Heterotic mini-landscape in blow-up

Nana Cabo Bizet

Center of Technological Applications
and Nuclear Development, La Habana, Cuba.

SUSY 2013 ICTP, August 27, 2013

Based on JHEP 190P0213 by Nana G. Cabo Bizet and Hans Peter Nilles.

We show the equivalence among heterotic string theory compactified on an orbifold deformed by vevs and on a smooth CY 3-fold.

We show the equivalence among heterotic string theory compactified on an orbifold deformed by vevs and on a smooth CY 3-fold.

Outline

Outline

1. Heterotic orbifold compactifications

2. T^6/Z_{6II} geometry

Outline

1. Heterotic orbifold compactifications
2. T^6/Z_{6II} geometry
3. Identifying the blow-up modes

Outline

1. Heterotic orbifold compactifications
2. T^6/Z_{6II} geometry
3. Identifying the blow-up modes
4. Matching the spectrum

Outline

1. Heterotic orbifold compactifications
2. T^6/Z_{6II} geometry
3. Identifying the blow-up modes
4. Matching the spectrum
5. Outlook and conclusions

Motivation

Heterotic string compactifications on Calabi-Yau manifolds with gauge bundles to 4d can lead to a promising UV completion of the $\mathcal{N} = 1$ supersymmetric standard model including gravity.

Motivation

Heterotic string compactifications on Calabi-Yau manifolds with gauge bundles to 4d can lead to a promising UV completion of the $\mathcal{N} = 1$ supersymmetric standard model including gravity.

Orbifolds are singular CY-varieties T^6/G with G a discrete symmetry of the torus and compactifications on them constitute a phenomenologically fertile region (for ex.mini-landscape) of the string landscape.[See Ramos-Sánchez's talk, See Oehlman's talk]

Motivation

Heterotic string compactifications on Calabi-Yau manifolds with gauge bundles to 4d can lead to a promising UV completion of the $\mathcal{N} = 1$ supersymmetric standard model including gravity.

Orbifolds are singular CY-varieties T^6/G with G a discrete symmetry of the torus and compactifications on them constitute a phenomenologically fertile region (for ex.mini-landscape) of the string landscape.[See Ramos-Sánchez's talk, See Oehlman's talk]

Moreover on orbifolds the spectrum and the interactions are exactly computable using conformal field theory methods.[See Mayorga-Peña's talk]

Motivation

Heterotic string compactifications on Calabi-Yau manifolds with gauge bundles to 4d can lead to a promising UV completion of the $\mathcal{N} = 1$ supersymmetric standard model including gravity.

Orbifolds are singular CY-varieties T^6/G with G a discrete symmetry of the torus and compactifications on them constitute a phenomenologically fertile region (for ex.mini-landscape) of the string landscape.[See Ramos-Sánchez's talk, See Oehlman's talk]

Moreover on orbifolds the spectrum and the interactions are exactly computable using conformal field theory methods.[See Mayorga-Peña's talk]

However orbifold compactifications represent only a measure zero subset in the moduli space of generically smooth CY compactifications.

Motivation

Heterotic string compactifications on Calabi-Yau manifolds with gauge bundles to 4d can lead to a promising UV completion of the $\mathcal{N}=1$ supersymmetric standard model including gravity.

Orbifolds are singular CY-varieties T^6/G with G a discrete symmetry of the torus and compactifications on them constitute a phenomenologically fertile region (for ex.mini-landscape) of the string landscape.[See Ramos-Sánchez's talk, See Oehlman's talk]

Moreover on orbifolds the spectrum and the interactions are exactly computable using conformal field theory methods.[See Mayorga-Peña's talk]

However orbifold compactifications represent only a measure zero subset in the moduli space of generically smooth CY compactifications.

Moreover heterotic orbifolds exhibit generally an anomalous U(1) symmetry generating a Fayet-Iliopoulus D-term [Fischler,Nilles,Polchinski,Raby,Susskind, 81]

$$D = \sum_{\phi} Q^{\phi}_{anom}|\phi|^2 + \xi, \ \xi \sim \mathrm{Tr} Q_{anom}.$$

Motivation

Heterotic string compactifications on Calabi-Yau manifolds with gauge bundles to 4d can lead to a promising UV completion of the $\mathcal{N}=1$ supersymmetric standard model including gravity.

Orbifolds are singular CY-varieties T^6/G with G a discrete symmetry of the torus and compactifications on them constitute a phenomenologically fertile region (for ex.mini-landscape) of the string landscape.[See Ramos-Sánchez's talk, See Oehlman's talk]

Moreover on orbifolds the spectrum and the interactions are exactly computable using conformal field theory methods.[See Mayorga-Peña's talk]

However orbifold compactifications represent only a measure zero subset in the moduli space of generically smooth CY compactifications.

Moreover heterotic orbifolds exhibit generally an anomalous U(1) symmetry generating a Fayet-Iliopoulus D-term [Fischler,Nilles,Polchinski,Raby,Susskind, 81]

$$D = \sum_{\phi} Q^{\phi}_{anom}|\phi|^2 + \xi, \ \xi \sim \mathrm{Tr} Q_{anom}.$$

4d $\mathcal{N}=1$ supersymmetry enforces $D=0$ and the scalar fields ϕ to acquire a vev. Scalar fields from the twisted sector can be identified with blow-up (bu) moduli smoothing the local singularities.

Motivation

Therefore the study of the transition between heterotic string on singular orbifold compactifications and smooth CY manifolds is essential and has been intensively studied starting with [Honecker, Trapletti, 06][Groot Nibbelink, Trapletti, Walter, 07] [G.Nibbelink *et al*,08][Blaszczyk, Nibbelink, Ruehle, Trapletti, Vaudrevange,10].

Motivation

Therefore the study of the transition between heterotic string on singular orbifold compactifications and smooth CY manifolds is essential and has been intensively studied starting with [Honecker, Trapletti, 06][Groot Nibbelink, Trapletti, Walter, 07] [G.Nibbelink *et al*,08][Blaszczyk, Nibbelink, Ruehle, Trapletti, Vaudrevange,10].

In our work we focus on the smoothing of the $T^6/\mathbb{Z}_{6II}$ orbifold with MSSM-like spectrum.

Motivation

Therefore the study of the transition between heterotic string on singular orbifold compactifications and smooth CY manifolds is essential and has been intensively studied starting with [Honecker, Trapletti, 06][Groot Nibbelink, Trapletti, Walter, 07] [G.Nibbelink *et al*,08][Blaszczyk, Nibbelink, Ruehle, Trapletti, Vaudrevange,10].

In our work we focus on the smoothing of the $T^6/\mathbb{Z}_{6II}$ orbifold with MSSM-like spectrum. We identify the deformed orbifold with the smooth CY, in particular:

- We identify orbifold twisted fields acquiring vevs which smooth the singularities: blow-up modes.

Motivation

Therefore the study of the transition between heterotic string on singular orbifold compactifications and smooth CY manifolds is essential and has been intensively studied starting with [Honecker, Trapletti, 06][Groot Nibbelink, Trapletti, Walter, 07] [G.Nibbelink *et al*,08][Blaszczyk, Nibbelink, Ruehle, Trapletti, Vaudrevange,10].

In our work we focus on the smoothing of the $T^6/\mathbb{Z}_{6II}$ orbifold with MSSM-like spectrum. We identify the deformed orbifold with the smooth CY, in particular:

- We identify orbifold twisted fields acquiring vevs which smooth the singularities: blow-up modes.
- The massless spectrum of the deformed orbifold and the massless spectrum of the smooth Calabi-Yau compactification are identified.

Motivation

Therefore the study of the transition between heterotic string on singular orbifold compactifications and smooth CY manifolds is essential and has been intensively studied starting with [Honecker, Trapletti, 06][Groot Nibbelink, Trapletti, Walter, 07] [G.Nibbelink *et al*,08][Blaszczyk, Nibbelink, Ruehle, Trapletti, Vaudrevange,10].

In our work we focus on the smoothing of the $T^6/\mathbb{Z}_{6II}$ orbifold with MSSM-like spectrum. We identify the deformed orbifold with the smooth CY, in particular:

- We identify orbifold twisted fields acquiring vevs which smooth the singularities: blow-up modes.
- The massless spectrum of the deformed orbifold and the massless spectrum of the smooth Calabi-Yau compactification are identified.
- The transition is understood studying the 4d anomaly cancellation mechanism from both perspectives.

Motivation

Therefore the study of the transition between heterotic string on singular orbifold compactifications and smooth CY manifolds is essential and has been intensively studied starting with [Honecker, Trapletti, 06][Groot Nibbelink, Trapletti, Walter, 07] [G.Nibbelink *et al*,08][Blaszczyk, Nibbelink, Ruehle, Trapletti, Vaudrevange,10].

In our work we focus on the smoothing of the $T^6/\mathbb{Z}_{6II}$ orbifold with MSSM-like spectrum. We identify the deformed orbifold with the smooth CY, in particular:

- We identify orbifold twisted fields acquiring vevs which smooth the singularities: blow-up modes.
- The massless spectrum of the deformed orbifold and the massless spectrum of the smooth Calabi-Yau compactification are identified.
- The transition is understood studying the 4d anomaly cancellation mechanism from both perspectives.

A similar program has been carried out in the $T^6/\mathbb{Z}_7$. [Blazscyk, C.B., Nilles and Ruehle,11]

Motivation

Therefore the study of the transition between heterotic string on singular orbifold compactifications and smooth CY manifolds is essential and has been intensively studied starting with [Honecker, Trapletti, 06][Groot Nibbelink, Trapletti, Walter, 07] [G.Nibbelink *et al*,08][Blaszczyk, Nibbelink, Ruehle, Trapletti, Vaudrevange,10].

In our work we focus on the smoothing of the $T^6/\mathbb{Z}_{6II}$ orbifold with MSSM-like spectrum. We identify the deformed orbifold with the smooth CY, in particular:

- We identify orbifold twisted fields acquiring vevs which smooth the singularities: blow-up modes.
- The massless spectrum of the deformed orbifold and the massless spectrum of the smooth Calabi-Yau compactification are identified.
- The transition is understood studying the 4d anomaly cancellation mechanism from both perspectives.

A similar program has been carried out in the $T^6/\mathbb{Z}_7$. [Blazscyk, C.B., Nilles and Ruehle,11]

A crucial difference in our case as in all realistic orbifolds is the presence of fixed tori.

Motivation

Therefore the study of the transition between heterotic string on singular orbifold compactifications and smooth CY manifolds is essential and has been intensively studied starting with [Honecker, Trapletti, 06][Groot Nibbelink, Trapletti, Walter, 07] [G.Nibbelink *et al*,08][Blaszczyk, Nibbelink, Ruehle, Trapletti, Vaudrevange,10].

In our work we focus on the smoothing of the $T^6/\mathbb{Z}_{6II}$ orbifold with MSSM-like spectrum. We identify the deformed orbifold with the smooth CY, in particular:

- We identify orbifold twisted fields acquiring vevs which smooth the singularities: blow-up modes.
- The massless spectrum of the deformed orbifold and the massless spectrum of the smooth Calabi-Yau compactification are identified.
- The transition is understood studying the 4d anomaly cancellation mechanism from both perspectives.

A similar program has been carried out in the $T^6/\mathbb{Z}_7$. [Blazscyk, C.B., Nilles and Ruehle,11]

A crucial difference in our case as in all realistic orbifolds is the presence of fixed tori.

A previous work on the $T^6/\mathbb{Z}_{6II}$ has been performed in [Groot Nibbelink, Held, Ruehle, Trapletti, Vaudrevange, 09] and in [Büchmuller, Louis, Schmidt, Valandro, 12] without Wilson lines. Here we study the correspondence further by completely matching of the spectrum and the anomalies.

