Dark Matter Primordial Black Holes

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SUSY2013, ICTP

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Based on

arXiv: 1309.XXXX

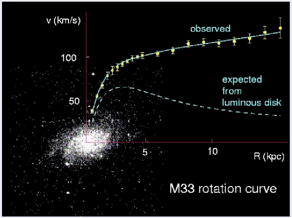
JCAP 1201 (2012) 035

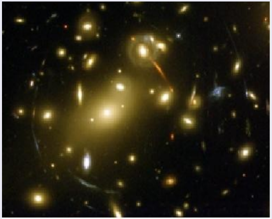
JCAP 1104 (2011) 005

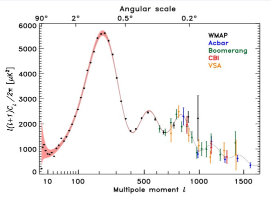
Outline

Dark Matter

Evidences







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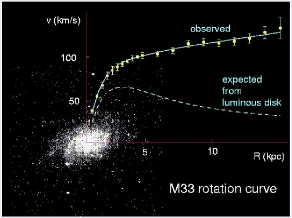

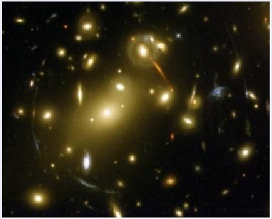

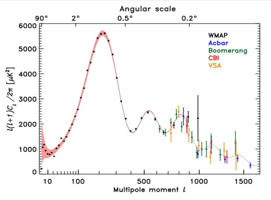

How much DM?

Planck.XVI, [arXiv: 1303.5076]

$$\Omega_{\rm DM} h^2 = 0.1196 \pm 0.0031$$

Properties

- sdtable
- neutral
- pressureless
- weakly interacting
- right relic density

Candidates

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- Sterile neutrinos
- WIMPs
- ...

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Any candidate in Standard Model?

Primordial Black Holes (PBHs)

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Definition

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RD era $\quad t \propto T^{-2} \longrightarrow M_{\rm PBH} = M_{\rm P}\left(\frac{T}{T_{\rm P}}\right)^{-2} \xrightarrow{T_{\rm RH}\simeq 10^{16}{\rm GeV}} M_{\rm min} = 1\,{\rm g}$

Hawking radiation

Temperature: $T_{\rm BH} \approx 10^{-7} \left(\frac{M}{M_\odot}\right)^{-1}$ K

$M > 10^{17}$	massless particles
$10^{15}\,{\rm g} \lesssim M \lesssim 10^{17}\,{\rm g}$	electrons
$10^{14}\,{\rm g} \lesssim M \lesssim 10^{15}\,{\rm g}$	muons
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Lifetime: $\tau_{\mathrm{BH}} \approx 10^{64} \left(\frac{M}{M_\odot} \right)^{3}$ y

M_{BH}	τ_{BH}
A man	$10^{-12}\,\mathrm{s}$
A building	$1\,\mathrm{s}$
$10^{15}\,\mathrm{g}$	$10^{10}\,\mathrm{y}$
The Earth	$10^{49}\,\mathrm{y}$
The Sun	$10^{66}\,\mathrm{y}$
The Galaxy	$10^{99}\,\mathrm{y}$

Why PBHs are useful?

- PBHs as a probe of the early Universe $(M < 10^{15}\,\mathrm{g})$
- PBHs as a probe of gravitational collapse $(M > 10^{15}\,\mathrm{g})$ ✓
  DM candidates $\Omega^0_{\mathrm{PBH}} \lesssim \Omega^0_{\mathrm{CDM}} (= 0.23)$
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How PBHs form?

- Soft equation of state
- Bubble collisions
- Collapse of cosmic loops
- Fluctuations by inflation ✓

Inflation

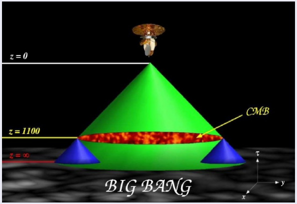
Accelerated expansion of the Universe $\ddot{a} > 0$

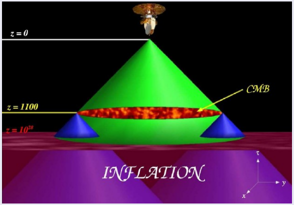
Why inflation?

