Checking for undesired vacua quickly at the 1-loop level

Introducing `Vevacious`

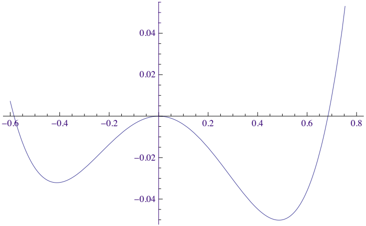
Ben O'Leary
in collaboration with
José Eliel Camargo Molina, Werner Porod, and Florian Staub

Julius-Maximilians-Universität Würzburg

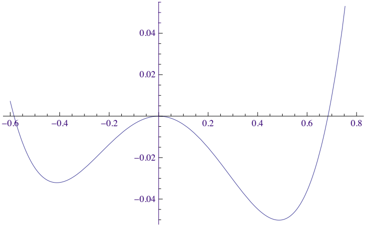
Higgs parallel session,
SUSY2013, ICTP Trieste,
August 26th, 2013

QFT potentials typically have multiple minima

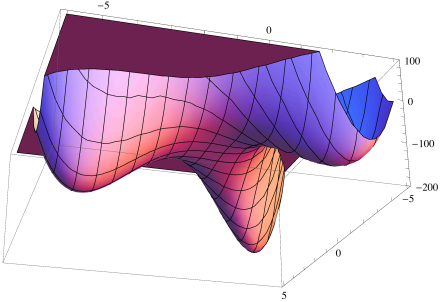
Even tree-level potentials for single scalars have in general multiple minima:



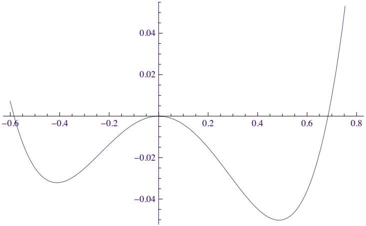
Even tree-level potentials for single scalars have in general multiple minima:



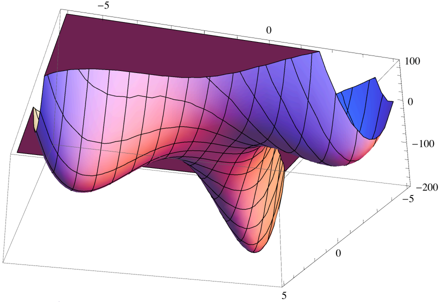
Multiple scalars in general yield many vacua:



Even tree-level potentials for single scalars have in general multiple minima:

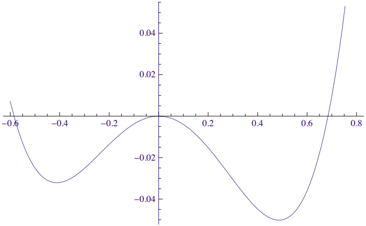


Multiple scalars in general yield many vacua:

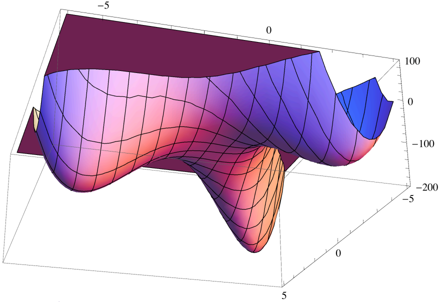


Finding global minimum not trivial!

Even tree-level potentials for single scalars have in general multiple minima:



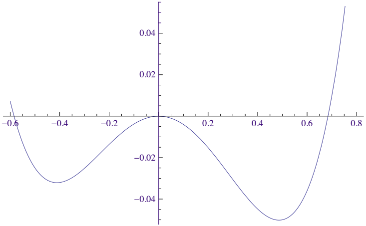
Multiple scalars in general yield many vacua:



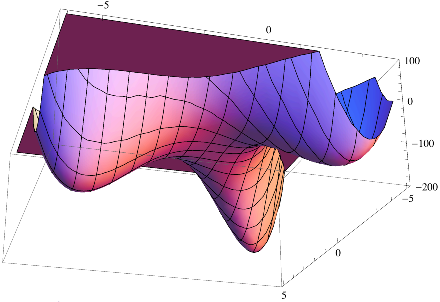
Finding global minimum not trivial!

- Charge- and/or color-breaking (CCB) minima ($\tilde{\tau}, \tilde{t}$ VEVs)?

Even tree-level potentials for single scalars have in general multiple minima:



Multiple scalars in general yield many vacua:



Finding global minimum not trivial!