Orbifolds: *A Mini-landscape*

Orbifolds: *A Mini-landscape*

Heterotic orbifold compactifications

The heterotic string theory can be compactified by modding out the heterotic space–time by a discrete group $H \subset (\mathbb{R}(9,1) \rtimes SO(9,1)) \times (E_8 \times E_8 \text{ lat. isom.})$.

H has the geometrical generators (θ, l), where θ denotes an isometric rotation of T^6 and l denotes translations in T^6. The transformation acts on the target space fields:

$$\begin{aligned} X^k &\rightarrow \theta^{kn} X^n + l^k,\ \tilde{\psi}^k \rightarrow \theta^{kn} \tilde{\psi}^n,\ k = 5, ..., 10, \\ X_L^I &\rightarrow X_L^I + V^I + A^I, I = 1, ..., 16, \end{aligned}$$

V, A represents the embedding in the gauge d.o.f. of the rotations and translations respectively. In a lattice basis $\{e_\alpha\}$ we have $l = n_\alpha e_\alpha$ and $A = n_\alpha A_\alpha$.

The gauge embedding is restricted by modular invariance and by consistent physical states transformation.

$\mathbb{Z}_N$ orbifolds: $\theta = \exp(2\pi i(v_1 J_{45} + v_2 J_{67} + v_3 J_{89}))$, where $J_{2i+2,2i+3}$ are the rotation generators in the three complex planes $i = 1, 2, 3$, with $\sum_i v_i = 0$.

Heterotic orbifold compactifications

The heterotic string theory can be compactified by modding out the heterotic space–time by a discrete group $H \subset (\mathbb{R}(9,1) \rtimes SO(9,1)) \times (E_8 \times E_8 \text{ lat. isom.})$.

H has the geometrical generators (θ, l), where θ denotes an isometric rotation of T^6 and l denotes translations in T^6. The transformation acts on the target space fields:

$$\begin{aligned} X^k &\to \theta^{kn} X^n + l^k,\ \tilde{\psi}^k \to \theta^{kn} \tilde{\psi}^n,\ k = 5, ..., 10, \\ X_L^I &\to X_L^I + V^I + A^I, I = 1, ..., 16, \end{aligned}$$

V, A represents the embedding in the gauge d.o.f. of the rotations and translations respectively. In a lattice basis $\{e_\alpha\}$ we have $l = n_\alpha e_\alpha$ and $A = n_\alpha A_\alpha$.

The gauge embedding is restricted by modular invariance and by consistent physical states transformation.

$\mathbb{Z}_N$ orbifolds: $\theta = \exp(2\pi i (v_1 J_{45} + v_2 J_{67} + v_3 J_{89}))$, where $J_{2i+2,2i+3}$ are the rotation generators in the three complex planes $i = 1, 2, 3$, with $\sum_i v_i = 0$.

Heterotic orbifold compactifications

The states are specified by their boundary conditions $g = (\theta^k, n_\alpha e_\alpha)$, which determine fixed points in the target space $f = \theta^k f + n_\alpha e_\alpha$. The $g-$boundary conditions for the bosonic coordinates read $X^i(\sigma+\pi) = \theta^k X^i(\sigma) + n_\alpha e_\alpha$.

Untwisted states $k = 0$: $|q\rangle_R \times \tilde{\alpha}|p\rangle_L$, with $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

Twisted states $k \neq 0$: $|q_{sh}\rangle_R \times \tilde{\alpha}|p_{sh}\rangle_L$, with $q_{sh} = q + kv$, $p_{sh} = p + V_g$, $V_g = kV + n_\alpha A_\alpha$, $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

The massless modes fulfill the level matching condition

$$\frac{p_{sh}^2}{2} + N - 1 + \delta c = \frac{q_{sh}^2}{2} - \frac{1}{2} + \delta c = 0.$$

with δc the vacuum energy.

g–twisted physical states have to fulfill the orbifold projection i.e. they have to be invariante under h with $[g, h] = 0$.

Heterotic orbifold compactifications

The states are specified by their boundary conditions $g = (\theta^k, n_\alpha e_\alpha)$, which determine fixed points in the target space $f = \theta^k f + n_\alpha e_\alpha$. The $g-$boundary conditions for the bosonic coordinates read $X^i(\sigma + \pi) = \theta^k X^i(\sigma) + n_\alpha e_\alpha$.

Untwisted states $k = 0$: $|q\rangle_R \times \tilde{\alpha}|p\rangle_L$, with $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

Twisted states $k \neq 0$: $|q_{sh}\rangle_R \times \tilde{\alpha}|p_{sh}\rangle_L$, with $q_{sh} = q + kv$, $p_{sh} = p + V_g$, $V_g = kV + n_\alpha A_\alpha$, $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

The massless modes fulfill the level matching condition

$$\frac{p_{sh}^2}{2} + N - 1 + \delta c = \frac{q_{sh}^2}{2} - \frac{1}{2} + \delta c = 0.$$

with δc the vacuum energy.

g–twisted physical states have to fulfill the orbifold projection i.e. they have to be invariante under h with $[g, h] = 0$.

Heterotic orbifold compactifications

The states are specified by their boundary conditions $g = (\theta^k, n_\alpha e_\alpha)$, which determine fixed points in the target space $f = \theta^k f + n_\alpha e_\alpha$. The $g-$boundary conditions for the bosonic coordinates read $X^i(\sigma + \pi) = \theta^k X^i(\sigma) + n_\alpha e_\alpha$.

Untwisted states $k = 0$: $|q\rangle_R \times \tilde{\alpha}|p\rangle_L$, with $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

Twisted states $k \neq 0$: $|q_{sh}\rangle_R \times \tilde{\alpha}|p_{sh}\rangle_L$, with $q_{sh} = q + kv$, $p_{sh} = p + V_g$, $V_g = kV + n_\alpha A_\alpha$, $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

The massless modes fulfill the level matching condition

$$\frac{p_{sh}^2}{2} + N - 1 + \delta c = \frac{q_{sh}^2}{2} - \frac{1}{2} + \delta c = 0.$$

with δc the vacuum energy.

g–twisted physical states have to fulfill the orbifold projection i.e. they have to be invariante under h with $[g, h] = 0$.

Heterotic orbifold compactifications

The states are specified by their boundary conditions $g = (\theta^k, n_\alpha e_\alpha)$, which determine fixed points in the target space $f = \theta^k f + n_\alpha e_\alpha$. The $g-$boundary conditions for the bosonic coordinates read $X^i(\sigma + \pi) = \theta^k X^i(\sigma) + n_\alpha e_\alpha$.

Untwisted states $k = 0$: $|q\rangle_R \times \tilde{\alpha}|p\rangle_L$, with $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

Twisted states $k \neq 0$: $|q_{sh}\rangle_R \times \tilde{\alpha}|p_{sh}\rangle_L$, with $q_{sh} = q + kv$, $p_{sh} = p + V_g$, $V_g = kV + n_\alpha A_\alpha$, $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

The massless modes fulfill the level matching condition

$$\frac{p_{sh}^2}{2} + N - 1 + \delta c = \frac{q_{sh}^2}{2} - \frac{1}{2} + \delta c = 0.$$

with δc the vacuum energy.

g–twisted physical states have to fulfill the orbifold projection i.e. they have to be invariante under h with $[g, h] = 0$.

Heterotic orbifold compactifications

The states are specified by their boundary conditions $g = (\theta^k, n_\alpha e_\alpha)$, which determine fixed points in the target space $f = \theta^k f + n_\alpha e_\alpha$. The $g-$boundary conditions for the bosonic coordinates read $X^i(\sigma + \pi) = \theta^k X^i(\sigma) + n_\alpha e_\alpha$.

Untwisted states $k = 0$: $|q\rangle_R \times \tilde{\alpha}|p\rangle_L$, with $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

Twisted states $k \neq 0$: $|q_{sh}\rangle_R \times \tilde{\alpha}|p_{sh}\rangle_L$, with $q_{sh} = q + kv$, $p_{sh} = p + V_g$, $V_g = kV + n_\alpha A_\alpha$, $q \in SO(8)$, $p \in \Gamma_8 \times \Gamma_8$.

The massless modes fulfill the level matching condition

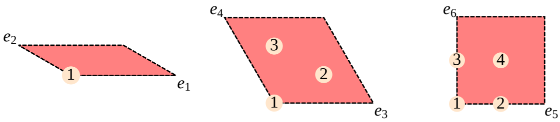
$$\frac{p_{sh}^2}{2} + N - 1 + \delta c = \frac{q_{sh}^2}{2} - \frac{1}{2} + \delta c = 0.$$

with δc the vacuum energy.

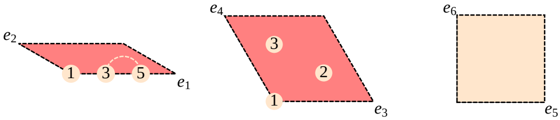
g–twisted physical states have to fulfill the orbifold projection i.e. they have to be invariante under h with $[g, h] = 0$.

T^6/Z_{6II} geometry

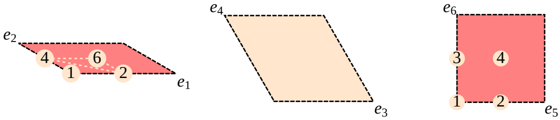
The orbifold action θ is generated by the shift $v = (1/6, 1/3, -1/2)$. The labels in the planes 1, 2 and 3 denote α, β and γ, respectively. There are:
There are 12 fixed points of the θ sector which are $\mathbb{C}^3/\mathbb{Z}_{6II}$ local singularities.



There are 6 fixed tori of the θ^2 and θ^4 sectors which are non-local $\mathbb{C}^2/\mathbb{Z}_3$ singularities.



There are 8 fixed tori of the θ^3 sector which are non-local $\mathbb{C}^2/\mathbb{Z}_2$ singularities.



$T^6/\mathbb{Z}_{6II}$ gauge embedding and spectrum

- The model is defined by the following gauge embedding

$$
\begin{aligned}
V &= \left(\frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, 0^5, \frac{1}{2}, -\frac{1}{6}, -\frac{1}{2}^{5}, \frac{1}{2}\right),\\
A_5 &= \left(-\frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, \frac{15}{4}, -\frac{19}{4}, -\frac{15}{4}^{4}, -\frac{11}{4}, \frac{19}{4}\right),\\
A_3 &= A_4 = \left(\frac{1}{6}, \frac{1}{6}, -\frac{1}{2}, \frac{1}{6}^{5}, \frac{5}{3}, -\frac{2}{3}, -\frac{5}{3}^{4}, -\frac{1}{3}, \frac{8}{3}\right).
\end{aligned}
$$

- This leads to the 4d gauge group $SU(3) \times SU(2) \times SU(6) \times U(1)^8$. The non-abelian charges of the massless spectrum are:

irrep.	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{3},\mathbf{1},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{6})$	$(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$	$(\mathbf{3},\mathbf{2},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{2},\mathbf{1})$
mult.	114	19	22	16	7	7	1	4

[Lebedev, Nilles, Raby, Ramos-Sánchez, Ratz, Vaudrevange and Wingerter,06]

- The anomaly polynomial can be written as

$$I = F_1(\text{tr}\, F^2_{su(6)} + \text{tr}\, F^2_{su(3)} + \text{tr}\, F^2_{su(2)} + \text{tr}\, R^2 + a_{ij} F_i F_j)$$

From this we can see that a single universal axion cancels the anomaly.

