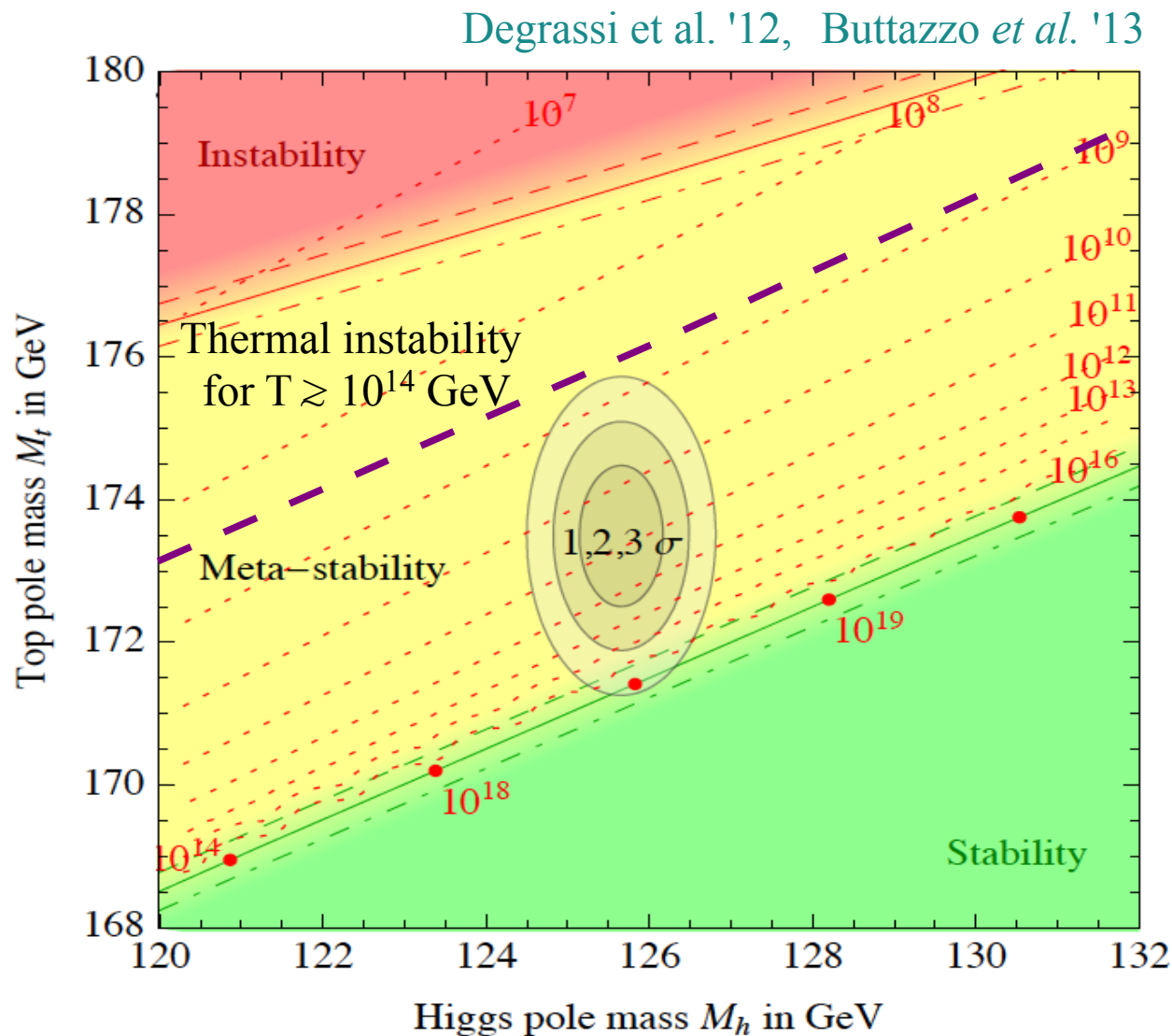


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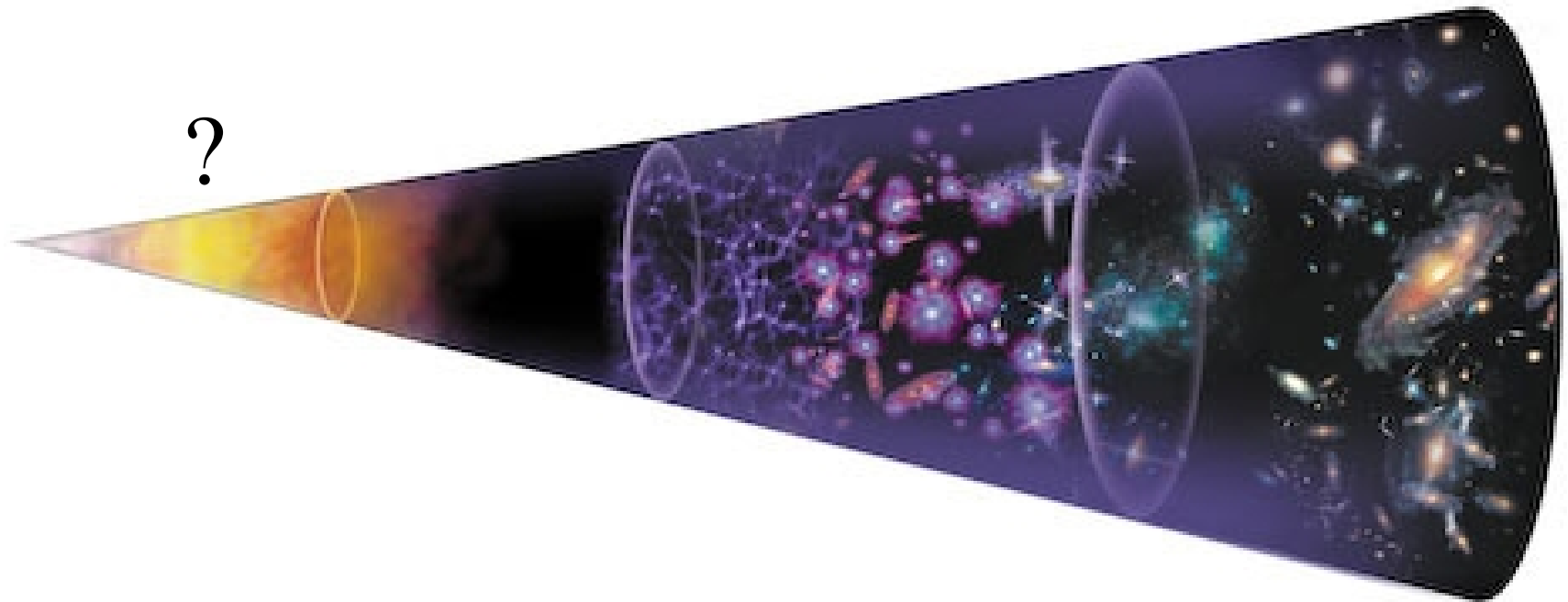
The error on  $m_h$  will soon go down  $\rightarrow$  main uncertainty induced by the top mass.

The  $m_t$  presently determined by ATLAS, CMS, Tevatron is not really the pole mass...