- Charge- and/or color-breaking (CCB) minima ($\tilde{\tau}, \tilde{t}$ VEVs)?
- Desired VEV combination may not be global minimum (even non-CCB: NMSSM)

Recent progress

Gröbner bases:

Gröbner bases:

- Decomposition of system using fancy algebra
- Has been used to investigate NMSSM
  (Maniatis, von Manteuffel, Nachtmann, arXiv:hep-ph/0608314, EJPC)
- Computationally expensive, especially in terms of RAM

Gröbner bases:

- Decomposition of system using fancy algebra
- Has been used to investigate NMSSM
  (Maniatis, von Manteuffel, Nachtmann, arXiv:hep-ph/0608314, EJPC)
- Computationally expensive, especially in terms of RAM

Homotopy continuation:

Gröbner bases:

- Decomposition of system using fancy algebra
- Has been used to investigate NMSSM
  (Maniatis, von Manteuffel, Nachtmann, arXiv:hep-ph/0608314, EJPC)
- Computationally expensive, especially in terms of RAM

Homotopy continuation:

- Gradual deformation of simple system of equations into target system
- Has been used to investigate SM with up to 5 extra scalars
  (Maniatis, Mehta, arXiv:1203.0409, EPJ+)
- $\exists$ public codes and programs: `PHCpack`, `Bertini`, `HOM4PS2`

Gröbner bases:

- Decomposition of system using fancy algebra
- Has been used to investigate NMSSM
  (Maniatis, von Manteuffel, Nachtmann, arXiv:hep-ph/0608314, EJPC)
- Computationally expensive, especially in terms of RAM

Homotopy continuation:

- Gradual deformation of simple system of equations into target system
- Has been used to investigate SM with up to 5 extra scalars
  (Maniatis, Mehta, arXiv:1203.0409, EPJ+)
- $\exists$ public codes and programs: `PHCpack`, `Bertini`, `HOM4PS2`

Thus far only implemented on a model-by-model basis!

Gröbner bases:

- Decomposition of system using fancy algebra
- Has been used to investigate NMSSM
  (Maniatis, von Manteuffel, Nachtmann, arXiv:hep-ph/0608314, EJPC)
- Computationally expensive, especially in terms of RAM

Homotopy continuation:

- Gradual deformation of simple system of equations into target system
- Has been used to investigate SM with up to 5 extra scalars
  (Maniatis, Mehta, arXiv:1203.0409, EPJ+)
- $\exists$ public codes and programs: `PHCpack`, `Bertini`, `HOM4PS2`

Thus far only implemented on a model-by-model basis! Only done at tree-level!

Download Vevacious! `http://vevacious.hepforge.org/`

`Vevacious` is a new, publicly-available code, that:

Download Vevacious! http://vevacious.hepforge.org/

Vevacious is a new, publicly-available code, that:

- takes a model file (automatically generated by SARAH)

`Vevacious` is a new, publicly-available code, that:

- takes a model file (automatically generated by `SARAH`)
- takes an `SLHA` file

Vevacious is a new, publicly-available code, that:

- takes a model file (automatically generated by SARAH)
- takes an SLHA file
- prepares and runs homotopy continuation code (HOM4PS2) to find *all* tree-level extrema

Vevacious is a new, publicly-available code, that:

- takes a model file (automatically generated by SARAH)
- takes an SLHA file
- prepares and runs homotopy continuation code (HOM4PS2) to find *all* tree-level extrema
- prepares and runs gradient minimization code (PyMinuit) to account for loop corrections

Vevacious is a new, publicly-available code, that:

- takes a model file (automatically generated by SARAH)
- takes an SLHA file
- prepares and runs homotopy continuation code (HOM4PS2) to find *all* tree-level extrema
- prepares and runs gradient minimization code (PyMinuit) to account for loop corrections
- calculates tunneling time to undesired minima if found (CosmoTransitions)

`Vevacious` is a new, publicly-available code, that:

- takes a model file (automatically generated by `SARAH`)
- takes an `SLHA` file
- prepares and runs homotopy continuation code (`HOM4PS2`) to find *all* tree-level extrema
- prepares and runs gradient minimization code (`PyMinuit`) to account for loop corrections
- calculates tunneling time to undesired minima if found (`CosmoTransitions`)

Fast enough for scans! MSSM with additional non-zero VEVs for $\tilde{\tau}_L, \tilde{\tau}_R, \tilde{t}_L, \tilde{t}_R$: global minimum found within 5s on my laptop. (Tunneling time calculation varies: less than a second, up to 10 minutes.)