Smooth Calabi-Yau 3-fold

blow-up

Smooth Calabi-Yau 3-fold

blow-up

Resolution of $T^6/\mathbb{Z}_{6II}$

- The $\mathbb{C}^n/\mathbb{Z}_N$ singularities and their resolutions can be described by toric geometry or equivalent by the abelian gauge linear sigma model construction [Groot Nibbelink, 11] [Blaszczyk, Groot Nibbelink, Ruehle, 11].

- The toric variety is given by $X_\Sigma = (\mathbb{C}^n - Z(\Sigma))/G$ with exclusion set $Z(\Sigma)$ to ensure proper G–orbits, $G = \ker\phi$ with

$$\phi_n : (\mathbb{C}^*)^n \to (\mathbb{C}^*)^k, \quad (t_1, ..., t_n) \to (\prod_{j=1}^{n} t_j^{v_{j1}}, ..., \prod_{j=1}^{n} t_j^{v_{jk}}).$$

- The vectors v_i represent ordinary divisors $D_i = \{z_i = 0\}$.

- For singular varieties the v_i do not generate a lattice $\mathbb{Z}^k$.

- The resolution is achieved by subdividing the diagram with vectors v_{E_r} corresponding to new coordinates y_r such that exceptional divisors appear as $E_r = \{y_r = 0\}$. Note that for a CY resolution all the v_{E_r} lie on a hyperplane.

Resolution of $T^6/\mathbb{Z}_{6II}$

- The $\mathbb{C}^n/\mathbb{Z}_N$ singularities and their resolutions can be described by toric geometry or equivalent by the abelian gauge linear sigma model construction [Groot Nibbelink, 11] [Blaszczyk, Groot Nibbelink, Ruehle, 11].

- The toric variety is given by $X_\Sigma = (\mathbb{C}^n - Z(\Sigma))/G$ with exclusion set $Z(\Sigma)$ to ensure proper G–orbits, $G = \ker\phi$ with

$$\phi_n : (\mathbb{C}^*)^n \to (\mathbb{C}^*)^k, \quad (t_1, ..., t_n) \to (\prod_{j=1}^{n} t_j^{v_{j1}}, ..., \prod_{j=1}^{n} t_j^{v_{jk}}).$$

- The vectors v_i represent ordinary divisors $D_i = \{z_i = 0\}$.

- For singular varieties the v_i do not generate a lattice $\mathbb{Z}^k$.

- The resolution is achieved by subdividing the diagram with vectors v_{E_r} corresponding to new coordinates y_r such that exceptional divisors appear as $E_r = \{y_r = 0\}$. Note that for a CY resolution all the v_{E_r} lie on a hyperplane.

Resolution of $T^6/\mathbb{Z}_{6II}$

- The $\mathbb{C}^n/\mathbb{Z}_N$ singularities and their resolutions can be described by toric geometry or equivalent by the abelian gauge linear sigma model construction [Groot Nibbelink, 11] [Blaszczyk, Groot Nibbelink, Ruehle, 11].

- The toric variety is given by $X_\Sigma = (\mathbb{C}^n - Z(\Sigma))/G$ with exclusion set $Z(\Sigma)$ to ensure proper G–orbits, $G = \ker\phi$ with

$$\phi_n : (\mathbb{C}^*)^n \to (\mathbb{C}^*)^k, \quad (t_1, ..., t_n) \to (\prod_{j=1}^{n} t_j^{v_{j1}}, ..., \prod_{j=1}^{n} t_j^{v_{jk}}).$$

- The vectors v_i represent ordinary divisors $D_i = \{z_i = 0\}$.

- For singular varieties the v_i do not generate a lattice $\mathbb{Z}^k$.

- The resolution is achieved by subdividing the diagram with vectors v_{E_r} corresponding to new coordinates y_r such that exceptional divisors appear as $E_r = \{y_r = 0\}$. Note that for a CY resolution all the v_{E_r} lie on a hyperplane.

Resolution of $T^6/\mathbb{Z}_{6II}$

- The $\mathbb{C}^n/\mathbb{Z}_N$ singularities and their resolutions can be described by toric geometry or equivalent by the abelian gauge linear sigma model construction [Groot Nibbelink, 11] [Blaszczyk, Groot Nibbelink, Ruehle, 11].

- The toric variety is given by $X_\Sigma = (\mathbb{C}^n - Z(\Sigma))/G$ with exclusion set $Z(\Sigma)$ to ensure proper G–orbits, $G = \ker\phi$ with

$$\phi_n : (\mathbb{C}^*)^n \to (\mathbb{C}^*)^k, \quad (t_1, ..., t_n) \to (\prod_{j=1}^{n} t_j^{v_{j1}}, ..., \prod_{j=1}^{n} t_j^{v_{jk}}).$$

- The vectors v_i represent ordinary divisors $D_i = \{z_i = 0\}$.

- For singular varieties the v_i do not generate a lattice $\mathbb{Z}^k$.

- The resolution is achieved by subdividing the diagram with vectors v_{E_r} corresponding to new coordinates y_r such that exceptional divisors appear as $E_r = \{y_r = 0\}$. Note that for a CY resolution all the v_{E_r} lie on a hyperplane.

Resolution of $T^6/\mathbb{Z}_{6II}$

- The $\mathbb{C}^n/\mathbb{Z}_N$ singularities and their resolutions can be described by toric geometry or equivalent by the abelian gauge linear sigma model construction [Groot Nibbelink, 11] [Blaszczyk, Groot Nibbelink, Ruehle, 11].

- The toric variety is given by $X_\Sigma = (\mathbb{C}^n - Z(\Sigma))/G$ with exclusion set $Z(\Sigma)$ to ensure proper G–orbits, $G = \ker\phi$ with

$$\phi_n : (\mathbb{C}^*)^n \to (\mathbb{C}^*)^k, \quad (t_1, ..., t_n) \to (\prod_{j=1}^{n} t_j^{v_{j1}}, ..., \prod_{j=1}^{n} t_j^{v_{jk}}).$$

- The vectors v_i represent ordinary divisors $D_i = \{z_i = 0\}$.

- For singular varieties the v_i do not generate a lattice $\mathbb{Z}^k$.

- The resolution is achieved by subdividing the diagram with vectors v_{E_r} corresponding to new coordinates y_r such that exceptional divisors appear as $E_r = \{y_r = 0\}$. Note that for a CY resolution all the v_{E_r} lie on a hyperplane.

Resolution of $T^6/\mathbb{Z}_{6II}$

- The $\mathbb{C}^n/\mathbb{Z}_N$ singularities and their resolutions can be described by toric geometry or equivalent by the abelian gauge linear sigma model construction [Groot Nibbelink, 11] [Blaszczyk, Groot Nibbelink, Ruehle, 11].

- The toric variety is given by $X_\Sigma = (\mathbb{C}^n - Z(\Sigma))/G$ with exclusion set $Z(\Sigma)$ to ensure proper G–orbits, $G = \ker\phi$ with

$$\phi_n : (\mathbb{C}^*)^n \to (\mathbb{C}^*)^k, \quad (t_1, ..., t_n) \to (\prod_{j=1}^{n} t_j^{v_{j1}}, ..., \prod_{j=1}^{n} t_j^{v_{jk}}).$$

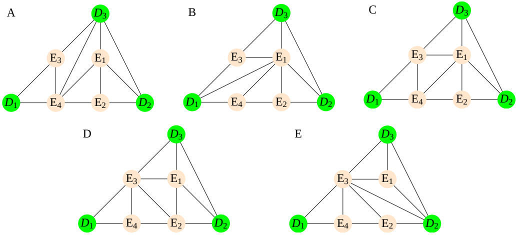
- The vectors v_i represent ordinary divisors $D_i = \{z_i = 0\}$.

- For singular varieties the v_i do not generate a lattice $\mathbb{Z}^k$.

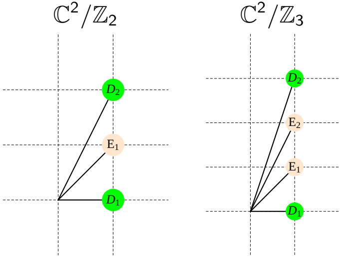
- The resolution is achieved by subdividing the diagram with vectors v_{E_r} corresponding to new coordinates y_r such that exceptional divisors appear as $E_r = \{y_r = 0\}$. Note that for a CY resolution all the v_{E_r} lie on a hyperplane.

Resolution of $T^6/\mathbb{Z}_{6II}$

The toric data of the local resolutions of the $\mathbb{C}^3/\mathbb{Z}_{6II}$ can be depicted by



Resolutions of codimension 2 singularities:



On the global $T^6/\mathbb{Z}_{6II}$ resolution [Lüst, Reffert, Scheideger and Stieberger, 06] the triple intersections of exceptional divisors give:

$$E^3_{1,\beta\gamma} = 6, E^3_{2,1\beta} = 8, E^3_{3,1\gamma} = 8, \\ E^3_{4,1\beta} = 8, E_{1,\beta\gamma}E^2_{2,1\beta} = -2, \\ E_{1,\beta\gamma}E^2_{3,1\gamma} = -2, E_{1,\beta\gamma}E^2_{4,1\beta} = -2, \\ E_{1,\beta\gamma}E_{2,1\beta}E_{4,1\beta} = 1, E^2_{2,1\beta}E_{4,1\beta} = -2.$$

Orbifold+vevs $\leftrightarrow$ CY 3-fold

The match

Orbifold+vevs $\leftrightarrow$ CY 3-fold

The match

Identifying the blow-up modes

The internal abelian flux is given by $\mathcal{F} = H_I V_r^I E_r$, it lies on the $E_8 \times E_8$ Cartan subalgebra $\{H_I\}$. Each V_r corresponds to the P_{sh} of an orbifold twisted state $\Phi_{k,\sigma}^{\text{bu-mode}}$: $\boxed{V_r \equiv P_{sh}}$.

The 16d vectors V_r obey the Bianchi identities (BI) $\int_S(\mathrm{tr}\,\mathcal{R}^2 - \mathrm{tr}\,\mathcal{F}^2) = 0$, with S a compact divisor. The BI read

$$(V_{2,1\beta}; V_{4,1\beta}) = 8/3, \sum_\beta (V_{2,3\beta}; V_{4,3\beta}) = 8, \; V_{3,1\gamma} \cdot \sum_\beta V_{1,\beta\gamma} = 1,$$

$$V_{4,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 8/9, \; V_{2,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 10/9,$$

with $(A; B) = A^2 + B^2 - AB$. Massless and non-oscillatory blow-up modes fulfill the equations $V_{1,\beta\gamma}^2 = \frac{25}{18}, \quad V_{3,\alpha\gamma}^2 = \frac{3}{2}, \quad V_{2,\alpha\beta}^2 = V_{4,\alpha\beta}^2 = \frac{14}{9}$.

We find a solution which leaves the non-Abelian orbifold gauge group intact due to $\alpha_i \cdot V_r = 0$. The spectrum is determined using the index theorem

$$\hat{N} = \frac{1}{6} \int_{\mathcal{M}} \left(\mathcal{F}^3 - \frac{1}{4}\,\mathrm{tr}\,\mathcal{R}^2 \mathcal{F} \right) = \sum_{\beta\gamma} \hat{N}(\widehat{\mathbb{C}^3/\mathbb{Z}_{6II}})|_{1\beta\gamma}$$

which give the following multiplicities for the 4d fermions

irrep	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{3},\mathbf{1},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{6})$	$(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$	$(\mathbf{3},\mathbf{2},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{2},\mathbf{1})$
mult.	40	9	8	2	4	4	0	3

Identifying the blow-up modes

The internal abelian flux is given by $\mathcal{F} = H_I V_r^I E_r$, it lies on the $E_8 \times E_8$ Cartan subalgebra $\{H_I\}$. Each V_r corresponds to the P_{sh} of an orbifold twisted state $\Phi^{\text{bu-mode}}_{k,\sigma}$: $\boxed{V_r \equiv P_{sh}}$.