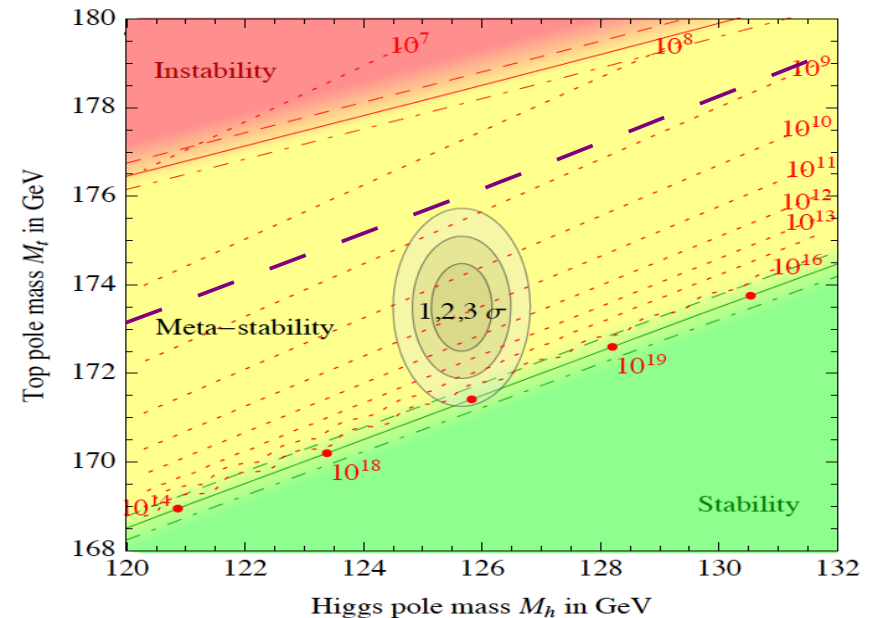
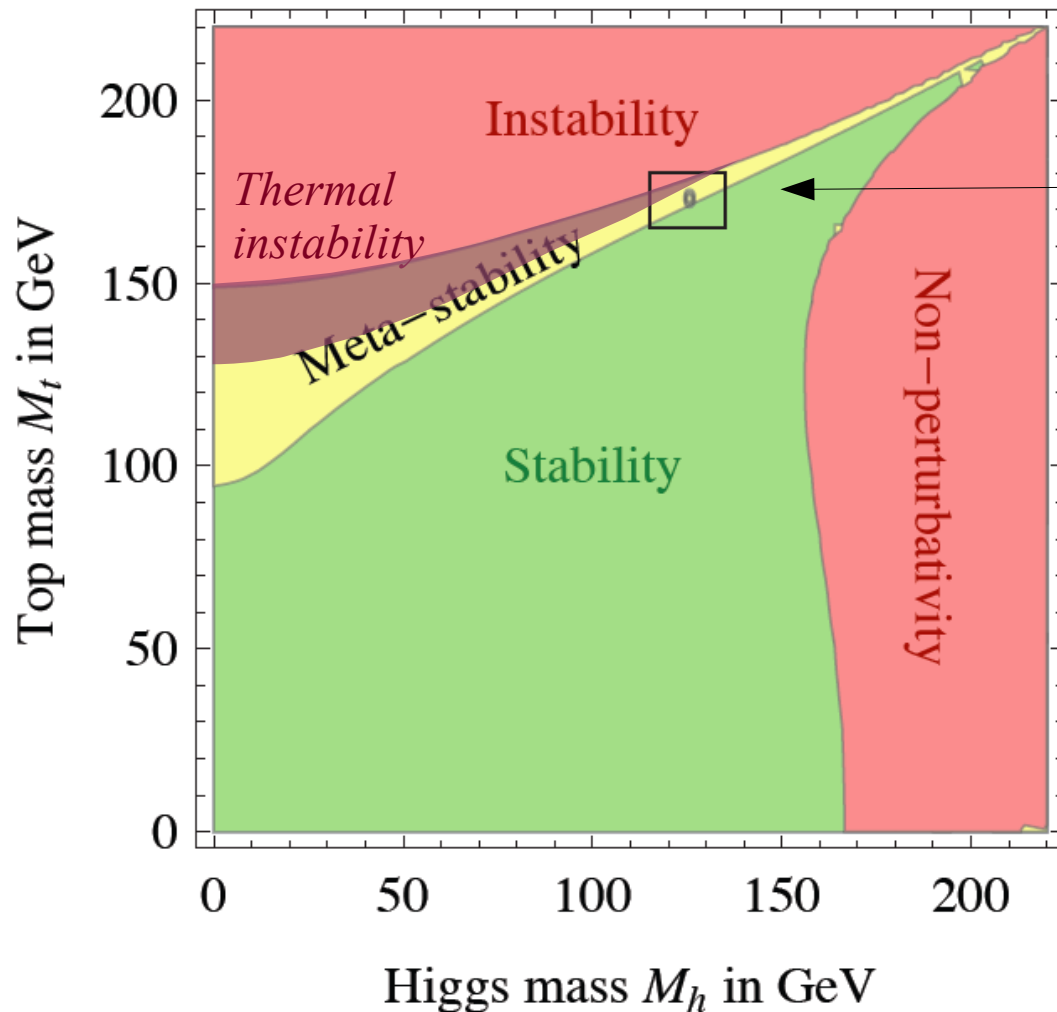
Hoang & Stewart, '07-'08  
Alekhin, Djouadi, Moch '12,

... but nice convergence to the same  $m_t$  of several indep. measurements by CMS & ATLAS  $\rightarrow$  small non-perturbative errors

► High-scale matching and SUSY



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Looking at the plane from a more distant perspective, it appears more clearly that “we live” in a quite “peculiar” region...

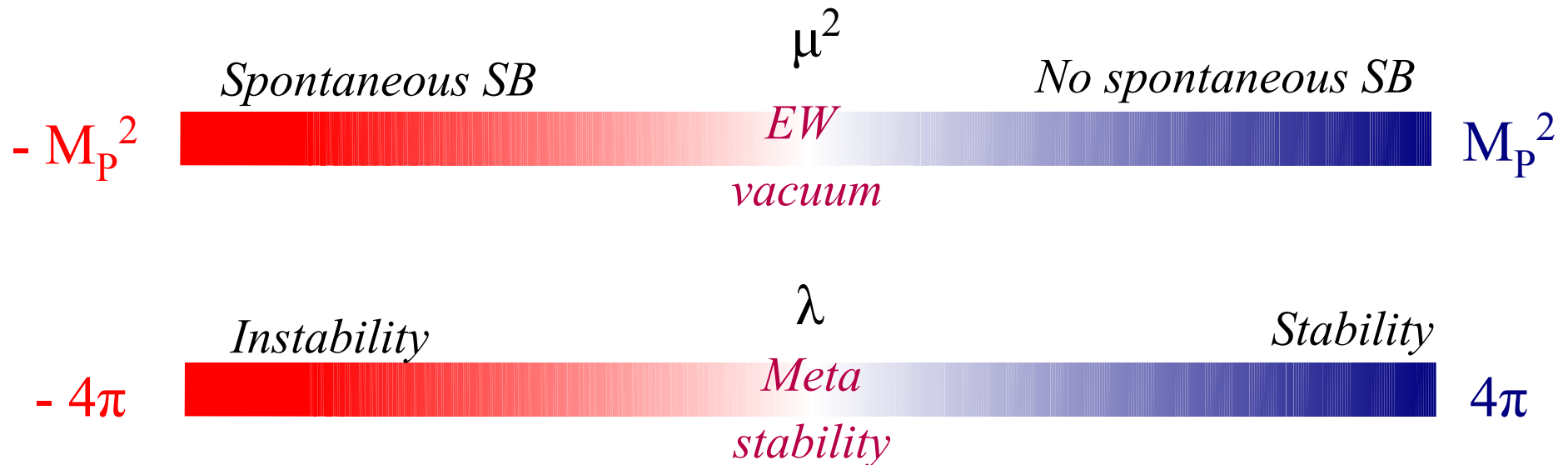
Moving  $m_t$  down by  $\sim 2$  GeV, we reach the even more peculiar configuration where  $\lambda(M_{pl})=0$

Froggatt, Nielsen, Takanishi, '01  
 Arkani-Hamed *et al.*, '08  
 Shaposhnikov, Wetterich, '10  
 +  
 many recent papers...

► High-scale matching and SUSY

It seems that the Higgs potential is “doubly tuned” around two “critical values”:

$$V(\phi) = -\mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2$$

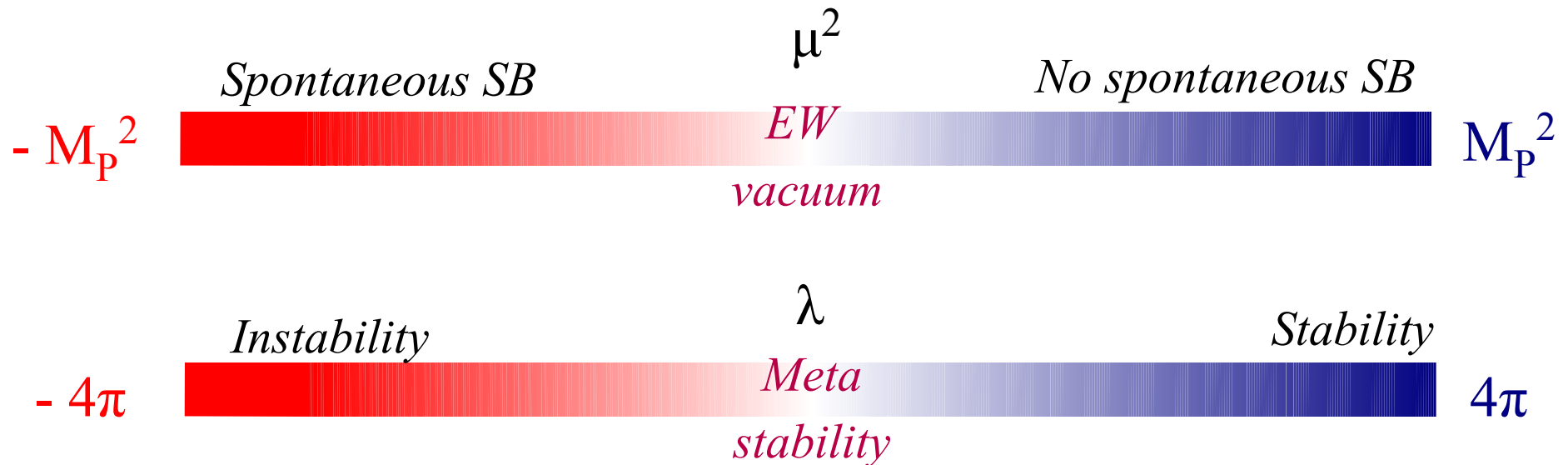




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Possible support in favor of the “Multiverse” [ → talk by Hall]

Close analogy with the cosmological constant? Maybe...

More dependence about the assumptions on the “pressure” in parameter space.