`Vevacious` is a new, publicly-available code, that:

- takes a model file (automatically generated by `SARAH`)
- takes an `SLHA` file
- prepares and runs homotopy continuation code (`HOM4PS2`) to find *all* tree-level extrema
- prepares and runs gradient minimization code (`PyMinuit`) to account for loop corrections
- calculates tunneling time to undesired minima if found (`CosmoTransitions`)

Fast enough for scans! MSSM with additional non-zero VEVs for $\tilde{\tau}_L, \tilde{\tau}_R, \tilde{t}_L, \tilde{t}_R$: global minimum found within 5s on my laptop. (Tunneling time calculation varies: less than a second, up to 10 minutes.)

`http://vevacious.hepforge.org/`

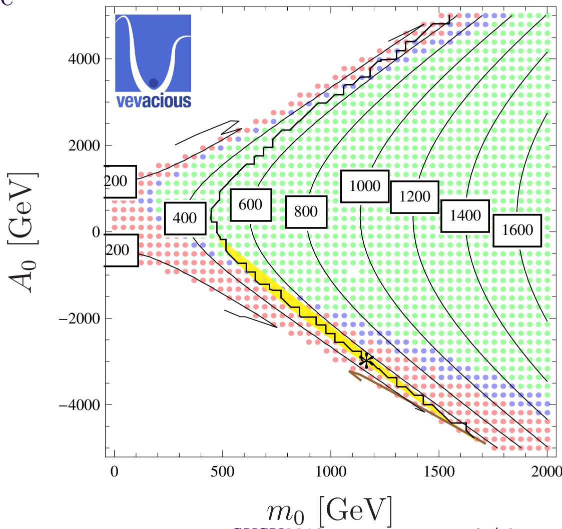
CCB restricts $\tilde{\tau}$ co-annihilation

$M_{1/2} = 1110$ GeV, $\tan\beta = 39.3$, $\mu > 0$; $m_{\tilde{\tau}_1}$ (GeV) contours
red: short-lived metastable ($\tau_{\text{tunnel}} < 1.4$ Gy)
blue: long-lived metastable
green: stable

yellow region:
correct relic density

star:
best-fit point of
`arXiv:1204.4199`
(Fittino)

wiggly line through
yellow:
neutralino LSP border



Vevacious is fast enough for scans

`Vevacious` is fast enough for scans

- Creating model file with `SARAH` takes minutes
- Evaluating stability of a parameter point depends on model

- Creating model file with `SARAH` takes minutes
- Evaluating stability of a parameter point depends on model

For example, MSSM (not just CMSSM), v_d, v_u+ 4 extra VEVs for $\tilde{\tau}_{L,R}, \tilde{t}_{L,R}$, on my laptop

- Creating model file with SARAH takes minutes
- Evaluating stability of a parameter point depends on model

For example, MSSM (not just CMSSM), $v_d, v_u +$ 4 extra VEVs for $\tilde{\tau}_{L,R}, \tilde{t}_{L,R}$, on my laptop

- Finding *all* tree-level extrema takes $< 0.5s$

Vevacious is fast enough for scans

- Creating model file with SARAH takes minutes
- Evaluating stability of a parameter point depends on model

For example, MSSM (not just CMSSM), v_d, v_u+ 4 extra VEVs for $\tilde{\tau}_{L,R}, \tilde{t}_{L,R}$, on my laptop

- Finding *all* tree-level extrema takes $< 0.5s$
- Determining 1-loop global minimum takes $3s$

Vevacious is fast enough for scans

- Creating model file with SARAH takes minutes
- Evaluating stability of a parameter point depends on model

For example, MSSM (not just CMSSM), $v_d, v_u +$ 4 extra VEVs for $\tilde{\tau}_{L,R}, \tilde{t}_{L,R}$, on my laptop

- Finding *all* tree-level extrema takes $< 0.5s$
- Determining 1-loop global minimum takes $3s$
- Estimating tunneling time stongly depends on relative depth and location of global minimum compared to input minimum: $15s$ typical, $500s$ for borderline cases