The 16d vectors V_r obey the Bianchi identities (BI) $\int_S (\operatorname{tr}\mathcal{R}^2 - \operatorname{tr}\mathcal{F}^2) = 0$, with S a compact divisor. The BI read

$$(V_{2,1\beta}; V_{4,1\beta}) = 8/3, \sum_\beta (V_{2,3\beta}; V_{4,3\beta}) = 8, \; V_{3,1\gamma} \cdot \sum_\beta V_{1,\beta\gamma} = 1,$$

$$V_{4,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 8/9, \; V_{2,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 10/9,$$

with $(A; B) = A^2 + B^2 - AB$. Massless and non-oscillatory blow-up modes fulfill the equations $V^2_{1,\beta\gamma} = \frac{25}{18}, \quad V^2_{3,\alpha\gamma} = \frac{3}{2}, \quad V^2_{2,\alpha\beta} = V^2_{4,\alpha\beta} = \frac{14}{9}$.

We find a solution which leaves the non-Abelian orbifold gauge group intact due to $\alpha_i \cdot V_r = 0$. The spectrum is determined using the index theorem

$$\hat{N} = \frac{1}{6}\int_{\mathcal{M}} \left(\mathcal{F}^3 - \frac{1}{4}\operatorname{tr}\mathcal{R}^2\mathcal{F}\right) = \sum_{\beta\gamma} \hat{N}(\widehat{\mathbb{C}^3/\mathbb{Z}_{6II}})|_{1\beta\gamma}$$

which give the following multiplicities for the 4d fermions

irrep	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{3},\mathbf{1},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{6})$	$(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$	$(\mathbf{3},\mathbf{2},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{2},\mathbf{1})$
mult.	40	9	8	2	4	4	0	3

Identifying the blow-up modes

The internal abelian flux is given by $\mathcal{F} = H_I V_r^I E_r$, it lies on the $E_8 \times E_8$ Cartan subalgebra $\{H_I\}$. Each V_r corresponds to the P_{sh} of an orbifold twisted state $\Phi_{k,\sigma}^{\text{bu-mode}}$: $\boxed{V_r \equiv P_{sh}}$.

The 16d vectors V_r obey the Bianchi identities (BI) $\int_S (\mathrm{tr}\,\mathcal{R}^2 - \mathrm{tr}\,\mathcal{F}^2) = 0$, with S a compact divisor. The BI read

$$(V_{2,1\beta}; V_{4,1\beta}) = 8/3, \sum_\beta (V_{2,3\beta}; V_{4,3\beta}) = 8,\ V_{3,1\gamma} \cdot \sum_\beta V_{1,\beta\gamma} = 1,$$

$$V_{4,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 8/9,\ V_{2,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 10/9,$$

with $(A;B) = A^2 + B^2 - AB$. Massless and non-oscillatory blow-up modes fulfill the equations $V_{1,\beta\gamma}^2 = \frac{25}{18}, \quad V_{3,\alpha\gamma}^2 = \frac{3}{2}, \quad V_{2,\alpha\beta}^2 = V_{4,\alpha\beta}^2 = \frac{14}{9}$.

We find a solution which leaves the non-Abelian orbifold gauge group intact due to $\alpha_i \cdot V_r = 0$. The spectrum is determined using the index theorem

$$\hat{N} = \frac{1}{6} \int_{\mathcal{M}} \left(\mathcal{F}^3 - \frac{1}{4}\,\mathrm{tr}\,\mathcal{R}^2 \mathcal{F} \right) = \sum_{\beta\gamma} \hat{N}(\widehat{\mathbb{C}^3/\mathbb{Z}_{6II}})|_{1\beta\gamma}$$

which give the following multiplicities for the 4d fermions

irrep	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{3},\mathbf{1},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{6})$	$(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$	$(\mathbf{3},\mathbf{2},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{2},\mathbf{1})$
mult.	40	9	8	2	4	4	0	3

Identifying the blow-up modes

The internal abelian flux is given by $\mathcal{F} = H_I V_r^I E_r$, it lies on the $E_8 \times E_8$ Cartan subalgebra $\{H_I\}$. Each V_r corresponds to the P_{sh} of an orbifold twisted state $\Phi^{\text{bu-mode}}_{k,\sigma}$: $\boxed{V_r \equiv P_{sh}}$.

The 16d vectors V_r obey the Bianchi identities (BI) $\int_S (\text{tr}\,\mathcal{R}^2 - \text{tr}\,\mathcal{F}^2) = 0$, with S a compact divisor. The BI read

$$(V_{2,1\beta}; V_{4,1\beta}) = 8/3, \sum_\beta (V_{2,3\beta}; V_{4,3\beta}) = 8, \; V_{3,1\gamma} \cdot \sum_\beta V_{1,\beta\gamma} = 1,$$

$$V_{4,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 8/9, \; V_{2,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 10/9,$$

with $(A; B) = A^2 + B^2 - AB$. Massless and non-oscillatory blow-up modes fulfill the equations $V^2_{1,\beta\gamma} = \frac{25}{18}$, $\quad V^2_{3,\alpha\gamma} = \frac{3}{2}$, $\quad V^2_{2,\alpha\beta} = V^2_{4,\alpha\beta} = \frac{14}{9}$.

We find a solution which leaves the non-Abelian orbifold gauge group intact due to $\alpha_i \cdot V_r = 0$. The spectrum is determined using the index theorem

$$\hat{N} = \frac{1}{6} \int_{\mathcal{M}} \left(\mathcal{F}^3 - \frac{1}{4} \text{tr}\,\mathcal{R}^2 \mathcal{F} \right) = \sum_{\beta\gamma} \hat{N}(\widehat{\mathbb{C}^3/\mathbb{Z}_{6II}})|_{1\beta\gamma}$$

which give the following multiplicities for the 4d fermions

irrep	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{3},\mathbf{1},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{6})$	$(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$	$(\mathbf{3},\mathbf{2},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{2},\mathbf{1})$
mult.	40	9	8	2	4	4	0	3

Identifying the blow-up modes

The internal abelian flux is given by $\mathcal{F} = H_I V_r^I E_r$, it lies on the $E_8 \times E_8$ Cartan subalgebra $\{H_I\}$. Each V_r corresponds to the P_{sh} of an orbifold twisted state $\Phi_{k,\sigma}^{\text{bu-mode}}$: $\boxed{V_r \equiv P_{sh}}$.

The 16d vectors V_r obey the Bianchi identities (BI) $\int_S (\operatorname{tr} \mathcal{R}^2 - \operatorname{tr} \mathcal{F}^2) = 0$, with S a compact divisor. The BI read

$$(V_{2,1\beta}; V_{4,1\beta}) = 8/3, \sum_\beta (V_{2,3\beta}; V_{4,3\beta}) = 8, \ V_{3,1\gamma} \cdot \sum_\beta V_{1,\beta\gamma} = 1,$$

$$V_{4,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 8/9, \ V_{2,1\beta} \cdot \sum_\gamma V_{1,\beta\gamma} = 10/9,$$

with $(A; B) = A^2 + B^2 - AB$. Massless and non-oscillatory blow-up modes fulfill the equations $V_{1,\beta\gamma}^2 = \frac{25}{18}, \quad V_{3,\alpha\gamma}^2 = \frac{3}{2}, \quad V_{2,\alpha\beta}^2 = V_{4,\alpha\beta}^2 = \frac{14}{9}$.

We find a solution which leaves the non-Abelian orbifold gauge group intact due to $\alpha_i \cdot V_r = 0$. The spectrum is determined using the index theorem

$$\hat{N} = \frac{1}{6} \int_{\mathcal{M}} \left(\mathcal{F}^3 - \frac{1}{4} \operatorname{tr} \mathcal{R}^2 \mathcal{F} \right) = \sum_{\beta\gamma} \hat{N}(\widehat{\mathbb{C}^3/\mathbb{Z}_{6II}})|_{1\beta\gamma}$$

which give the following multiplicities for the 4d fermions

irrep	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{3},\mathbf{1},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{6})$	$(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$	$(\mathbf{3},\mathbf{2},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{2},\mathbf{1})$
mult.	40	9	8	2	4	4	0	3

Matching the spectrum

In order to match the spectrum in $T^6/\mathbb{Z}_{6II}$ with the spectrum in $\mathcal{M}$ it is crucial to find the right field redefinitions. Consider the twisted fields Φ^{orb}_γ and $\Phi^{\text{bu-mode}}_i$ with constructing elements given by fields $g = (\theta^k, n_\alpha e_\alpha)$ and $g_i = (\theta^{k_i}, m^i_\alpha e_\alpha)$ respectively. A generic field redefinition of the state γ involving the mentioned blow-up modes is

$$\Phi^{\text{bu}}_\gamma = \Phi^{\text{orb}}_\gamma \prod_i (\Phi^{\text{bu-mode}}_i)^{-r^\gamma_i}$$

The momentum of the state γ in blow-up P^γ_{bu} is given by $P^\gamma_{\text{bu}} = p - \sum_i r^\gamma_i p_i + \delta$, with $\delta = \left(k - \sum_i r^\gamma_i k_i\right) V + \left(n_\alpha - \sum_i r^\gamma_i m^i_\alpha\right) A_\alpha$. Such that $\delta \in \Gamma_8 \times \Gamma_8$.
An example is the orbifold-resolution map for the $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ representation

Multip.	Blow-up state	Redefinition	irrep.
-2	Φ^I_{11}	$(\psi_{48}, \psi_{60}) \to \Phi^I_{11}$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
-1	Φ^I_{20}	$\psi_{189} \to \Phi^I_{20}$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
0	Φ^{III}_{13}	$\psi_{93} \to \Phi^{III}_{13}$, $\psi_{173} \to \bar{\Phi}^{III}_{13}$	$(\mathbf{3}, \mathbf{2}, \mathbf{1})$

The field redefinitions performed are

$$\begin{aligned}
\Phi^I_{20} &= \psi_{189}(\Phi^{\text{bu-mode}}_{412})^{-1}, \\
\Phi^I_{11} &= \psi_{48}(\Phi^{\text{bu-mode}}_{113})^{-1},\ \Phi^I_{11} = \psi_{60}(\Phi^{\text{bu-mode}}_{111})^{-1}, \\
\Phi^{III}_{13} &= \psi_{93}\Phi^{\text{bu-mode}}_{112}\Phi^{\text{bu-mode}}_{122}\Phi^{\text{bu-mode}}_{231},\ \Phi^{III}_{13} = \psi_{173}(\Phi^{\text{bu-mode}}_{112})^{-1}(\Phi^{\text{bu-mode}}_{124})^{-1}(\Phi^{\text{bu-mode}}_{231})^{-1}.
\end{aligned}$$

Matching the spectrum

In order to match the spectrum in $T^6/\mathbb{Z}_{6II}$ with the spectrum in $\mathcal{M}$ it is crucial to find the right field redefinitions. Consider the twisted fields Φ_γ^{orb} and $\Phi_i^{\text{bu-mode}}$ with constructing elements given by fields $g = (\theta^k, n_\alpha e_\alpha)$ and $g_i = (\theta^{k_i}, m^i_\alpha e_\alpha)$ respectively. A generic field redefinition of the state γ involving the mentioned blow-up modes is

$$\boxed{\Phi_\gamma^{\text{bu}} = \Phi_\gamma^{\text{orb}} \prod_i (\Phi_i^{\text{bu-mode}})^{-r_i^\gamma}}$$