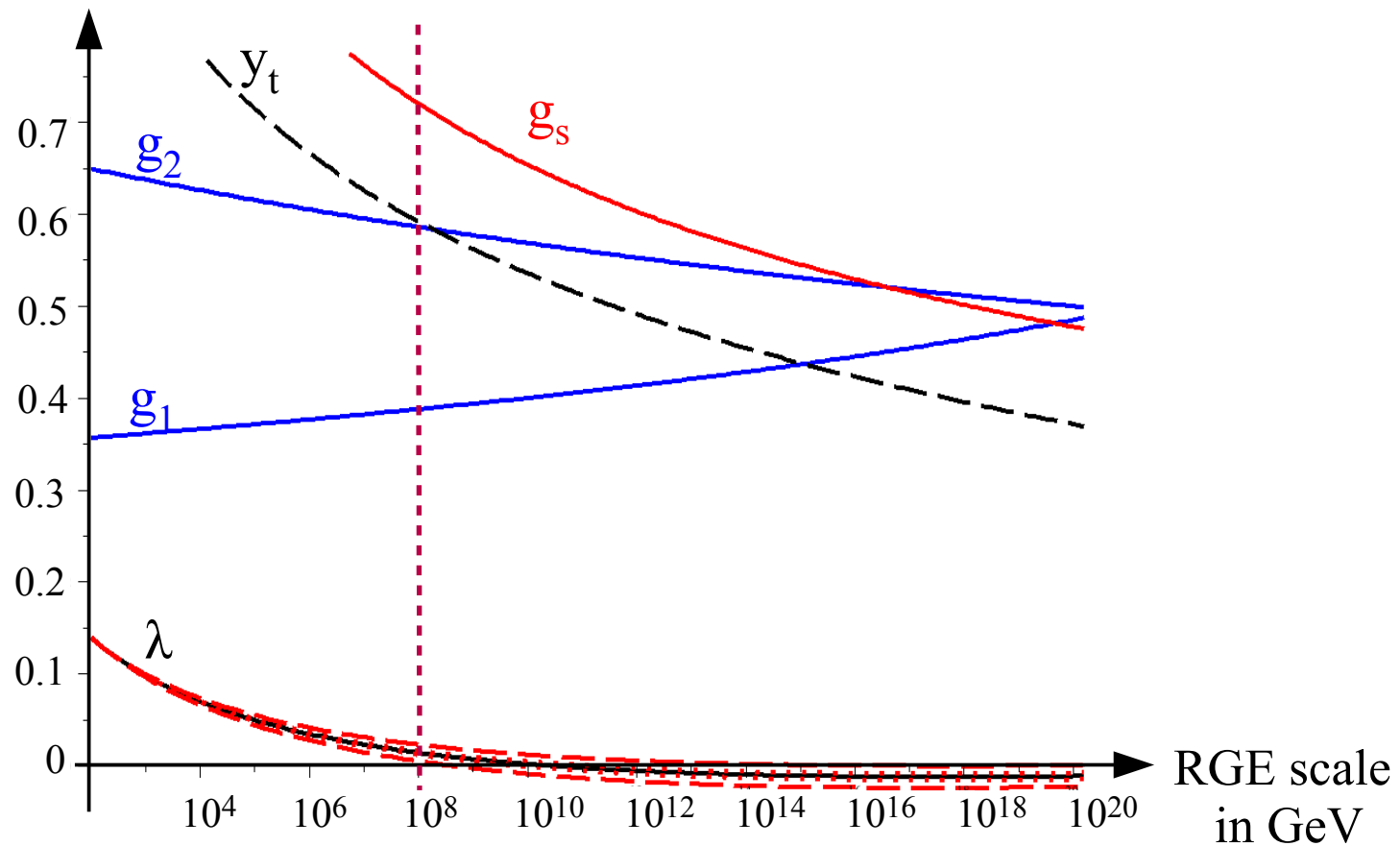
► High-scale matching and SUSY

What's special about  $\lambda(M_{\text{pl}})=0$ ?

Despite also the beta function vanishes, is not a true fixed point (other coupl.  $\neq 0$ ).

Maybe more interesting the overall smallness of  $\lambda$  compared to the other couplings.

At a scale  $\Lambda \gtrsim 10^8 \text{ GeV}$   $\lambda$  becomes of the same order of its typical e.w. quantum corrections: *hints of a radiatively generated coupling?*



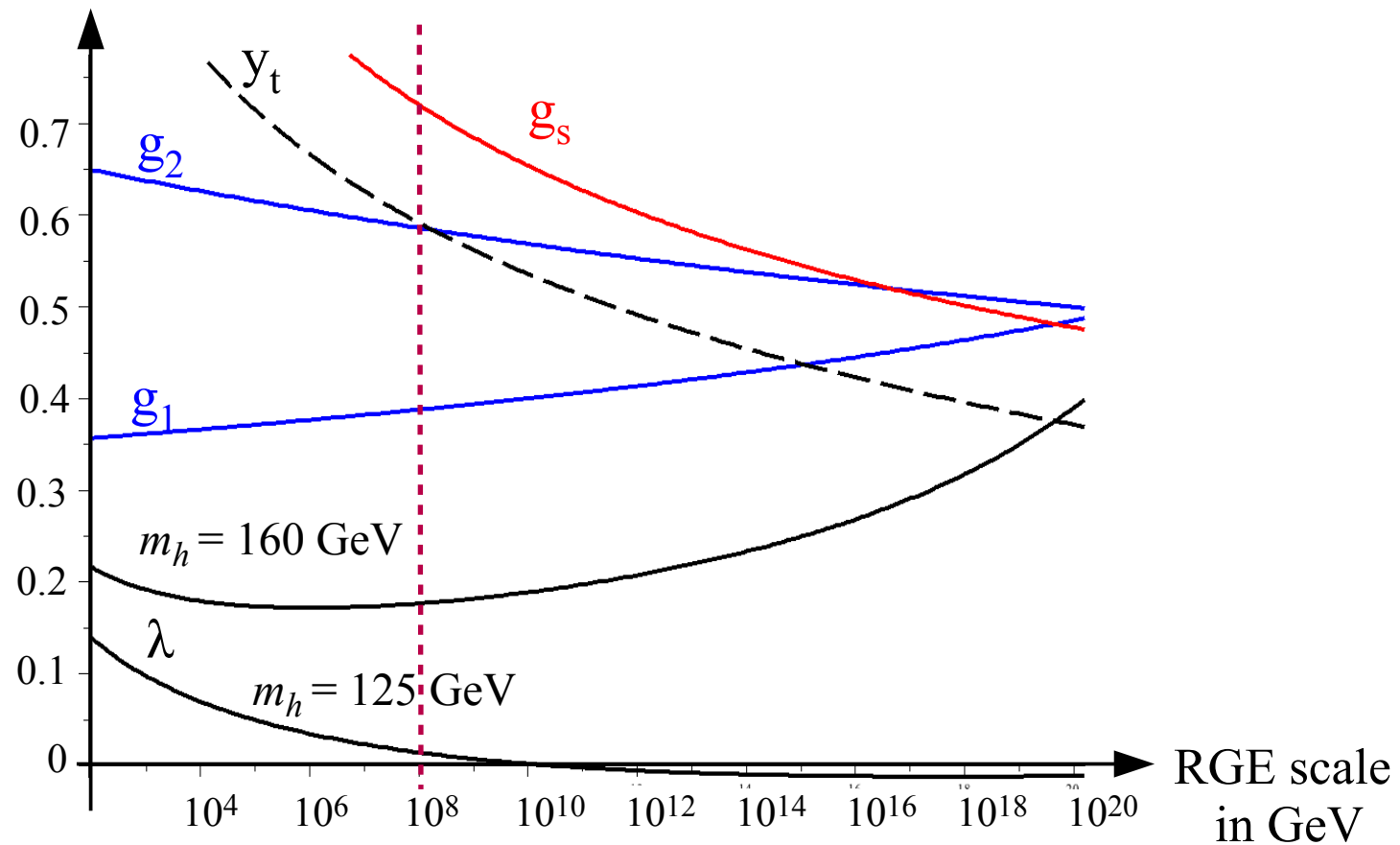
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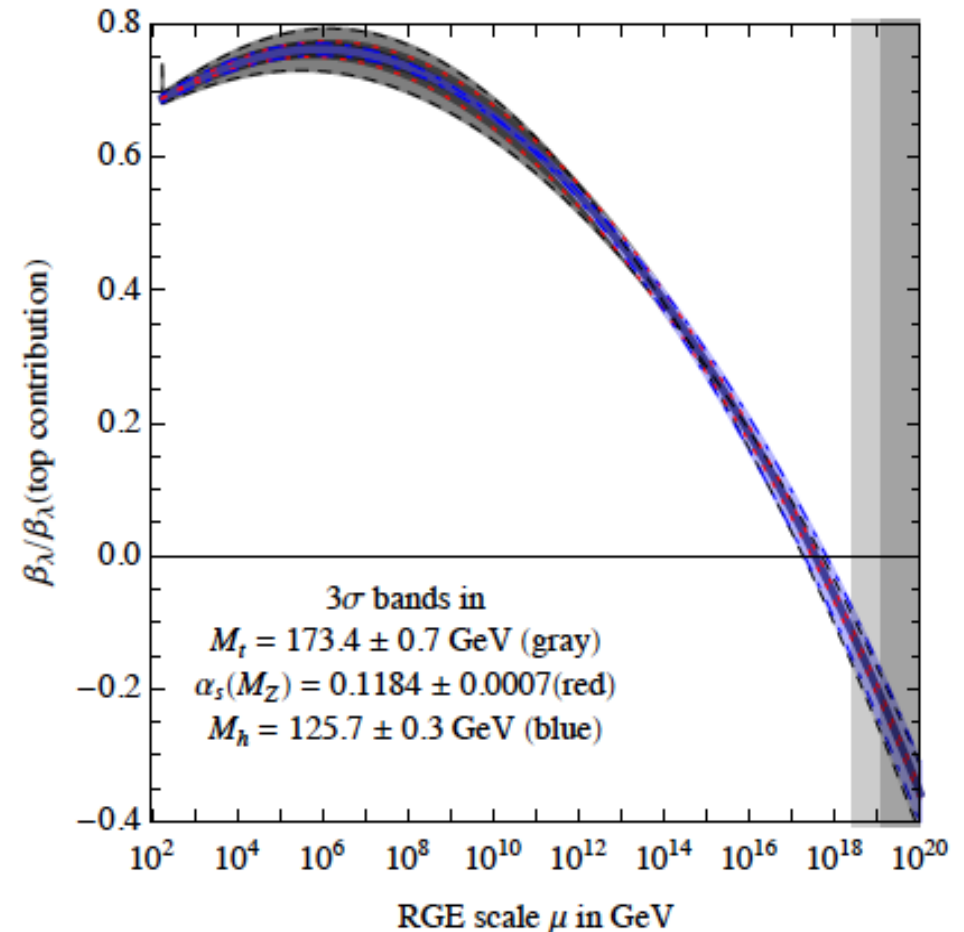
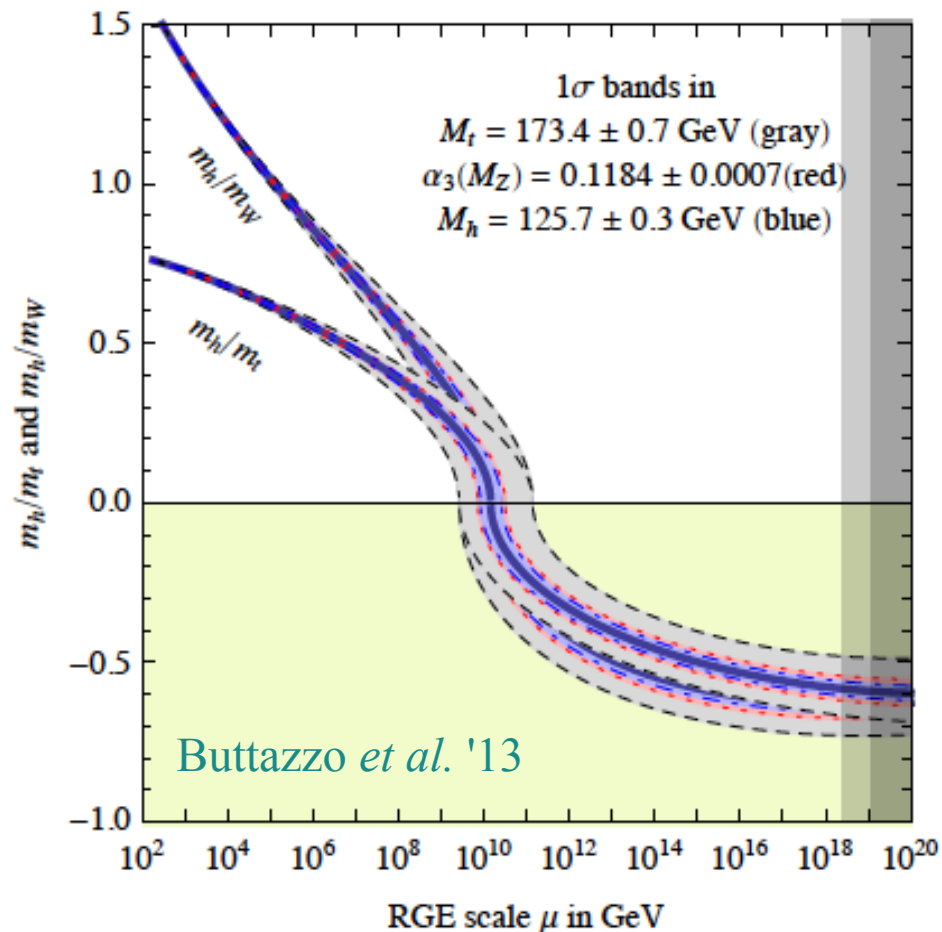