- Flatness problem

$$\Omega_0 - 1 = (\Omega_i - 1)\left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2$$

- Horizon problem





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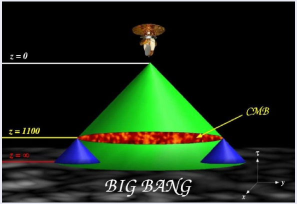

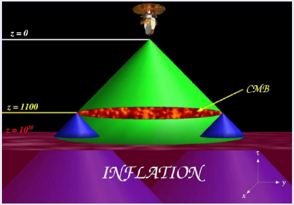

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \xrightarrow{\ddot{a}>0} w < -\frac{1}{3}$$

Scenario

Equation of motion $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \xrightarrow{V(\phi) \gg \dot{\phi}^2} 3H\dot{\phi} \simeq -V'(\phi)$

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Slow-roll parameters

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2$$

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Inflation parameters

$$n_s = 1 - 6\epsilon + 2\eta$$

$$\alpha_s = -24\epsilon^2 + 16\epsilon\eta - 2\xi^2$$

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Scale dependent spectral index

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$$n(k) = n_{\rm s}(k_0) + \frac{1}{2!}\alpha_{\rm s}(k_0)\ln\left(\frac{k}{k_0}\right) + \ldots$$

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Planck+WP+highL

Planck.XXII ,[arXiv: 1303.5082]

$$n_s(k_{\rm pivot}) = 0.9570 \pm 0.0075$$

$$\alpha_s(k_{\rm pivot}) = -0.022^{+0.011}_{-0.010}$$

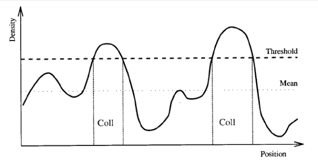
$$\ln\left(10^{10}\mathcal{P}_{\mathcal{R}_c}(k_{\rm pivot})\right) = 2.198 \pm 0.056$$

$$r_{0.002} < 0.23 \quad (68\%\ \mathrm{CL})$$

$$k_{\rm pivot} = 0.05\ \mathrm{Mpc}^{-1}$$

Press-Schechter Formalism

The Press-Schechter formalism is a model for predicting the number density of bound objects of a certain mass.



$$f(\geq M) = 2\,\gamma \int_{\delta_{\rm th}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_\delta(R)} \exp\left(-\frac{\delta^2(R)}{2\sigma_\delta^2(R)}\right) d\delta(M)$$

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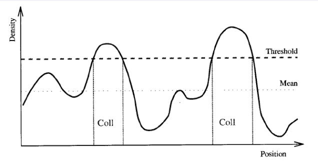

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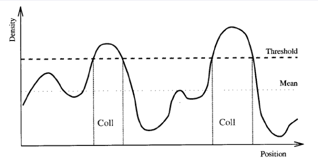

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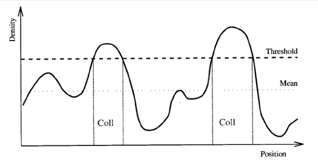

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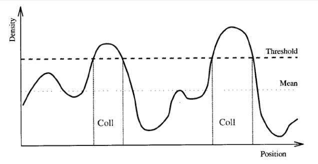

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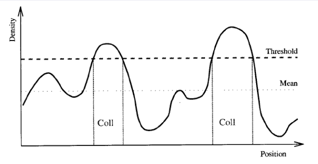

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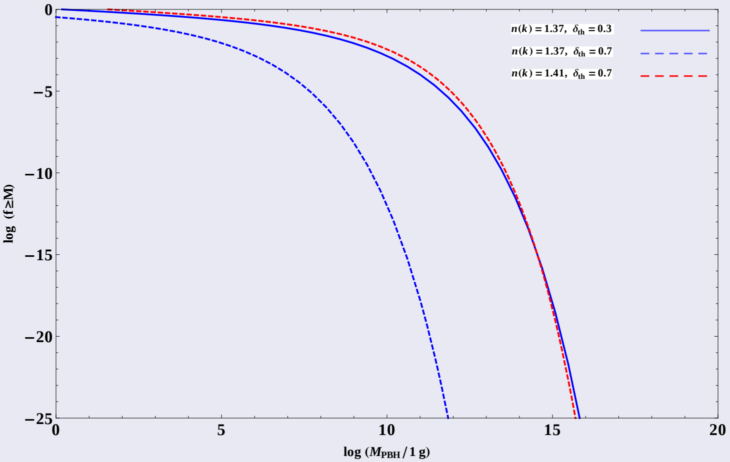
$$M_{\rm PBH} = \gamma M_{\rm PH} \xrightarrow{\gamma = w^{3/2}} \frac{R}{1\,\mathrm{Mpc}} = 5.5\times 10^{-24}\gamma^{-\frac{1}{2}}\left(\frac{M_{\rm PBH}}{1\,\mathrm{g}}\right)^{1/2}\left(\frac{g_*}{3.36}\right)^{1/6}$$