The momentum of the state γ in blow-up P_{bu}^γ is given by $P_{\text{bu}}^\gamma = p - \sum_i r_i^\gamma p_i + \delta$, with $\delta = \left(k - \sum_i r_i^\gamma k_i\right) V + \left(n_\alpha - \sum_i r_i^\gamma m^i_\alpha\right) A_\alpha$. Such that $\delta \in \Gamma_8 \times \Gamma_8$.
An example is the orbifold-resolution map for the $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ representation

Multip.	Blow-up state	Redefinition	irrep.
-2	Φ^I_{11}	$(\psi_{48}, \psi_{60}) \to \Phi^I_{11}$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
-1	Φ^I_{20}	$\psi_{189} \to \Phi^I_{20}$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
0	Φ^{III}_{13}	$\psi_{93} \to \Phi^{III}_{13}$, $\psi_{173} \to \bar{\Phi}^{III}_{13}$	$(\mathbf{3}, \mathbf{2}, \mathbf{1})$

The field redefinitions performed are

$$\begin{aligned}
\Phi^I_{20} &= \psi_{189}(\Phi^{\text{bu-mode}}_{412})^{-1}, \\
\Phi^I_{11} &= \psi_{48}(\Phi^{\text{bu-mode}}_{113})^{-1},\ \Phi^I_{11} = \psi_{60}(\Phi^{\text{bu-mode}}_{111})^{-1}, \\
\Phi^{III}_{13} &= \psi_{93}\Phi^{\text{bu-mode}}_{112}\Phi^{\text{bu-mode}}_{122}\Phi^{\text{bu-mode}}_{231},\ \Phi^{III}_{13} = \psi_{173}(\Phi^{\text{bu-mode}}_{112})^{-1}(\Phi^{\text{bu-mode}}_{124})^{-1}(\Phi^{\text{bu-mode}}_{231})^{-1}.
\end{aligned}$$

Matching the spectrum

In order to match the spectrum in $T^6/\mathbb{Z}_{6II}$ with the spectrum in $\mathcal{M}$ it is crucial to find the right field redefinitions. Consider the twisted fields Φ_γ^{orb} and $\Phi_i^{\text{bu-mode}}$ with constructing elements given by fields $g = (\theta^k, n_\alpha e_\alpha)$ and $g_i = (\theta^{k_i}, m_\alpha^i e_\alpha)$ respectively. A generic field redefinition of the state γ involving the mentioned blow-up modes is

$$\Phi_\gamma^{\text{bu}} = \Phi_\gamma^{\text{orb}} \prod_i (\Phi_i^{\text{bu-mode}})^{-r_i^\gamma}$$

The momentum of the state γ in blow-up P_{bu}^γ is given by $P_{\text{bu}}^\gamma = p - \sum_i r_i^\gamma p_i + \delta$, with $\delta = \left(k - \sum_i r_i^\gamma k_i\right) V + \left(n_\alpha - \sum_i r_i^\gamma m_\alpha^i\right) A_\alpha$. Such that $\delta \in \Gamma_8 \times \Gamma_8$.
An example is the orbifold-resolution map for the $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ representation

Multip.	Blow-up state	Redefinition	irrep.
-2	Φ_{11}^I	$(\psi_{48}, \psi_{60}) \to \Phi_{11}^I$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
-1	Φ_{20}^I	$\psi_{189} \to \Phi_{20}^I$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
0	Φ_{13}^{III}	$\psi_{93} \to \Phi_{13}^{III}$, $\psi_{173} \to \bar{\Phi}_{13}^{III}$	$(\mathbf{3}, \mathbf{2}, \mathbf{1})$

The field redefinitions performed are

$$\begin{aligned}
\Phi_{20}^I &= \psi_{189}(\Phi_{412}^{\text{bu-mode}})^{-1}, \\
\Phi_{11}^I &= \psi_{48}(\Phi_{113}^{\text{bu-mode}})^{-1},\ \Phi_{11}^I = \psi_{60}(\Phi_{111}^{\text{bu-mode}})^{-1}, \\
\Phi_{13}^{III} &= \psi_{93}\Phi_{112}^{\text{bu-mode}}\Phi_{122}^{\text{bu-mode}}\Phi_{231}^{\text{bu-mode}},\ \Phi_{13}^{III} = \psi_{173}(\Phi_{112}^{\text{bu-mode}})^{-1}(\Phi_{124}^{\text{bu-mode}})^{-1}(\Phi_{231}^{\text{bu-mode}})^{-1}.
\end{aligned}$$

Matching the spectrum

In order to match the spectrum in $T^6/\mathbb{Z}_{6II}$ with the spectrum in $\mathcal{M}$ it is crucial to find the right field redefinitions. Consider the twisted fields $\Phi^{\text{orb}}_{\gamma}$ and $\Phi^{\text{bu-mode}}_{i}$ with constructing elements given by fields $g = (\theta^k, n_\alpha e_\alpha)$ and $g_i = (\theta^{k_i}, m^i_\alpha e_\alpha)$ respectively. A generic field redefinition of the state γ involving the mentioned blow-up modes is

$$\boxed{\Phi^{\text{bu}}_{\gamma} = \Phi^{\text{orb}}_{\gamma} \prod_i (\Phi^{\text{bu-mode}}_{i})^{-r^{\gamma}_i}}$$

The momentum of the state γ in blow-up P^{γ}_{bu} is given by $P^{\gamma}_{\text{bu}} = p - \sum_i r^{\gamma}_i p_i + \delta$, with $\delta = \left(k - \sum_i r^{\gamma}_i k_i\right) V + \left(n_\alpha - \sum_i r^{\gamma}_i m^i_\alpha\right) A_\alpha$. Such that $\delta \in \Gamma_8 \times \Gamma_8$.

An example is the orbifold-resolution map for the $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ representation

Multip.	Blow-up state	Redefinition	irrep.
-2	Φ^{I}_{11}	$(\psi_{48}, \psi_{60}) \to \Phi^{I}_{11}$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
-1	Φ^{I}_{20}	$\psi_{189} \to \Phi^{I}_{20}$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
0	Φ^{III}_{13}	$\psi_{93} \to \Phi^{III}_{13}$, $\psi_{173} \to \bar{\Phi}^{III}_{13}$	$(\mathbf{3}, \mathbf{2}, \mathbf{1})$

The field redefinitions performed are

$$\begin{aligned}
\Phi^{I}_{20} &= \psi_{189} (\Phi^{\text{bu-mode}}_{412})^{-1}, \\
\Phi^{I}_{11} &= \psi_{48} (\Phi^{\text{bu-mode}}_{113})^{-1},\ \Phi^{I}_{11} = \psi_{60} (\Phi^{\text{bu-mode}}_{111})^{-1}, \\
\Phi^{III}_{13} &= \psi_{93} \Phi^{\text{bu-mode}}_{112} \Phi^{\text{bu-mode}}_{122} \Phi^{\text{bu-mode}}_{231},\ \Phi^{III}_{13} = \psi_{173} (\Phi^{\text{bu-mode}}_{112})^{-1} (\Phi^{\text{bu-mode}}_{124})^{-1} (\Phi^{\text{bu-mode}}_{231})^{-1}.
\end{aligned}$$

Matching the spectrum

In order to match the spectrum in $T^6/\mathbb{Z}_{6II}$ with the spectrum in $\mathcal{M}$ it is crucial to find the right field redefinitions. Consider the twisted fields Φ_γ^{orb} and $\Phi_i^{\text{bu-mode}}$ with constructing elements given by fields $g = (\theta^k, n_\alpha e_\alpha)$ and $g_i = (\theta^{k_i}, m_\alpha^i e_\alpha)$ respectively. A generic field redefinition of the state γ involving the mentioned blow-up modes is

$$\Phi_\gamma^{\text{bu}} = \Phi_\gamma^{\text{orb}} \prod_i (\Phi_i^{\text{bu-mode}})^{-r_i^\gamma}$$

The momentum of the state γ in blow-up P_{bu}^γ is given by $P_{\text{bu}}^\gamma = p - \sum_i r_i^\gamma p_i + \delta$, with $\delta = \left(k - \sum_i r_i^\gamma k_i\right) V + \left(n_\alpha - \sum_i r_i^\gamma m_\alpha^i\right) A_\alpha$. Such that $\delta \in \Gamma_8 \times \Gamma_8$.
An example is the orbifold-resolution map for the $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ representation

Multip.	Blow-up state	Redefinition	irrep.
-2	Φ_{11}^I	$(\psi_{48}, \psi_{60}) \to \Phi_{11}^I$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
-1	Φ_{20}^I	$\psi_{189} \to \Phi_{20}^I$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
0	Φ_{13}^{III}	$\psi_{93} \to \Phi_{13}^{III}$, $\psi_{173} \to \bar{\Phi}_{13}^{III}$	$(\mathbf{3}, \mathbf{2}, \mathbf{1})$

The field redefinitions performed are

$$\begin{aligned}
\Phi_{20}^I &= \psi_{189}(\Phi_{412}^{\text{bu-mode}})^{-1}, \\
\Phi_{11}^I &= \psi_{48}(\Phi_{113}^{\text{bu-mode}})^{-1},\ \Phi_{11}^I = \psi_{60}(\Phi_{111}^{\text{bu-mode}})^{-1}, \\
\Phi_{13}^{III} &= \psi_{93}\Phi_{112}^{\text{bu-mode}}\Phi_{122}^{\text{bu-mode}}\Phi_{231}^{\text{bu-mode}},\ \Phi_{13}^{III} = \psi_{173}(\Phi_{112}^{\text{bu-mode}})^{-1}(\Phi_{124}^{\text{bu-mode}})^{-1}(\Phi_{231}^{\text{bu-mode}})^{-1}.
\end{aligned}$$

Matching the spectrum

In order to match the spectrum in $T^6/\mathbb{Z}_{6II}$ with the spectrum in $\mathcal{M}$ it is crucial to find the right field redefinitions. Consider the twisted fields Φ_γ^{orb} and $\Phi_i^{\text{bu-mode}}$ with constructing elements given by fields $g = (\theta^k, n_\alpha e_\alpha)$ and $g_i = (\theta^{k_i}, m_\alpha^i e_\alpha)$ respectively. A generic field redefinition of the state γ involving the mentioned blow-up modes is

$$\boxed{\Phi_\gamma^{\text{bu}} = \Phi_\gamma^{\text{orb}} \prod_i (\Phi_i^{\text{bu-mode}})^{-r_i^\gamma}}$$

The momentum of the state γ in blow-up P_{bu}^γ is given by $P_{\text{bu}}^\gamma = p - \sum_i r_i^\gamma p_i + \delta$, with $\delta = \left(k - \sum_i r_i^\gamma k_i\right) V + \left(n_\alpha - \sum_i r_i^\gamma m_\alpha^i\right) A_\alpha$. Such that $\delta \in \Gamma_8 \times \Gamma_8$.
An example is the orbifold-resolution map for the $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ representation

Multip.	Blow-up state	Redefinition	irrep.
-2	Φ_{11}^I	$(\psi_{48}, \psi_{60}) \to \Phi_{11}^I$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
-1	Φ_{20}^I	$\psi_{189} \to \Phi_{20}^I$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
0	Φ_{13}^{III}	$\psi_{93} \to \Phi_{13}^{III}$, $\psi_{173} \to \bar{\Phi}_{13}^{III}$	$(\mathbf{3}, \mathbf{2}, \mathbf{1})$