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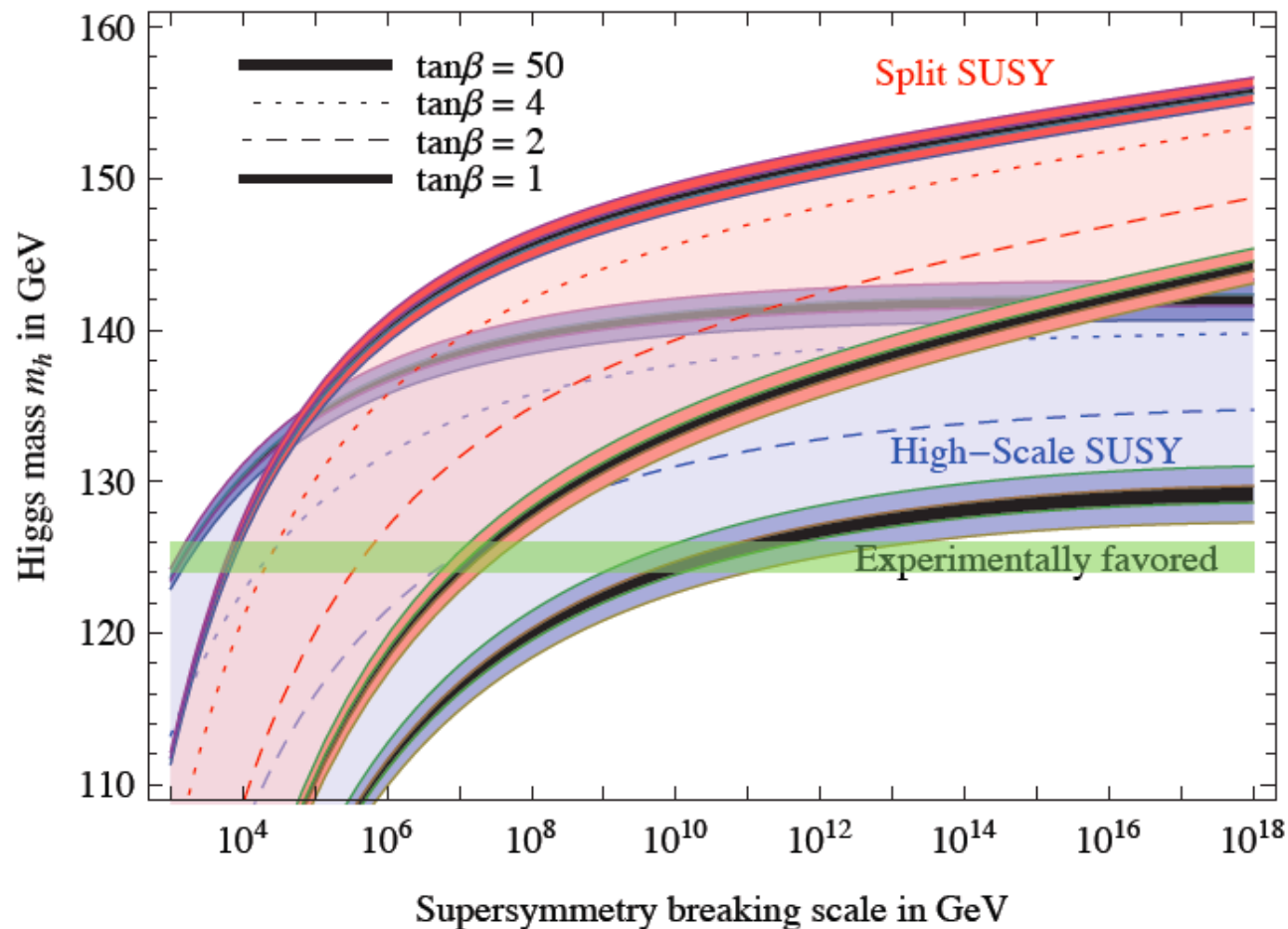
**N.B.:** using “natural units” the smallness of  $\lambda$  seems more “accidental”

## ► High-scale matching and SUSY

The smallness of  $\lambda$  certainly fits well with the possibility of a high-scale matching with a weakly coupled theory, such as various versions of SUSY  
 [*high-scale/split/mini-split...*]

Giudice & Strumia '11-'12

Arvanitaki, Craig, Dimopoulos, Villadoro, '12



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A closer look to the simplest case:

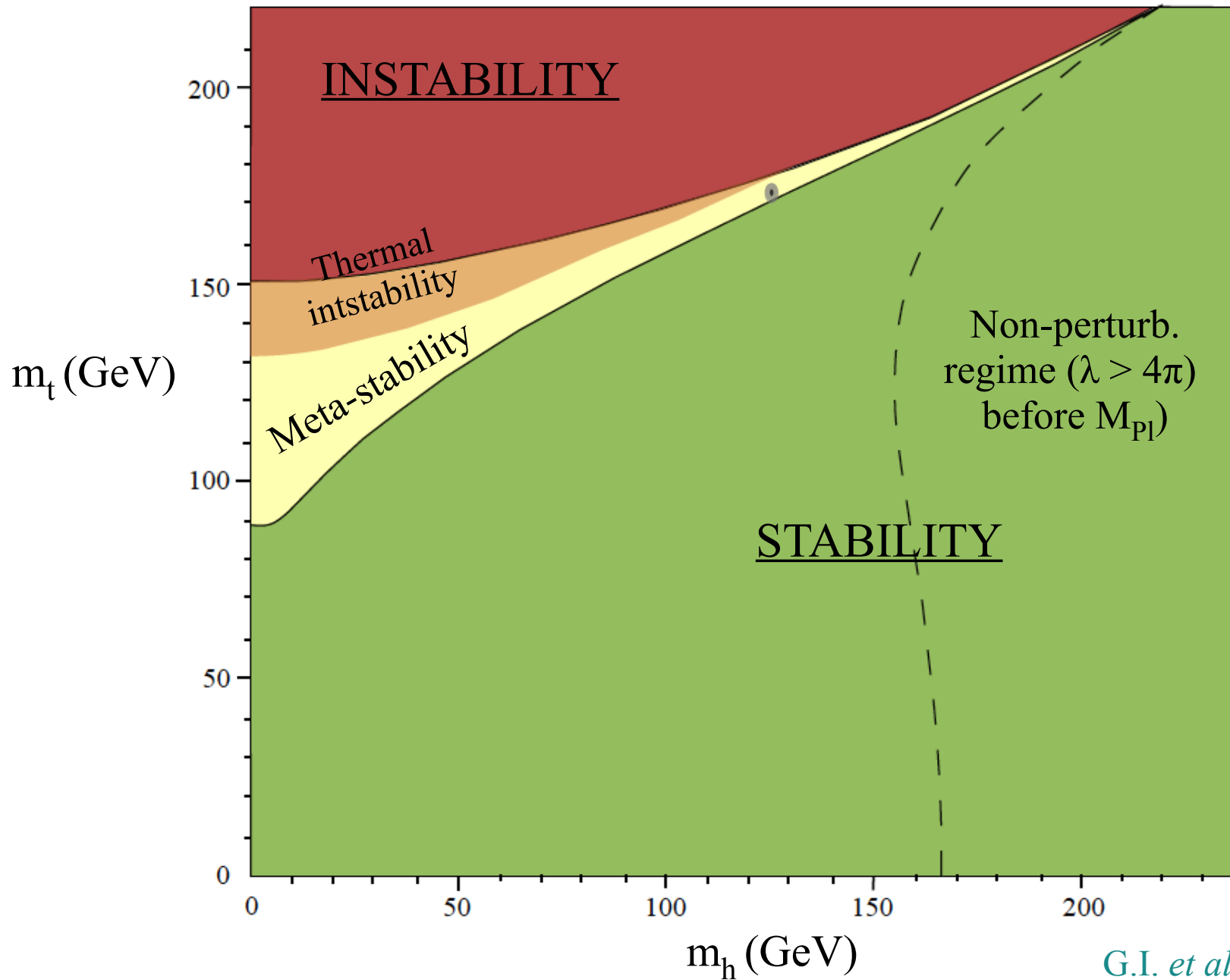
(unsplitted) high-scale SUSY  $\rightarrow$  common SUSY mass  $M_S$

To a good accuracy, the prediction for  $m_h$  is obtained imposing this UV matching condition,

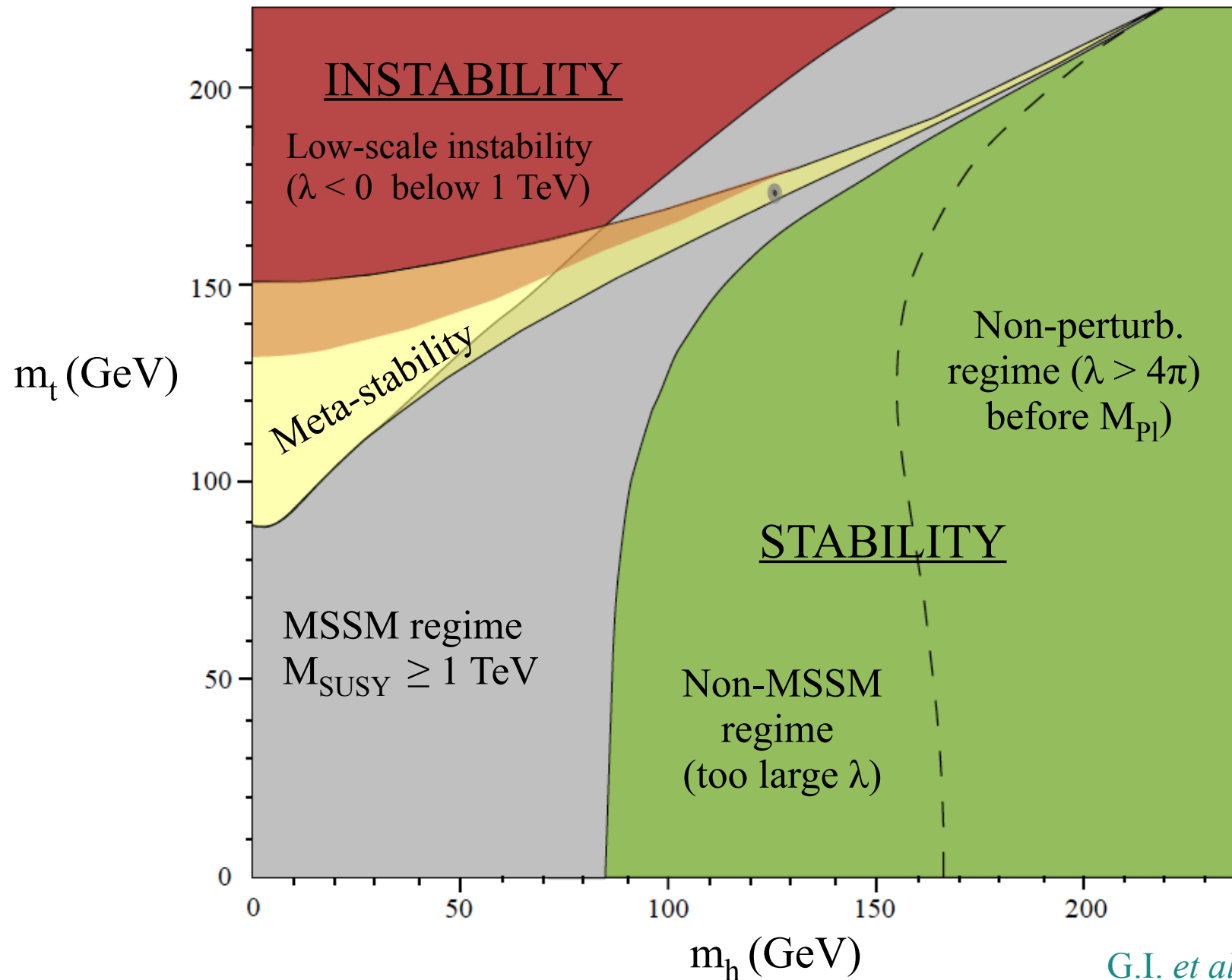
$$\lambda(M_S) = \frac{1}{8} [g^2(M_S) + g'^2(M_S)] \cos(2\beta) + \frac{3}{16\pi^2} y_t(M_S)^4 \tilde{X}_t \quad 0 < \tilde{X}_t < 6$$