$f(\geq M)$ diagram for the mass range $10^0 - 10^{20}\,\mathrm{g}$



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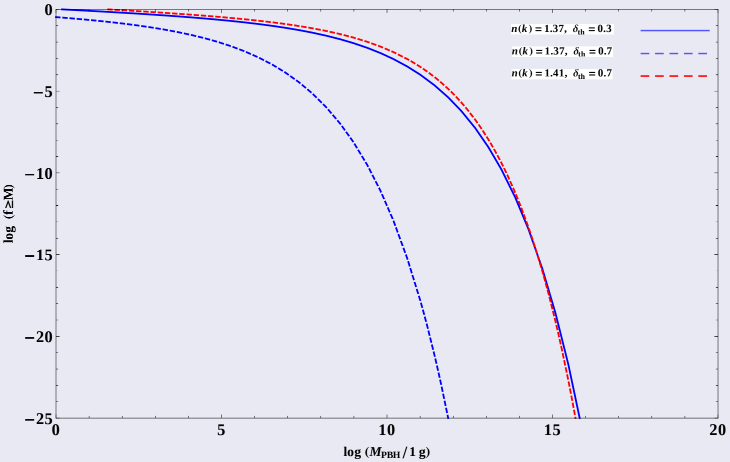

Result

$n_s(k_{\rm PBH}) \geq 1.37$ for long–lived PBHs

Solution ?

Running of the running of the spectral index

$$n(k) = n_s(k_0) + \frac{1}{2!}\,\alpha_s(k_0)\ln\left(\frac{k}{k_0}\right)$$

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$$0 < \beta_s < 0.0126 \quad (2\,\sigma)$$

$$k \in \left[2.4 \times 10^{-4},\, 10\right] \mathrm{Mpc}^{-1}$$

$$\alpha_s \neq 0 \Rightarrow \beta_s \lesssim 0.00169$$

$$\alpha_s = 0 \Rightarrow \beta_s \lesssim 0.00176$$

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Planck+WP+highL (68% CL)

$n_s(k_{\rm pivot}) = 0.9476^{+0.086}_{-0.088}$

$\alpha_s(k_{\rm pivot}) = 0.001^{+0.013}_{-0.014}$

$\beta_s(k_{\rm pivot}) = 0.022^{+0.016}_{-0.013}$

Hilltop/inflection point inflation

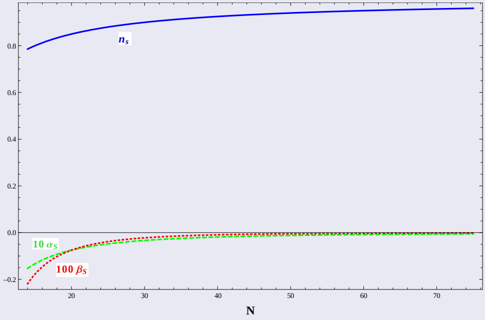
$$V(\phi) = V_0\left[1 - \left(\frac{\phi}{\mu}\right)^p\right]$$ ☒

for $p > 2$

$$n_s - 1 \simeq -\frac{p-1}{p-2}\frac{2}{N} < 0$$

$$\alpha_s \simeq -\frac{p-1}{p-2}\frac{2}{N^2} < 0$$

$$\beta_s \simeq -\frac{p-1}{p-2}\frac{2}{N^3} < 0$$



This model is not ruled out by *Planck*.

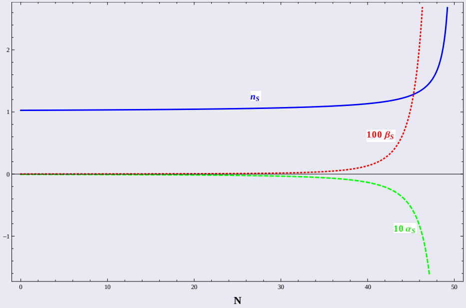
Inverse power low inflation

$$V(\phi) = V_0 + \frac{\Lambda_3^{p+4}}{\phi^p}$$ ⊠

$$n_s - 1 \simeq \frac{p+1}{p+2}\frac{2}{N_{\rm tot}\left(1-\frac{N}{N_{\rm tot}}\right)}$$

$$\alpha_s \simeq -\frac{p+2}{p+1}\frac{(n_{\rm s}-1)^2}{2}$$

$$\beta_s \simeq \left(\frac{p+2}{p+1}\right)^2\frac{(n_{\rm s}-1)^3}{2}$$



This model is disfavoured by *Planck* for any p.