The field redefinitions performed are

$$\begin{aligned}
\Phi_{20}^I &= \psi_{189}(\Phi_{412}^{\text{bu-mode}})^{-1}, \\
\Phi_{11}^I &= \psi_{48}(\Phi_{113}^{\text{bu-mode}})^{-1},\ \Phi_{11}^I = \psi_{60}(\Phi_{111}^{\text{bu-mode}})^{-1}, \\
\Phi_{13}^{III} &= \psi_{93}\Phi_{112}^{\text{bu-mode}}\Phi_{122}^{\text{bu-mode}}\Phi_{231}^{\text{bu-mode}},\ \Phi_{13}^{III} = \psi_{173}(\Phi_{112}^{\text{bu-mode}})^{-1}(\Phi_{124}^{\text{bu-mode}})^{-1}(\Phi_{231}^{\text{bu-mode}})^{-1}.
\end{aligned}$$

Matching the spectrum

In order to match the spectrum in $T^6/\mathbb{Z}_{6II}$ with the spectrum in $\mathcal{M}$ it is crucial to find the right field redefinitions. Consider the twisted fields Φ_γ^{orb} and $\Phi_i^{\text{bu-mode}}$ with constructing elements given by fields $g = (\theta^k, n_\alpha e_\alpha)$ and $g_i = (\theta^{k_i}, m_\alpha^i e_\alpha)$ respectively. A generic field redefinition of the state γ involving the mentioned blow-up modes is

$$\boxed{\Phi_\gamma^{\text{bu}} = \Phi_\gamma^{\text{orb}} \prod_i (\Phi_i^{\text{bu-mode}})^{-r_i^\gamma}}$$

The momentum of the state γ in blow-up P_{bu}^γ is given by $P_{\text{bu}}^\gamma = p - \sum_i r_i^\gamma p_i + \delta$, with $\delta = \left(k - \sum_i r_i^\gamma k_i\right) V + \left(n_\alpha - \sum_i r_i^\gamma m_\alpha^i\right) A_\alpha$. Such that $\delta \in \Gamma_8 \times \Gamma_8$.
An example is the orbifold-resolution map for the $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ representation

Multip.	Blow-up state	Redefinition	irrep.
-2	Φ_{11}^I	$(\psi_{48}, \psi_{60}) \to \Phi_{11}^I$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
-1	Φ_{20}^I	$\psi_{189} \to \Phi_{20}^I$	$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})$
0	Φ_{13}^{III}	$\psi_{93} \to \Phi_{13}^{III},\ \psi_{173} \to \bar{\Phi}_{13}^{III}$	$(\mathbf{3}, \mathbf{2}, \mathbf{1})$

The field redefinitions performed are

$$\begin{aligned}
\Phi_{20}^I &= \psi_{189} (\Phi_{412}^{\text{bu-mode}})^{-1}, \\
\Phi_{11}^I &= \psi_{48} (\Phi_{113}^{\text{bu-mode}})^{-1},\ \Phi_{11}^I = \psi_{60} (\Phi_{111}^{\text{bu-mode}})^{-1}, \\
\Phi_{13}^{III} &= \psi_{93} \Phi_{112}^{\text{bu-mode}} \Phi_{122}^{\text{bu-mode}} \Phi_{231}^{\text{bu-mode}},\ \Phi_{13}^{III} = \psi_{173} (\Phi_{112}^{\text{bu-mode}})^{-1} (\Phi_{124}^{\text{bu-mode}})^{-1} (\Phi_{231}^{\text{bu-mode}})^{-1}.
\end{aligned}$$

Matching the spectrum

The $(\mathbf{3},\mathbf{1},\mathbf{1})$ and $(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$ acquire masses due to Yukawa couplings with blow-up modes. Look for example at the mass terms

$$\begin{pmatrix} \psi_6 \\ \psi_{112} \\ \psi_{92} \\ \psi_{116} \\ \psi_{105} \end{pmatrix}^{\mathsf{T}} \begin{pmatrix} 0 & 0 & a_1\langle\psi_{126}\rangle & a_2\langle\psi_{134}\rangle & a_3\langle\psi_{150}\rangle & a_4\langle\psi_{153}\rangle \\ 0 & 0 & a_5\langle\psi_{70}\rangle & a_5\langle\psi_{77}\rangle & a_6\langle\psi_{70}\rangle & a_6\langle\psi_{77}\rangle \\ 0 & 0 & a_7\langle\psi_{70}\rangle & a_7\langle\psi_{77}\rangle & a_8\langle\psi_{70}\rangle & a_8\langle\psi_{77}\rangle \\ e_1\langle\psi_{154}\rangle & e_1\langle\psi_{155}\rangle & e_2\langle\psi_{15}\rangle & e_2\langle\psi_{22}\rangle & e_1\langle\psi_{15}\rangle & e_1\langle\psi_{22}\rangle \\ e_3\langle\psi_{154}\rangle & e_3\langle\psi_{155}\rangle & e_4\langle\psi_{15}\rangle & e_4\langle\psi_{22}\rangle & e_3\langle\psi_{15}\rangle & e_3\langle\psi_{22}\rangle \end{pmatrix} \begin{pmatrix} \psi_{30} \\ \psi_{36} \\ \psi_{125} \\ \psi_{133} \\ \psi_{149} \\ \psi_{152} \end{pmatrix}.$$

The field redefinitions define the map $\psi_6, ..., \psi_{105} \to \Phi_4^I$ and $\psi_{30},...,\psi_{152} \to \bar{\Phi}_4^I$.

Every pair of fields forming a mass term from the orbifold superpotential Yukawa couplings are redefined to conjugated pairs in blow-up.

With a similar procedure we have matched the complete massless spectrum, containing the $(\mathbf{1},\mathbf{2},\mathbf{1})$, $(\mathbf{1},\mathbf{1},\mathbf{6})$, $(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$ and $(\mathbf{1},\mathbf{1},\mathbf{1})$ irreps.

Matching the spectrum

The $\mathbf{(3,1,1)}$ and $(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$ acquire masses due to Yukawa couplings with blow-up modes. Look for example at the mass terms

$$\begin{pmatrix} \psi_6 \\ \psi_{112} \\ \psi_{92} \\ \psi_{116} \\ \psi_{105} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & a_1\langle\psi_{126}\rangle & a_2\langle\psi_{134}\rangle & a_3\langle\psi_{150}\rangle & a_4\langle\psi_{153}\rangle \\ 0 & 0 & a_5\langle\psi_{70}\rangle & a_5\langle\psi_{77}\rangle & a_6\langle\psi_{70}\rangle & a_6\langle\psi_{77}\rangle \\ 0 & 0 & a_7\langle\psi_{70}\rangle & a_7\langle\psi_{77}\rangle & a_8\langle\psi_{70}\rangle & a_8\langle\psi_{77}\rangle \\ e_1\langle\psi_{154}\rangle & e_1\langle\psi_{155}\rangle & e_2\langle\psi_{15}\rangle & e_2\langle\psi_{22}\rangle & e_1\langle\psi_{15}\rangle & e_1\langle\psi_{22}\rangle \\ e_3\langle\psi_{154}\rangle & e_3\langle\psi_{155}\rangle & e_4\langle\psi_{15}\rangle & e_4\langle\psi_{22}\rangle & e_3\langle\psi_{15}\rangle & e_3\langle\psi_{22}\rangle \end{pmatrix} \begin{pmatrix} \psi_{30} \\ \psi_{36} \\ \psi_{125} \\ \psi_{133} \\ \psi_{149} \\ \psi_{152} \end{pmatrix}.$$

The field redefinitions define the map $\psi_6, \ldots, \psi_{105} \to \Phi_4^I$ and $\psi_{30},\ldots,\psi_{152} \to \bar{\Phi}_4^I$.

Every pair of fields forming a mass term from the orbifold superpotential Yukawa couplings are redefined to conjugated pairs in blow-up.

With a similar procedure we have matched the complete massless spectrum, containing the $(\mathbf{1},\mathbf{2},\mathbf{1})$, $(\mathbf{1},\mathbf{1},\mathbf{6})$, $(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$ and $(\mathbf{1},\mathbf{1},\mathbf{1})$ irreps.

Matching the spectrum

The $(\mathbf{3},\mathbf{1},\mathbf{1})$ and $(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$ acquire masses due to Yukawa couplings with blow-up modes. Look for example at the mass terms

$$\begin{pmatrix} \psi_6 \\ \psi_{112} \\ \psi_{92} \\ \psi_{116} \\ \psi_{105} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & a_1\langle\psi_{126}\rangle & a_2\langle\psi_{134}\rangle & a_3\langle\psi_{150}\rangle & a_4\langle\psi_{153}\rangle \\ 0 & 0 & a_5\langle\psi_{70}\rangle & a_5\langle\psi_{77}\rangle & a_6\langle\psi_{70}\rangle & a_6\langle\psi_{77}\rangle \\ 0 & 0 & a_7\langle\psi_{70}\rangle & a_7\langle\psi_{77}\rangle & a_8\langle\psi_{70}\rangle & a_8\langle\psi_{77}\rangle \\ e_1\langle\psi_{154}\rangle & e_1\langle\psi_{155}\rangle & e_2\langle\psi_{15}\rangle & e_2\langle\psi_{22}\rangle & e_1\langle\psi_{15}\rangle & e_1\langle\psi_{22}\rangle \\ e_3\langle\psi_{154}\rangle & e_3\langle\psi_{155}\rangle & e_4\langle\psi_{15}\rangle & e_4\langle\psi_{22}\rangle & e_3\langle\psi_{15}\rangle & e_3\langle\psi_{22}\rangle \end{pmatrix} \begin{pmatrix} \psi_{30} \\ \psi_{36} \\ \psi_{125} \\ \psi_{133} \\ \psi_{149} \\ \psi_{152} \end{pmatrix}.$$

The field redefinitions define the map $\psi_6, ..., \psi_{105} \to \Phi_4^I$ and $\psi_{30},...,\psi_{152} \to \bar{\Phi}_4^I$.

Every pair of fields forming a mass term from the orbifold superpotential Yukawa couplings are redefined to conjugated pairs in blow-up.

With a similar procedure we have matched the complete massless spectrum, containing the $(\mathbf{1},\mathbf{2},\mathbf{1})$, $(\mathbf{1},\mathbf{1},\mathbf{6})$, $(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$ and $(\mathbf{1},\mathbf{1},\mathbf{1})$ irreps.

Matching the spectrum

The $(\mathbf{3},\mathbf{1},\mathbf{1})$ and $(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$ acquire masses due to Yukawa couplings with blow-up modes. Look for example at the mass terms

$$\begin{pmatrix} \psi_6 \\ \psi_{112} \\ \psi_{92} \\ \psi_{116} \\ \psi_{105} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & a_1\langle\psi_{126}\rangle & a_2\langle\psi_{134}\rangle & a_3\langle\psi_{150}\rangle & a_4\langle\psi_{153}\rangle \\ 0 & 0 & a_5\langle\psi_{70}\rangle & a_5\langle\psi_{77}\rangle & a_6\langle\psi_{70}\rangle & a_6\langle\psi_{77}\rangle \\ 0 & 0 & a_7\langle\psi_{70}\rangle & a_7\langle\psi_{77}\rangle & a_8\langle\psi_{70}\rangle & a_8\langle\psi_{77}\rangle \\ e_1\langle\psi_{154}\rangle & e_1\langle\psi_{155}\rangle & e_2\langle\psi_{15}\rangle & e_2\langle\psi_{22}\rangle & e_1\langle\psi_{15}\rangle & e_1\langle\psi_{22}\rangle \\ e_3\langle\psi_{154}\rangle & e_3\langle\psi_{155}\rangle & e_4\langle\psi_{15}\rangle & e_4\langle\psi_{22}\rangle & e_3\langle\psi_{15}\rangle & e_3\langle\psi_{22}\rangle \end{pmatrix} \begin{pmatrix} \psi_{30} \\ \psi_{36} \\ \psi_{125} \\ \psi_{133} \\ \psi_{149} \\ \psi_{152} \end{pmatrix}.$$

The field redefinitions define the map $\psi_6, \ldots, \psi_{105} \to \Phi_4^I$ and $\psi_{30},\ldots,\psi_{152} \to \bar{\Phi}_4^I$.