and running down to the e.w. scale using SM RGE

► High-scale matching and SUSY

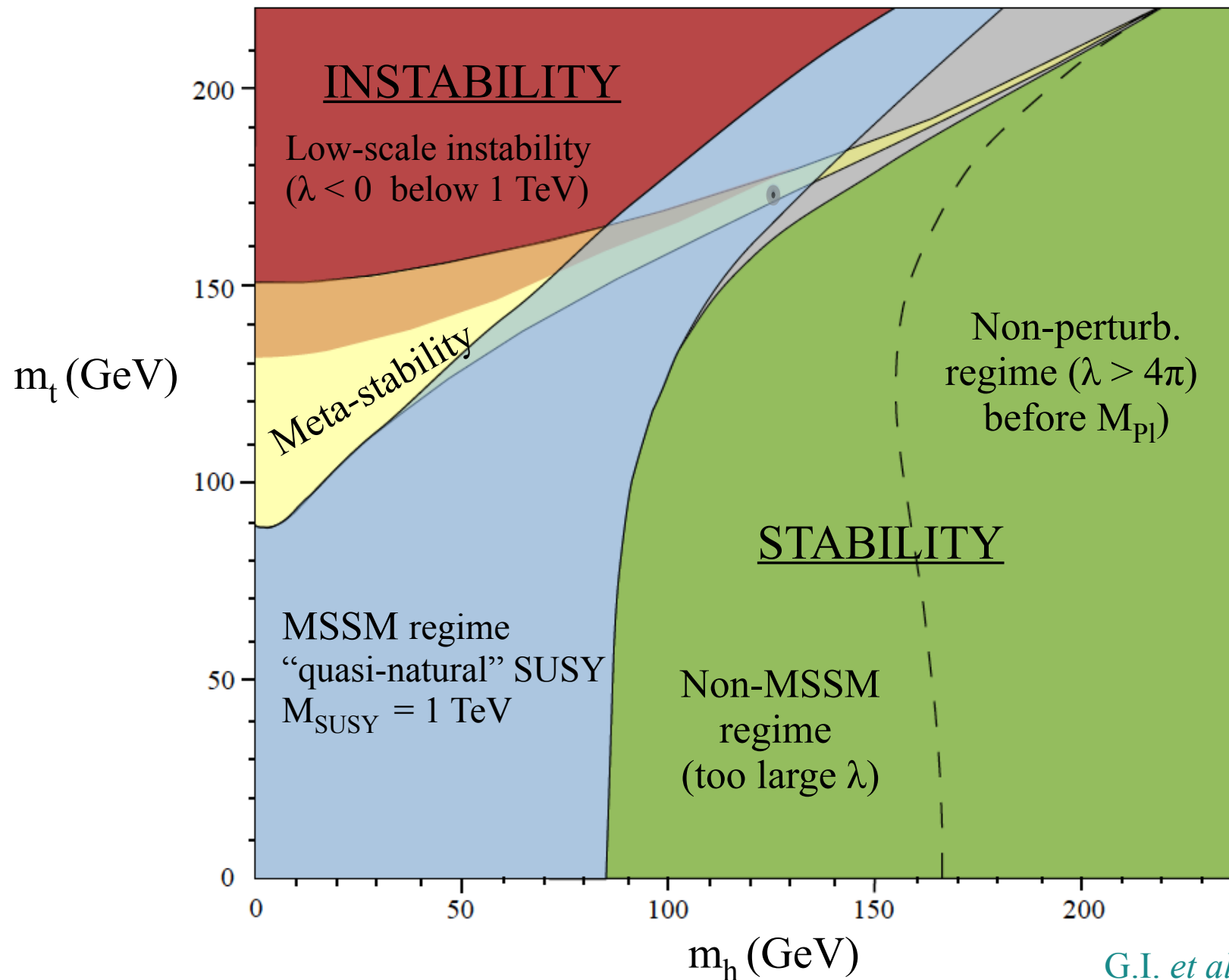


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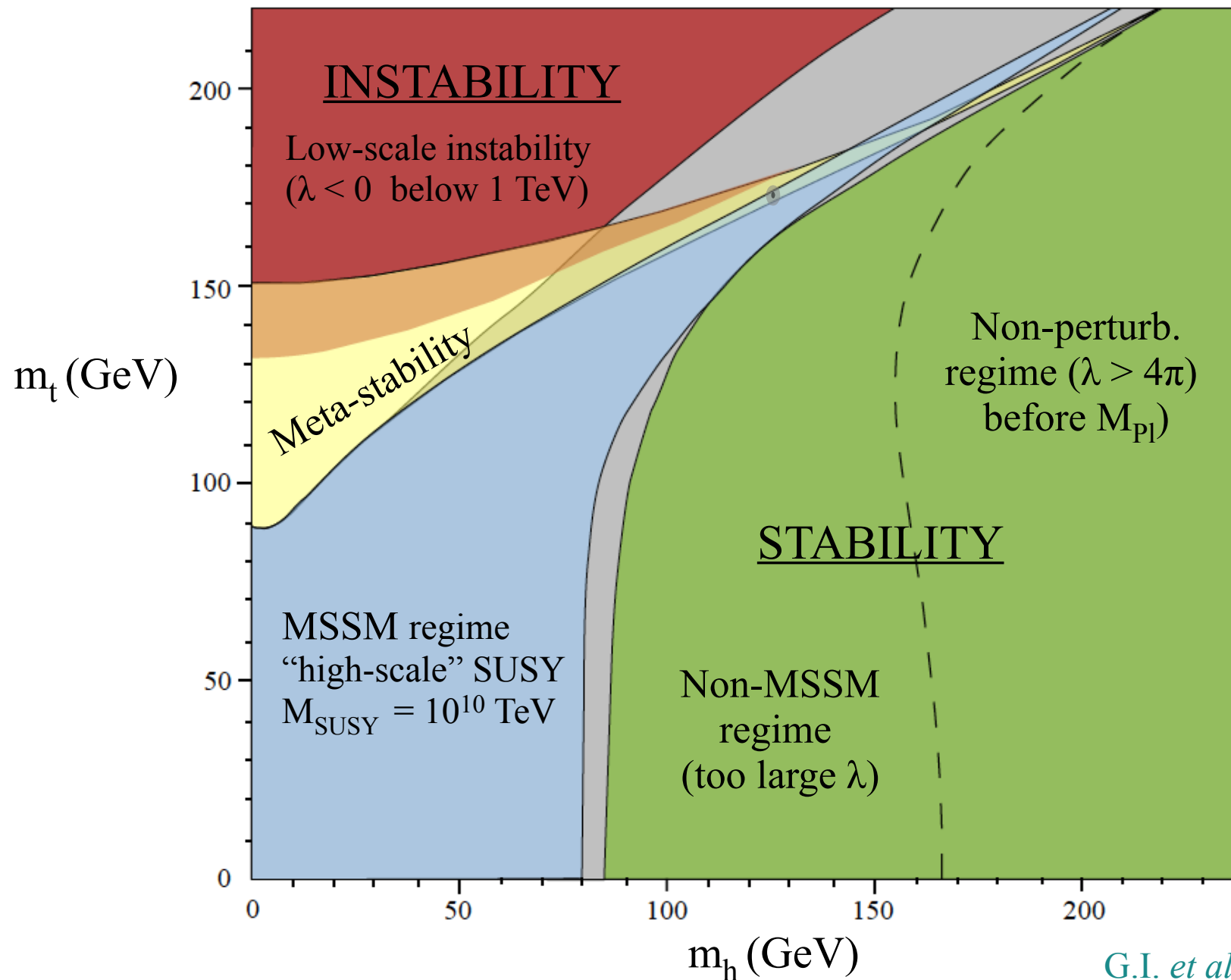