Running-mass inflation

The inflation potential is dominated by the soft SUSY breaking mass term generated by V_0 and its radiative corrections

$$V(\phi) = V_0 + \frac{1}{2} m_\phi^2(\phi)\phi^2 + ...$$

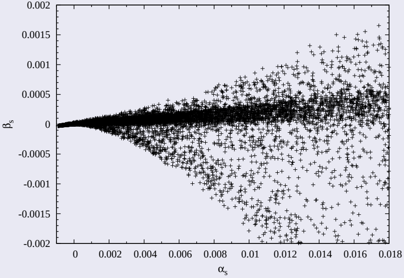
RGE $$\frac{dm^2}{d\ln\phi} \equiv \beta_m \qquad \text{with} \qquad \beta_m = -\frac{2C}{\pi}\alpha\,\tilde{m}^2 + \frac{D}{16\pi^2}|\lambda_Y|^2 m_{\mathrm{s}}^2$$

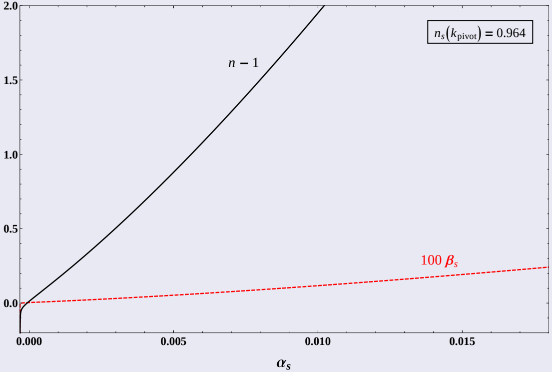
Over a sufficiently small range of ϕ, or small inflaton coupling, we can do the Taylor expansion:

$$V = V_0 + \frac{1}{2} m_\phi^2(\phi_*)\phi^2 + \frac{1}{2}\left.\frac{dm_\phi^2}{d\ln\phi}\right|_{\phi_*} \ln\left(\frac{\phi}{\phi_*}\right) + \frac{1}{4}\left.\frac{d^2 m_\phi^2}{d(\ln\phi)^2}\right|_{\phi_*} \ln^2\left(\frac{\phi}{\phi_*}\right)$$

where ϕ_* is the local extremum of the potential.

$$\alpha_s \geq -\frac{(n_s - 1)^2}{4}$$





Large Field Models $\quad |\triangle\phi| \gtrsim M_{\rm P}\,, \quad r \neq 0$

Chaotic inflation

$$V(\phi) = \Lambda^4 \left(\frac{\phi}{\mu}\right)^p \quad \boxtimes$$

$$\begin{aligned} n_s - 1 &= -\frac{2(p+2)}{4N+p} \\ \alpha_s &= -\frac{2}{p+2}(n_{\rm s}-1)^2 \\ \beta_s &= \frac{8}{(p+2)^2}(n_{\rm s}-1)^3 \\ r &= -\frac{7p}{p+2}(n_{\rm s}-1) \end{aligned}$$

This model is ruled out with *Planck* data for $p = 4$.

Natural inflation

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right] \quad \boxtimes$$

$$\begin{aligned} n_s - 1 &\propto -\frac{2}{N} < 0 \\ \alpha_s &\propto -\frac{2}{N^2} < 0 \\ \beta_s &\propto -\frac{2}{N^3} < 0 \end{aligned}$$

This model agrees with *Planck*+WP data for $f \gtrsim 5\, M_{\rm P}$.

Negative exponential inflation

$$V(\phi) = V_0 \left[1 - \exp\left(\frac{-q\phi}{M_{\rm P}}\right)\right] , \quad q > 0 \quad \boxtimes$$

$$n_s - 1 \simeq -2/(N+1) < 0$$
$$\alpha_s \simeq -2/(N+1)^2 < 0$$
$$\beta_s \simeq -4/(N+1)^3 < 0$$

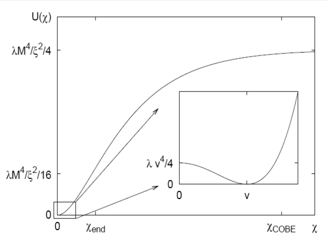
This model is ruled out by *Planck*.