Every pair of fields forming a mass term from the orbifold superpotential Yukawa couplings are redefined to conjugated pairs in blow-up.

With a similar procedure we have matched the complete massless spectrum, containing the $(\mathbf{1},\mathbf{2},\mathbf{1})$, $(\mathbf{1},\mathbf{1},\mathbf{6})$, $(\mathbf{1},\mathbf{1},\bar{\mathbf{6}})$ and $(\mathbf{1},\mathbf{1},\mathbf{1})$ irreps.

Matching the anomalies

4d anomalous U(1)s in CY 3-fold with $U(1)$ bundles were studied in
[Blumenhagen, Honecker, Weigand, 05] [Groot Nibbelink,Nilles,Trapletti,07].
In our case the anomalies in the deformed Calabi-Yau 3-fold compactification are the pure $U(1)$, $U(1)\times SU(6)^2$, $U(1)\times SU(3)^2$, $U(1)\times SU(2)^2$ and $U(1)\times \text{grav}^2$ anomalies:

$$I^{\text{bu}} = \sum_{ij} e_{ijk} F_i F_j F_k + \sum_i b_i F_i \text{tr} F^2_{su(6)} + \sum_i c_i F_i (\text{tr} F^2_{su(2)} + \text{tr} F^2_{su(3)}) + \sum_i d_i F_i \text{tr} R^2 .$$

The 4d anomaly polynomial in the resolution I^{bu} coincides with $I^{\text{orb}} + I^{\text{red}}$ such that

$$F^{\text{orb}} X_4^{\text{orb}} + \sum_a q_I^a F^I X_{4,a}^{\text{red}} = X_2^{\text{uni}} X_4^{\text{uni}} + \sum_r X_2^r X_4^r \,.$$

This anomaly is canceled via counter–terms involving the blow-up modes $\Phi_r^{\text{bu-mode}} \sim e^{i\tau_r}$ as

$$a^{\text{orb}} X_4^{\text{orb}} + \sum_r \tau_r X_{4,r}^{\text{red}} = a^{\text{uni}} X_4^{\text{uni}} + \sum_r \beta_r X_4^r \,.$$

Resolution axions are identified with blow–up modes as

$$\beta_r \sim \tau_r \,, \qquad a^{\text{uni}} = -6 a^{\text{orb}} + \sum_r c_r \tau_r \,.$$

Matching the anomalies

4d anomalous U(1)s in CY 3-fold with $U(1)$ bundles were studied in [Blumenhagen, Honecker, Weigand, 05] [Groot Nibbelink,Nilles,Trapletti,07].

In our case the anomalies in the deformed Calabi-Yau 3-fold compactification are the pure $U(1)$, $U(1)\times SU(6)^2$, $U(1)\times SU(3)^2$, $U(1)\times SU(2)^2$ and $U(1)\times \mathrm{grav}^2$ anomalies:

$$I^{\mathrm{bu}} = \sum_{ij} e_{ijk} F_i F_j F_k + \sum_i b_i F_i \mathrm{tr} F^2_{su(6)} + \sum_i c_i F_i (\mathrm{tr} F^2_{su(2)} + \mathrm{tr} F^2_{su(3)}) + \sum_i d_i F_i \mathrm{tr} R^2 .$$

The 4d anomaly polynomial in the resolution I^{bu} coincides with $I^{\mathrm{orb}} + I^{\mathrm{red}}$ such that

$$F^{\mathrm{orb}} X_4^{\mathrm{orb}} + \sum_a q_I^a F^I X_{4,a}^{\mathrm{red}} = X_2^{\mathrm{uni}} X_4^{\mathrm{uni}} + \sum_r X_2^r X_4^r .$$

This anomaly is canceled via counter–terms involving the blow-up modes $\Phi_r^{\mathrm{bu\text{-}mode}} \sim e^{i\tau_r}$ as

$$a^{\mathrm{orb}} X_4^{\mathrm{orb}} + \sum_r \tau_r X_{4,r}^{\mathrm{red}} = a^{\mathrm{uni}} X_4^{\mathrm{uni}} + \sum_r \beta_r X_4^r .$$

Resolution axions are identified with blow–up modes as

$$\beta_r \sim \tau_r , \qquad a^{\mathrm{uni}} = -6 a^{\mathrm{orb}} + \sum_r c_r \tau_r .$$

Matching the anomalies

4d anomalous U(1)s in CY 3-fold with $U(1)$ bundles were studied in
[Blumenhagen, Honecker, Weigand, 05] [Groot Nibbelink,Nilles,Trapletti,07].

In our case the anomalies in the deformed Calabi-Yau 3-fold compactification are the pure $U(1)$, $U(1) \times SU(6)^2$, $U(1) \times SU(3)^2$, $U(1) \times SU(2)^2$ and $U(1) \times \mathrm{grav}^2$ anomalies:

$$I^{\rm bu} = \sum_{ij} e_{ijk} F_i F_j F_k + \sum_i b_i F_i \mathrm{tr} F^2_{su(6)} + \sum_i c_i F_i (\mathrm{tr} F^2_{su(2)} + \mathrm{tr} F^2_{su(3)}) + \sum_i d_i F_i \mathrm{tr} R^2 .$$

The 4d anomaly polynomial in the resolution $I^{\rm bu}$ coincides with $I^{\rm orb} + I^{\rm red}$ such that

$$F^{\rm orb} X_4^{\rm orb} + \sum_a q_I^a F^I X_{4,a}^{\rm red} = X_2^{\rm uni} X_4^{\rm uni} + \sum_r X_2^r X_4^r .$$

This anomaly is canceled via counter–terms involving the blow-up modes $\Phi_r^{\text{bu-mode}} \sim e^{i\tau_r}$ as

$$a^{\rm orb} X_4^{\rm orb} + \sum_r \tau_r X_{4,r}^{\rm red} = a^{\rm uni} X_4^{\rm uni} + \sum_r \beta_r X_4^r .$$

Resolution axions are identified with blow–up modes as

$$\beta_r \sim \tau_r , \qquad a^{\rm uni} = -6 a^{\rm orb} + \sum_r c_r \tau_r .$$

Matching the anomalies

4d anomalous U(1)s in CY 3-fold with $U(1)$ bundles were studied in [Blumenhagen, Honecker, Weigand, 05] [Groot Nibbelink,Nilles,Trapletti,07].

In our case the anomalies in the deformed Calabi-Yau 3-fold compactification are the pure $U(1)$, $U(1)\times SU(6)^2$, $U(1)\times SU(3)^2$, $U(1)\times SU(2)^2$ and $U(1)\times \text{grav}^2$ anomalies:

$$I^{\text{bu}} = \sum_{ij} e_{ijk} F_i F_j F_k + \sum_i b_i F_i \text{tr} F^2_{su(6)} + \sum_i c_i F_i (\text{tr} F^2_{su(2)} + \text{tr} F^2_{su(3)}) + \sum_i d_i F_i \text{tr} R^2 .$$

The 4d anomaly polynomial in the resolution I^{bu} coincides with $I^{\text{orb}} + I^{\text{red}}$ such that

$$F^{\text{orb}} X_4^{\text{orb}} + \sum_a q_I^a F^I X_{4,a}^{\text{red}} = X_2^{\text{uni}} X_4^{\text{uni}} + \sum_r X_2^r X_4^r \, .$$

This anomaly is canceled via counter–terms involving the blow-up modes $\Phi_r^{\text{bu-mode}} \sim e^{i\tau_r}$ as

$$a^{\text{orb}} X_4^{\text{orb}} + \sum_r \tau_r X_{4,r}^{\text{red}} = a^{\text{uni}} X_4^{\text{uni}} + \sum_r \beta_r X_4^r \, .$$

Resolution axions are identified with blow–up modes as

$$\beta_r \sim \tau_r \, , \qquad a^{\text{uni}} = -6 a^{\text{orb}} + \sum_r c_r \tau_r \, .$$

Matching the anomalies

4d anomalous U(1)s in CY 3-fold with $U(1)$ bundles were studied in
[Blumenhagen, Honecker, Weigand, 05] [Groot Nibbelink,Nilles,Trapletti,07].
In our case the anomalies in the deformed Calabi-Yau 3-fold compactification are the pure $U(1)$, $U(1)\times SU(6)^2$, $U(1)\times SU(3)^2$, $U(1)\times SU(2)^2$ and $U(1)\times \text{grav}^2$ anomalies:

$$I^{\text{bu}} = \sum_{ij} e_{ijk} F_i F_j F_k + \sum_i b_i F_i \text{tr} F^2_{su(6)} + \sum_i c_i F_i (\text{tr} F^2_{su(2)} + \text{tr} F^2_{su(3)}) + \sum_i d_i F_i \text{tr} R^2 .$$

The 4d anomaly polynomial in the resolution I^{bu} coincides with $I^{\text{orb}} + I^{\text{red}}$ such that

$$F^{\text{orb}} X_4^{\text{orb}} + \sum_a q_I^a F^I X_{4,a}^{\text{red}} = X_2^{\text{uni}} X_4^{\text{uni}} + \sum_r X_2^r X_4^r \, .$$

This anomaly is canceled via counter–terms involving the blow-up modes $\Phi_r^{\text{bu-mode}} \sim e^{i\tau_r}$ as

$$a^{\text{orb}} X_4^{\text{orb}} + \sum_r \tau_r X_{4,r}^{\text{red}} = a^{\text{uni}} X_4^{\text{uni}} + \sum_r \beta_r X_4^r \, .$$

Resolution axions are identified with blow–up modes as

$$\beta_r \sim \tau_r \, , \qquad a^{\text{uni}} = -6 a^{\text{orb}} + \sum_r c_r \tau_r \, .$$

Matching the anomalies

4d anomalous U(1)s in CY 3-fold with $U(1)$ bundles were studied in
[Blumenhagen, Honecker, Weigand, 05] [Groot Nibbelink,Nilles,Trapletti,07].
In our case the anomalies in the deformed Calabi-Yau 3-fold compactification are the pure $U(1)$, $U(1)\times SU(6)^2$, $U(1)\times SU(3)^2$, $U(1)\times SU(2)^2$ and $U(1)\times \mathrm{grav}^2$ anomalies:

$$I^{\mathrm{bu}} = \sum_{ij} e_{ijk} F_i F_j F_k + \sum_i b_i F_i \mathrm{tr} F^2_{su(6)} + \sum_i c_i F_i (\mathrm{tr} F^2_{su(2)} + \mathrm{tr} F^2_{su(3)}) + \sum_i d_i F_i \mathrm{tr} R^2 .$$

The 4d anomaly polynomial in the resolution I^{bu} coincides with $I^{\mathrm{orb}} + I^{\mathrm{red}}$ such that

$$F^{\mathrm{orb}} X_4^{\mathrm{orb}} + \sum_a q_I^a F^I X_{4,a}^{\mathrm{red}} = X_2^{\mathrm{uni}} X_4^{\mathrm{uni}} + \sum_r X_2^r X_4^r \, .$$

This anomaly is canceled via counter–terms involving the blow-up modes $\Phi_r^{\text{bu-mode}} \sim e^{i\tau_r}$ as

$$a^{\mathrm{orb}} X_4^{\mathrm{orb}} + \sum_r \tau_r X_{4,r}^{\mathrm{red}} = a^{\mathrm{uni}} X_4^{\mathrm{uni}} + \sum_r \beta_r X_4^r \, .$$

Resolution axions are identified with blow–up modes as

$$\beta_r \sim \tau_r \, , \qquad a^{\mathrm{uni}} = -6 a^{\mathrm{orb}} + \sum_r c_r \tau_r \, .$$

Matching the anomalies

4d anomalous U(1)s in CY 3-fold with $U(1)$ bundles were studied in [Blumenhagen, Honecker, Weigand, 05] [Groot Nibbelink,Nilles,Trapletti,07].