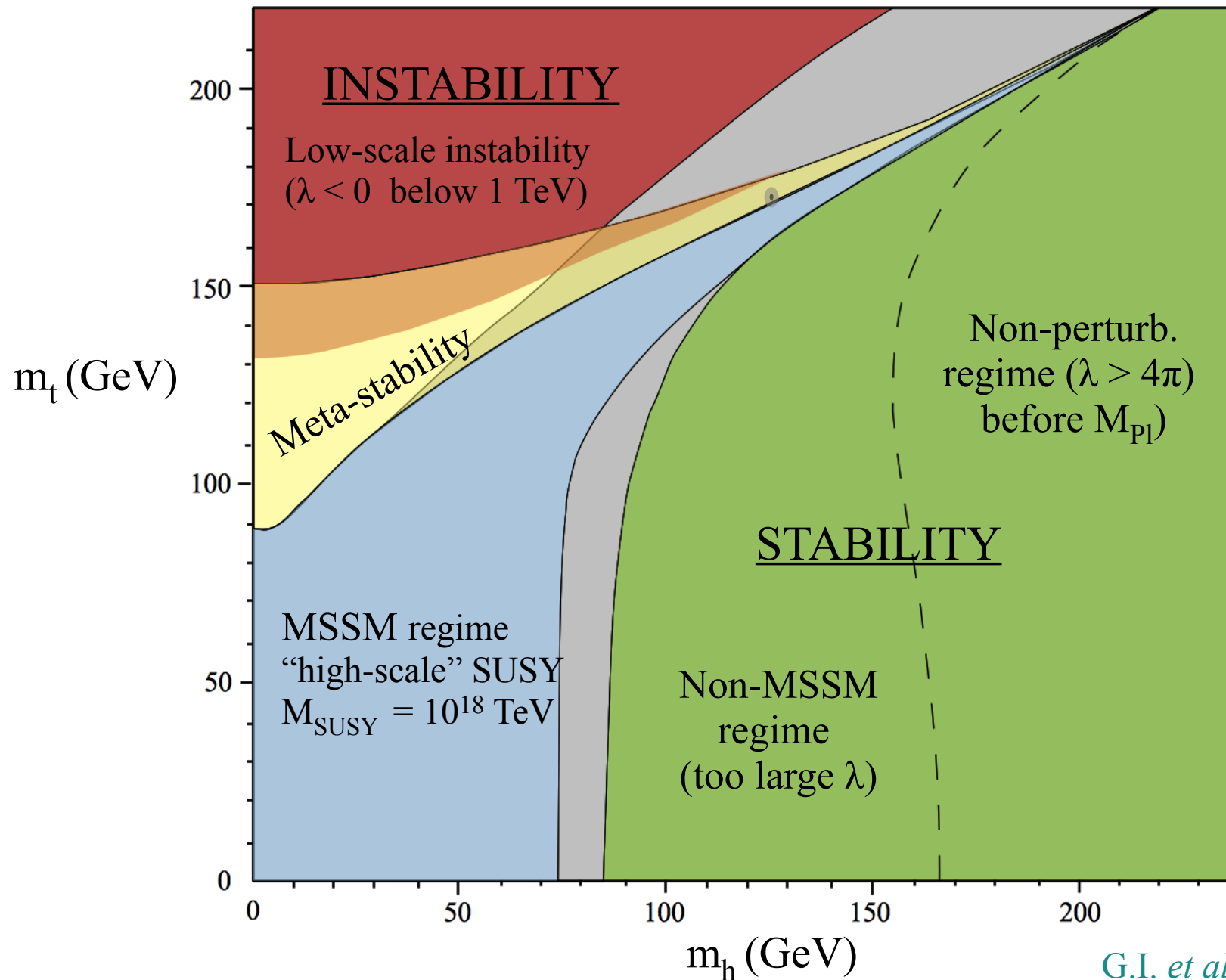
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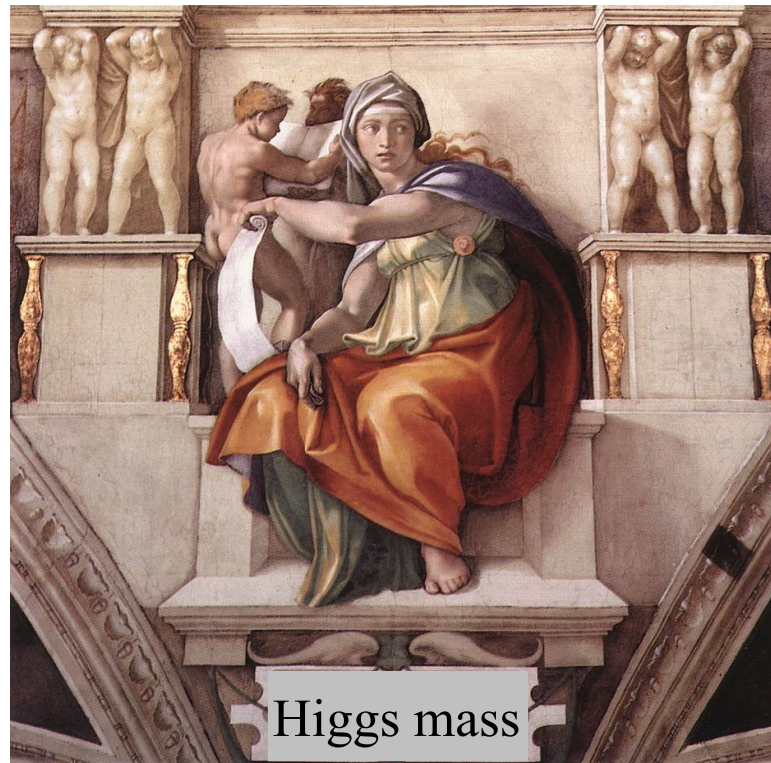


## ► Conclusions

- A SM-like Higgs with  $m_h \sim 125$  GeV does not allow us to derive model-independent conclusions about the scale of New Physics: **the SM Higgs potential is unstable but sufficiently long-lived.**
- Clear indication of a small  $\lambda$  at high energies: **SUSY remains an excellent candidate as UV completion of the SM**, but  $m_h$  alone leaves open a wide range of values for the SUSY breaking scale.
- The peculiar “doubly-critical” structure of the Higgs potential may be the indication some (*non-completely understood yet*) statistical phenomenon, as expected in the “Multiverse”.

► Conclusions

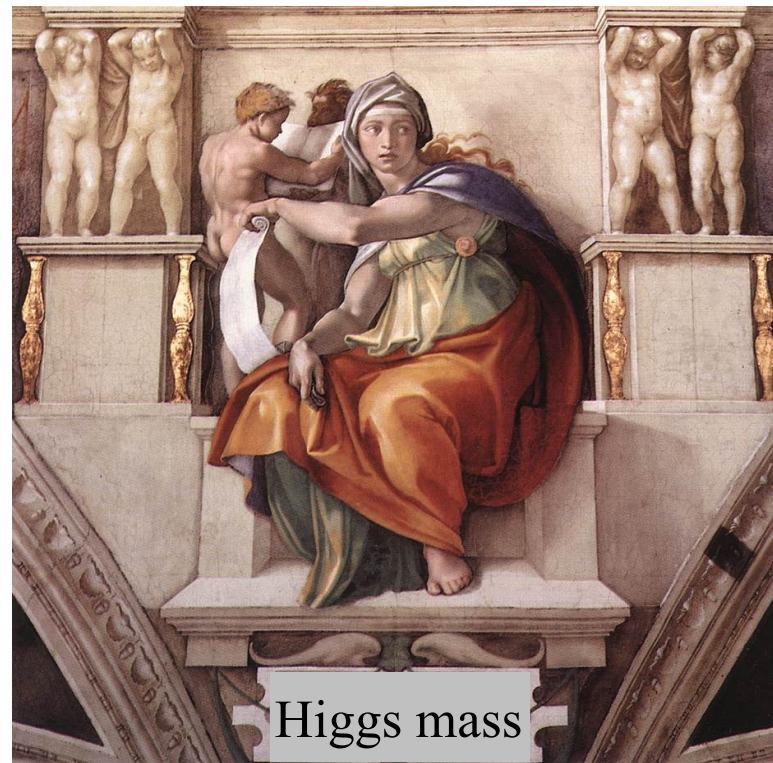
*My understanding of the “Higgs-mass oracle”:*



“ SM valid up to high scales No SUSY will show at LHC13 ”

► Conclusions

*My understanding of the “Higgs-mass oracle”:*



“ SM valid up to high scales No SUSY will show at LHC13 ”

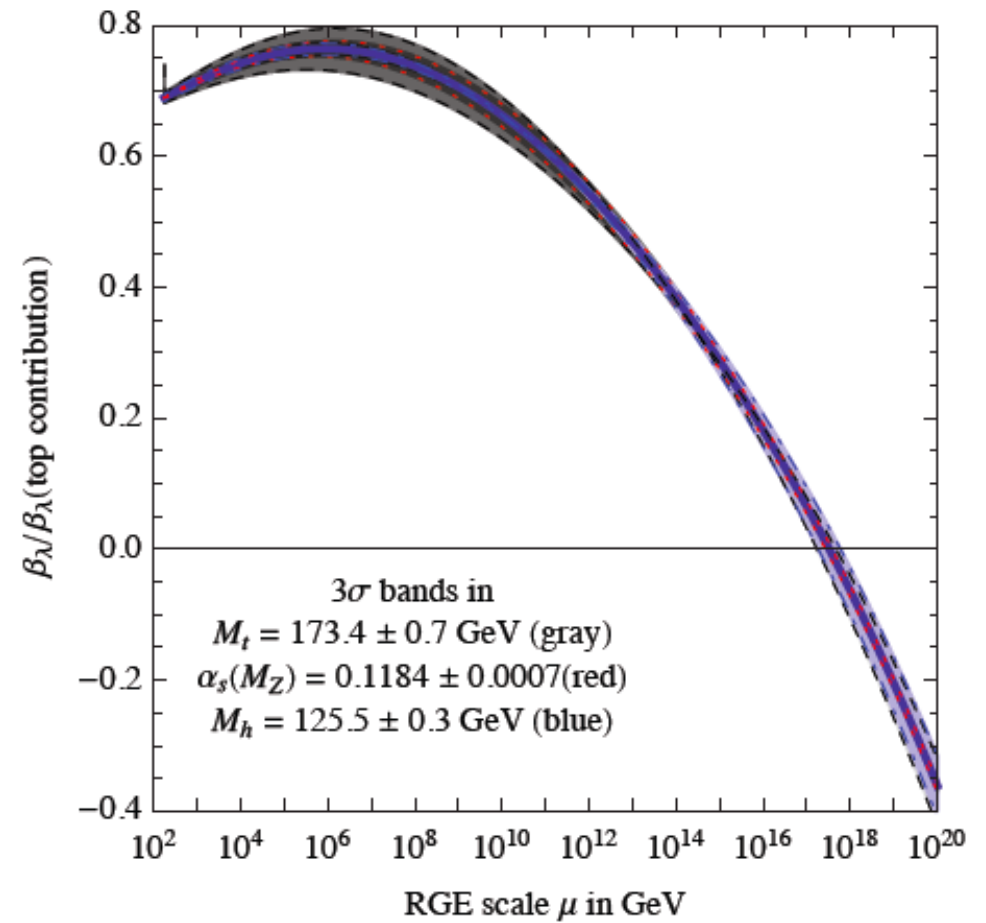
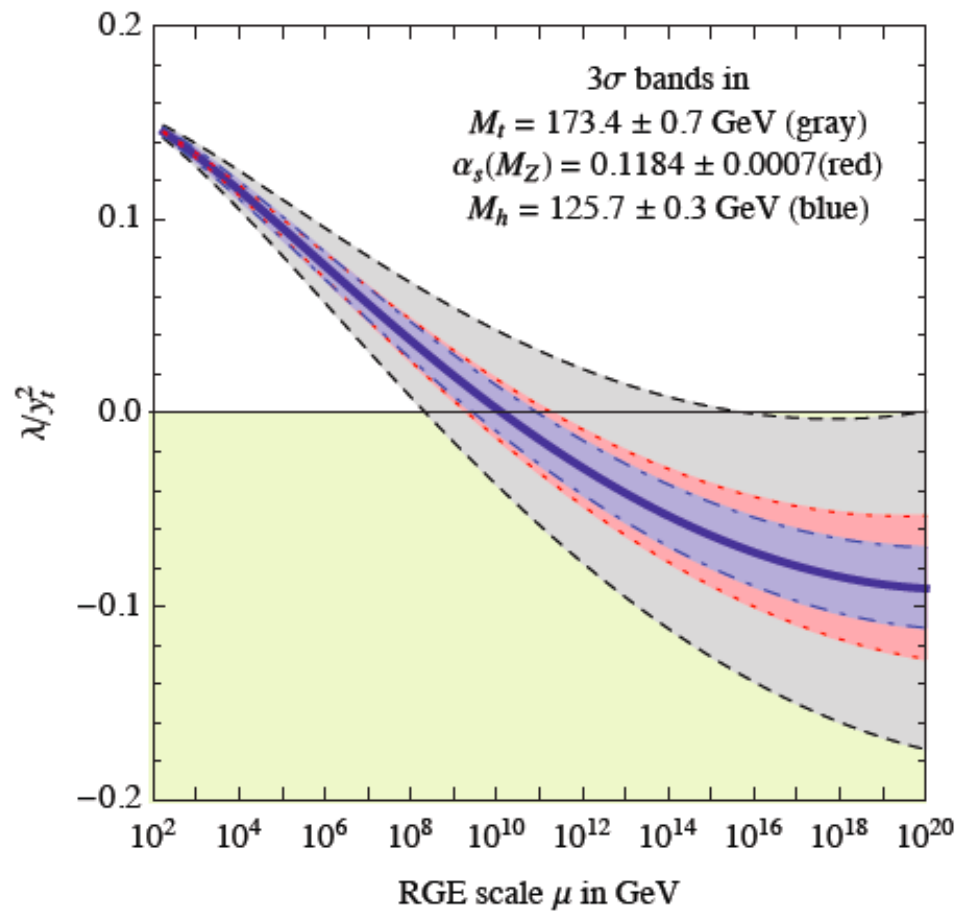