- Higgs Inflation

$$S_{\rm J} = \int {\rm d}^4x\sqrt{-g}\left\{-\frac{M^2+\xi\, h^2}{2}R + \frac{\partial_\mu h\,\partial^\mu h}{2} - \frac{\lambda}{4}(h^2 - h_0^2)^2\right\}$$

$$U(\chi) = \frac{\lambda M_{\rm P}^4}{4\xi^2}\left[1 - \exp\left(-\frac{2\chi}{\sqrt{6}M_{\rm P}}\right)\right]^2 \quad \text{where } h \simeq \frac{M_{\rm P}}{\sqrt{\xi}}\exp\left(\frac{\chi}{\sqrt{6}M_{\rm P}}\right) \quad \boxtimes$$

$$n_s - 1 \simeq -\frac{8}{3}\frac{M_{\rm P}^2}{\xi\, h^2} < 0$$
$$\alpha_s = -(n_{\rm s} - 1)^2/2 < 0$$
$$\beta_s = (n_{\rm s} - 1)^3/2 < 0$$

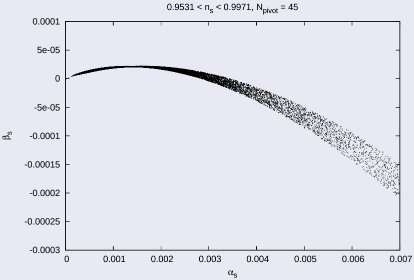


This model is fully consistent with *Planck* constraints.

Generalized exponential inflation

$$V(\phi) = \Lambda^4 e^{(\phi/\mu)^p} \quad \boxtimes$$

for $1 < p < 2$



Arctan inflation

$$V = V_0 \left[1 + \frac{2}{\pi} \arctan\left(\frac{\phi}{\mu}\right)\right] \quad \boxtimes$$

$$n_s - 1 \simeq -\frac{4}{3N+\pi} < 0$$

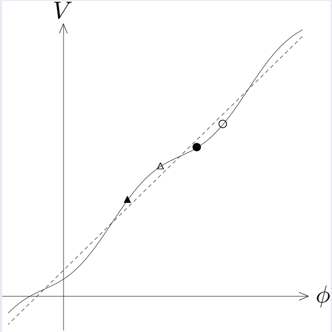
$$\alpha_s = -\frac{3}{4}(n_s - 1)^2 < 0$$

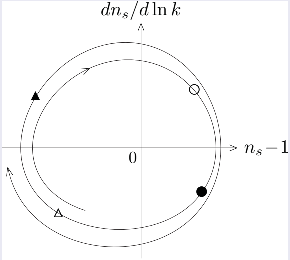
$$\beta_s = \frac{9}{8}(n_s - 1)^3 < 0$$

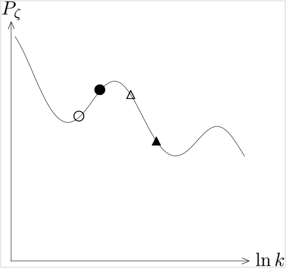
Modulated Inflation

$$V(\phi) = V_0(\phi) + V_{\rm mod}(\phi) \quad \text{where} \quad |V_0(\phi)| \gg |V_{\rm mod}(\phi)|$$

$$V(\phi) = V_0(\phi) + \Lambda^4 \cos\left(\frac{\phi}{f} + \theta\right)$$







Kobayashi and Takahashi, JCAP 1101 (2011) 026

Conclusions

- The fluctuation which arise at inflation are the most likely source of PBHs and for their formation, the fluctuation amplitude should increase with decreasing scale.
- The value of spectral index should be larger than 1.37 in the scale corresponding to PBHs with mass larger than $10^{15}\,\mathrm{g}$.
- The COBE normalization on Lyman-α range puts an upper bound on the running of the running spectral index, $\beta_s < 0.0126\,(2\,\sigma)$.
- Except running-mass inflation model, most of the single field inflation models can not accommodate long–lived PBHs formation.
- If future data confirm with high precision that $\alpha_s \lesssim -0.01$, all simple single–field models of inflation would be excluded. ✓
  Similarly, proving conclusively that the second derivative of the spectral index is positive would exclude all the large–field models we investigated. ?

DM PBHs may had masses similar to that of mount Everest.

Thank you

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Carr *et.al*, PRD 81 (2010) 104019