In our case the anomalies in the deformed Calabi-Yau 3-fold compactification are the pure $U(1)$, $U(1) \times SU(6)^2$, $U(1) \times SU(3)^2$, $U(1) \times SU(2)^2$ and $U(1) \times \mathrm{grav}^2$ anomalies:

$$I^{\mathrm{bu}} = \sum_{ij} e_{ijk} F_i F_j F_k + \sum_i b_i F_i \mathrm{tr} F^2_{su(6)} + \sum_i c_i F_i (\mathrm{tr} F^2_{su(2)} + \mathrm{tr} F^2_{su(3)}) + \sum_i d_i F_i \mathrm{tr} R^2 .$$

The 4d anomaly polynomial in the resolution I^{bu} coincides with $I^{\mathrm{orb}} + I^{\mathrm{red}}$ such that

$$F^{\mathrm{orb}} X_4^{\mathrm{orb}} + \sum_a q_I^a F^I X_{4,a}^{\mathrm{red}} = X_2^{\mathrm{uni}} X_4^{\mathrm{uni}} + \sum_r X_2^r X_4^r \, .$$

This anomaly is canceled via counter–terms involving the blow-up modes $\Phi_r^{\text{bu-mode}} \sim e^{i\tau_r}$ as

$$a^{\mathrm{orb}} X_4^{\mathrm{orb}} + \sum_r \tau_r X_{4,r}^{\mathrm{red}} = a^{\mathrm{uni}} X_4^{\mathrm{uni}} + \sum_r \beta_r X_4^r \, .$$

Resolution axions are identified with blow–up modes as

$$\beta_r \sim \tau_r \, , \qquad a^{\mathrm{uni}} = -6 a^{\mathrm{orb}} + \sum_r c_r \tau_r \, .$$

Matching the anomalies

4d anomalous U(1)s in CY 3-fold with $U(1)$ bundles were studied in [Blumenhagen, Honecker, Weigand, 05] [Groot Nibbelink,Nilles,Trapletti,07].

In our case the anomalies in the deformed Calabi-Yau 3-fold compactification are the pure $U(1)$, $U(1) \times SU(6)^2$, $U(1) \times SU(3)^2$, $U(1) \times SU(2)^2$ and $U(1) \times \text{grav}^2$ anomalies:

$$I^{\text{bu}} = \sum_{ij} e_{ijk} F_i F_j F_k + \sum_i b_i F_i \text{tr} F^2_{su(6)} + \sum_i c_i F_i (\text{tr} F^2_{su(2)} + \text{tr} F^2_{su(3)}) + \sum_i d_i F_i \text{tr} R^2 .$$

The 4d anomaly polynomial in the resolution I^{bu} coincides with $I^{\text{orb}} + I^{\text{red}}$ such that

$$F^{\text{orb}} X_4^{\text{orb}} + \sum_a q_I^a F^I X_{4,a}^{\text{red}} = X_2^{\text{uni}} X_4^{\text{uni}} + \sum_r X_2^r X_4^r \,.$$

This anomaly is canceled via counter–terms involving the blow-up modes $\Phi_r^{\text{bu-mode}} \sim e^{i\tau_r}$ as

$$a^{\text{orb}} X_4^{\text{orb}} + \sum_r \tau_r X_{4,r}^{\text{red}} = a^{\text{uni}} X_4^{\text{uni}} + \sum_r \beta_r X_4^r \,.$$

Resolution axions are identified with blow–up modes as

$$\beta_r \sim \tau_r \,, \qquad a^{\text{uni}} = -6 a^{\text{orb}} + \sum_r c_r \tau_r \,.$$

Conclusions

We established the correspondence between heterotic string on a deformed $T^6/\mathbb{Z}_{6II}$ orbifold in the mini-landscape with an heterotic compactification in a smooth CY 3-fold.

- Twisted fields which gain vevs to ensure a D–flat vacuum are identified with blow-up modes.
- The massless spectrum is matched using field redefinitions, after taking into account the orbifold mass terms.
- The anomaly cancellation mechanism in 4d is understood in both approaches. The blow–up modes transmute in the non–universal axions on the resolution.

Outlook:

- Explore the implications of recently studied orbifold selection rules to the possible orbifold-resolution maps.
- Constructing an algebraic description of global CY with gauge bundles.
- Study the deformations of the $(0, 2)$ orbifold SCFT by vevs of blow-up modes.
- Exploring the phenomenological consequences in more realistic models with non-anomalous U(1)s.

Conclusions

We established the correspondence between heterotic string on a deformed $T^6/\mathbb{Z}_{6II}$ orbifold in the mini-landscape with an heterotic compactification in a smooth CY 3-fold.

- Twisted fields which gain vevs to ensure a D–flat vacuum are identified with blow-up modes.
- The massless spectrum is matched using field redefinitions, after taking into account the orbifold mass terms.
- The anomaly cancellation mechanism in 4d is understood in both approaches. The blow–up modes transmute in the non–universal axions on the resolution.

Outlook:

- Explore the implications of recently studied orbifold selection rules to the possible orbifold-resolution maps.
- Constructing an algebraic description of global CY with gauge bundles.
- Study the deformations of the $(0, 2)$ orbifold SCFT by vevs of blow-up modes.
- Exploring the phenomenological consequences in more realistic models with non-anomalous U(1)s.

Conclusions

We established the correspondence between heterotic string on a deformed $T^6/\mathbb{Z}_{6II}$ orbifold in the mini-landscape with an heterotic compactification in a smooth CY 3-fold.

- Twisted fields which gain vevs to ensure a D–flat vacuum are identified with blow-up modes.
- The massless spectrum is matched using field redefinitions, after taking into account the orbifold mass terms.
- The anomaly cancellation mechanism in 4d is understood in both approaches. The blow–up modes transmute in the non–universal axions on the resolution.

Outlook:

- Explore the implications of recently studied orbifold selection rules to the possible orbifold-resolution maps.
- Constructing an algebraic description of global CY with gauge bundles.
- Study the deformations of the $(0,2)$ orbifold SCFT by vevs of blow-up modes.
- Exploring the phenomenological consequences in more realistic models with non-anomalous U(1)s.

Conclusions

We established the correspondence between heterotic string on a deformed $T^6/\mathbb{Z}_{6II}$ orbifold in the mini-landscape with an heterotic compactification in a smooth CY 3-fold.

- Twisted fields which gain vevs to ensure a D–flat vacuum are identified with blow-up modes.
- The massless spectrum is matched using field redefinitions, after taking into account the orbifold mass terms.
- The anomaly cancellation mechanism in 4d is understood in both approaches. The blow–up modes transmute in the non–universal axions on the resolution.

Outlook:

- Explore the implications of recently studied orbifold selection rules to the possible orbifold-resolution maps.
- Constructing an algebraic description of global CY with gauge bundles.
- Study the deformations of the $(0, 2)$ orbifold SCFT by vevs of blow-up modes.
- Exploring the phenomenological consequences in more realistic models with non-anomalous U(1)s.

Conclusions

We established the correspondence between heterotic string on a deformed $T^6/\mathbb{Z}_{6II}$ orbifold in the mini-landscape with an heterotic compactification in a smooth CY 3-fold.

- Twisted fields which gain vevs to ensure a D–flat vacuum are identified with blow-up modes.
- The massless spectrum is matched using field redefinitions, after taking into account the orbifold mass terms.
- The anomaly cancellation mechanism in 4d is understood in both approaches. The blow–up modes transmute in the non–universal axions on the resolution.

Outlook:

- Explore the implications of recently studied orbifold selection rules to the possible orbifold-resolution maps.
- Constructing an algebraic description of global CY with gauge bundles.
- Study the deformations of the $(0, 2)$ orbifold SCFT by vevs of blow-up modes.
- Exploring the phenomenological consequences in more realistic models with non-anomalous U(1)s.

Thank you

References

W.Fischler, H.P.Nilles, J.Polchinski, S.Raby and L.Susskind
"Vanishing Renormalization of the D term in Supersymmetric U(1) Theories"

M.Blaszczyk, S.Groot Nibbelink, F.Ruehle
"Green-Schwarz Mechanism in Heterotic (2,0) Gauged Linear Sigma Models: Torsion and NS5 Branes"

Tatsuo Kobayashi, Susha L. Parameswaran, Saul Ramos-Sanchez, Ivonne Zavala
"R-charge conservation and more in factorizable an Non-factorizable orbifolds."

Oleg Lebedev, Hans Peter Nilles, Stuart Raby, Saúl Ramos-Sanchez, Michael Ratz, Patrick K.S. Vaudrevange, Akin Wingerter.
"A Mini-Landscape of Exact MSSM Spectra in Heterotic Orbifolds"

S. Groot Nibbelink, M. Trapletti and M. Walter(2007)
"Resolutions of C_n/Z_n Orbifolds, their U(1) Bundles and Applications to String Model Building"

References

G.Honecker and M.Trapletti.
"Merging Heterotic Orbifolds and K3 Compactifications with Line Bundles."

S. Groot Nibbelink, M. Trapletti and M. Walter(2007)
"Resolutions of C_n/Z_n Orbifolds, their U(1) Bundles and Applications to String Model Building"

S.Groot Nibbelink, J.Held, F.Ruehle, M.Trapletti and P.Vaudrevange(2009)
" Heterotic $\mathcal{Z}_{6II}$ MSSM Orbifolds in Blowup."

Wilfried Buchmuller, Jan Louis, Jonas Schmidt, Roberto Valandro
"Voisin-Borcea Manifolds and Heterotic Orbifold Models"

D.Lüst, S.Reffert, E.Scheideger and S.Stieberger
"Resolved toroidal orbifolds and their orientifolds"

S.Groot Nibbelink
"Heterotic orbifold resolutions as (2,0) gauged linear models."

S.Groot Nibbelink,H.P.Nilles and M.Trapletti(2007).
"Multiple anomalous U(1)s in heterotic blow-ups."

S. Groot Nibbelink, T.W. Ha, and M. Trapletti(2008)
"Toric Resolutions of Heterotic Orbifolds".

References

S. Groot Nibbelink, D. Klevers, F. Ploger, M. Trapletti, and P.K.S. Vaudrevange(2008).
"Compact heterotic orbifolds in blow-up"

M.Blaszczyk, S.Groot Nibbelink, F.Ruehle, M.Trapletti, P.K.S. Vaudrevange(2010).
"Heterotic MSSM on a Resolved Orbifold"

R. Blumenhagen, G. Honecker and T. Weigand,
Loop-corrected compactifications of the heterotic string with line bundles.

M.Blaszczyk, N.G.Cabo Bizet, H.P.Nilles, F.Ruehle
"A perfect match of MSSM-like orbifold and resolution models via anomalies."

Saúl Ramos-Sánchez
" The mu problem and the NMSSM with strings", Talk at SUSY 2013.

Paul Oehlman
"A Zip-code for quarks and leptons", Talk at SUSY 2013.

Damian Mayorga Peña
" Discrete R-symmetries from Heterotic Orbifolds", Talk at SUSY 2013.