SM valid up to high scales.  
No SUSY will show at LHC13.



SM valid up to high scales?  
No: SUSY will show at LHC13!

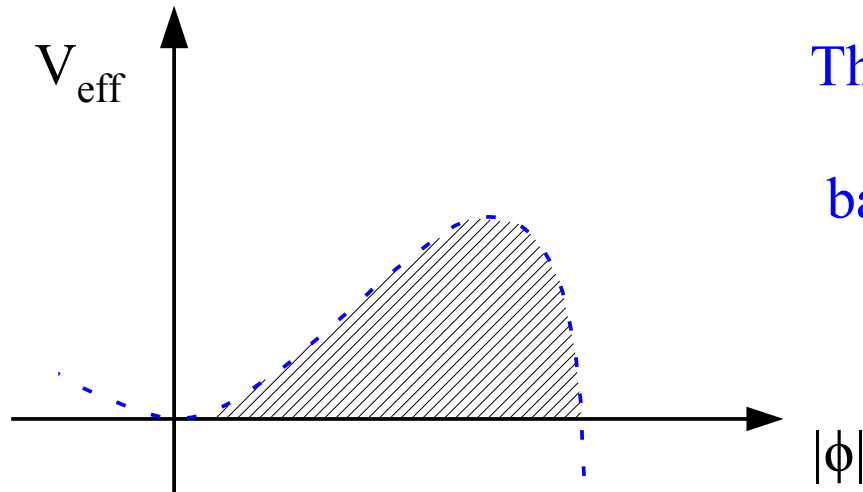




Rattazzi @ LHC-IT (Genova, 2013)



★ The quantum-tunneling rate:



The quantum tunneling occurs via bubble formation in the homogeneous background of the false (e.w.) minimum

Coleman '79

At the semi-classical level, the tunneling probability can be written as:

$$p \approx K e^{-S_0[\hbar]}$$

Volume factor

$K \propto T_U^4$  not exactly calculable within the semi-classical approx.

Euclidean action

$$\int \frac{1}{2} (\partial_\mu h)^2 + V(h)$$

*Bounce*

solution of the e.o.m. that interpolates between the false and the true vacuum

**N.B.:** within a QFT (system with infinite d.o.f.) the tunneling is suppressed even in absence of a potential barrier (kinematic barrier due to the boundary conditions)

★ The quantum-tunneling rate:

If we neglect the mass term, the tree-level Higgs potential is scale invariant & its bounces have a rather simple form:

$$h(r) = \left( \frac{2}{|\lambda|} \right)^{1/2} \frac{2R}{r^2 + R^2} \quad r = x_\mu x_\mu \quad O(4) \text{ invariant bounces minimize the action}$$

$R =$  arbitrary scale parameter

$$S_0[h] = \frac{8\pi^2}{3|\lambda|} \quad \rightarrow \quad P_{\text{semicl.}} \approx (T_U/R)^4 e^{-8\pi^2/3|\lambda|}$$

If  $|\lambda|$  remains sufficiently small, the tunneling rate can be very suppressed

**N.B.:** the tunneling rate is a pure non-perturbative phenomenon - cannot be computed to any finite order in “ordinary” perturbation theory [*wrong choice of the vacuum*]

★ The quantum-tunneling rate:

To go beyond the semi-classical level we need to take into account the quantum fluctuations around the (non-constant) bounce solution

Callan, Coleman '79

Non-trivial problem which has been solved (semi-analytically) in the SM case:

G.I., Ridolfi, Strumia '01



- Quantum corrections break scale invariance
- The tunneling is dominated by bounces of size  $R$ , such that  $\lambda(1/R)$  reaches its minimum value:

$$p = \max_R \frac{V_U}{R^4} \exp \left[ -\frac{8\pi^2}{3|\lambda(\mu)|} - \Delta S(\mu R) \right]$$

$\mu$  independent

$\Delta S \approx 0$  if we set  $\mu = 1/R$

★ The quantum-tunneling rate:

To go beyond the semi-classical level we need to take into account the quantum fluctuations around the (non-constant) bounce solution

Callan, Coleman '79

Non-trivial problem which has been solved (semi-analytically) in the SM case:

G.I., Ridolfi, Strumia '01



- Quantum corrections break scale invariance
- The tunneling is dominated by bounces of size  $R$ , such that  $\lambda(1/R)$  reaches its minimum value
- The critical  $R$  determine the reference scale of the volume pre-factor:

$$p \approx \max_R \frac{V_U}{R^4} \exp \left[ - \frac{8 \pi^2}{3 |\lambda(1/R)|} \right]$$

The leading gravitational effects are also calculable when  $1/R$  is not far from (but below)  $M_{\text{pl}}$

G.I., Rychkov, Strumia, Tetradis '08

Buttazzo *et al.* '13:

## Renormalisation Group Equations

	LO 1 loop	NLO 2 loop	NNLO 3 loop	NNNLO 4 loop
$g_3$	full [50,51]	$\mathcal{O}(\alpha_3^2)$ [52,53] $\mathcal{O}(\alpha_3\alpha_{1,2})$ [58] full [60]	$\mathcal{O}(\alpha_3^3)$ [54,55] $\mathcal{O}(\alpha_3^2\alpha_t)$ [59] full [61,62]	$\mathcal{O}(\alpha_3^4)$ [56,57]
$g_{1,2}$	full [50,51]	full [60]	full [61,62]	—
$y_t$	full [63]	$\mathcal{O}(\alpha_t^2, \alpha_3\alpha_t)$ [64] full [67]	full [65,66]	—
$\lambda, m^2$	full [63]	full [68,69]	full [70,71]	—

## Threshold corrections at the weak scale

	LO 0 loop	NLO 1 loop	NNLO 2 loop	NNNLO 3 loop
$g_2$	$2M_W/V$	full [72,73]	Work in progress	—
$g_Y$	$2\sqrt{M_Z^2 - M_W^2}/V$	full [72,73]	Work in progress	—
$y_t$	$\sqrt{2}M_t/V$	$\mathcal{O}(\alpha_3)$ [74] $\mathcal{O}(\alpha)$ [78]	$\mathcal{O}(\alpha_3^2, \alpha_3\alpha_{1,2})$ [33] full [This work]	$\mathcal{O}(\alpha_3^3)$ [75–77]
$\lambda$	$M_h^2/2V^2$	full [79]	for $g_{1,2} = 0$ [4] full [This work]	—
$m^2$	$M_h^2$	full [79]	full [This work]	—

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