Analytic conditions

$$V^{\text{tree}} = \frac{1}{32}\left(g_1^2(v_d^2 - v_u^2 + v_{\tilde{\tau}_L}^2 - 2v_{\tilde{\tau}_R}^2)^2 + g_2^2(v_d^2 - v_u^2 - v_{\tilde{\tau}_L}^2)^2\right) - B_\mu v_d v_u + \frac{1}{2}\left(|\mu|^2(v_d^2 + v_u^2) + m_{H_d}^2 v_d^2 + m_{H_u}^2 v_u^2 + m_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 v_{\tilde{\tau}_R}^2\right) + \frac{1}{4}\left(Y_\tau^2(v_d^2 v_{\tilde{\tau}_L}^2 + v_d^2 v_{\tilde{\tau}_R}^2 + v_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_R}^2\right) + \frac{Y_\tau}{\sqrt{2}} v_{\tilde{\tau}_L} v_{\tilde{\tau}_R}\left(A_\tau v_d - \mu v_u\right) + \ldots$$

Analytic conditions

$$V^{\text{tree}} = \frac{1}{32}\left(g_1^2(v_d^2 - v_u^2 + v_{\tilde{\tau}_L}^2 - 2v_{\tilde{\tau}_R}^2)^2 + g_2^2(v_d^2 - v_u^2 - v_{\tilde{\tau}_L}^2)^2\right) - B_\mu v_d v_u + \frac{1}{2}\left(|\mu|^2(v_d^2 + v_u^2) + m_{H_d}^2 v_d^2 + m_{H_u}^2 v_u^2 + m_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 v_{\tilde{\tau}_R}^2\right) + \frac{1}{4}\left(Y_\tau^2(v_d^2 v_{\tilde{\tau}_L}^2 + v_d^2 v_{\tilde{\tau}_R}^2 + v_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_R}^2\right) + \frac{Y_\tau}{\sqrt{2}} v_{\tilde{\tau}_L} v_{\tilde{\tau}_R}\left(A_\tau v_d - \mu v_u\right) + \ldots$$

Some conditions in the literature have often been (mis-)used:

$$V^{\text{tree}} = \frac{1}{32}\left(g_1^2(v_d^2 - v_u^2 + v_{\tilde{\tau}_L}^2 - 2v_{\tilde{\tau}_R}^2)^2 + g_2^2(v_d^2 - v_u^2 - v_{\tilde{\tau}_L}^2)^2\right) - B_\mu v_d v_u + \frac{1}{2}\left(|\mu|^2(v_d^2 + v_u^2) + m_{H_d}^2 v_d^2 + m_{H_u}^2 v_u^2 + m_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 v_{\tilde{\tau}_R}^2\right) + \frac{1}{4}\left(Y_\tau^2(v_d^2 v_{\tilde{\tau}_L}^2 + v_d^2 v_{\tilde{\tau}_R}^2 + v_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_R}^2\right) + \frac{Y_\tau}{\sqrt{2}} v_{\tilde{\tau}_L} v_{\tilde{\tau}_R}\left(A_\tau v_d - \mu v_u\right) + \ldots$$

Some conditions in the literature have often been (mis-)used:

- $A_\tau^2 < 3(m_{H_d}^2 + |\mu|^2 + m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2)$ [“A_τ”]

$$V^{\text{tree}} = \frac{1}{32}\left(g_1^2(v_d^2 - v_u^2 + v_{\tilde{\tau}_L}^2 - 2v_{\tilde{\tau}_R}^2)^2 + g_2^2(v_d^2 - v_u^2 - v_{\tilde{\tau}_L}^2)^2\right) - B_\mu v_d v_u + \frac{1}{2}\left(|\mu|^2(v_d^2 + v_u^2) + m_{H_d}^2 v_d^2 + m_{H_u}^2 v_u^2 + m_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 v_{\tilde{\tau}_R}^2\right) + \frac{1}{4}\left(Y_\tau^2(v_d^2 v_{\tilde{\tau}_L}^2 + v_d^2 v_{\tilde{\tau}_R}^2 + v_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_R}^2\right) + \frac{Y_\tau}{\sqrt{2}} v_{\tilde{\tau}_L} v_{\tilde{\tau}_R}\left(A_\tau v_d - \mu v_u\right) + \ldots$$

Some conditions in the literature have often been (mis-)used:

- $A_\tau^2 < 3(m_{H_d}^2 + |\mu|^2 + m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2)$ ["A_τ"]
- $A_t^2 < 3(m_{H_u}^2 + |\mu|^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2)$ ["A_t"]

$$V^{\text{tree}} = \frac{1}{32}\left(g_1^2(v_d^2 - v_u^2 + v_{\tilde{\tau}_L}^2 - 2v_{\tilde{\tau}_R}^2)^2 + g_2^2(v_d^2 - v_u^2 - v_{\tilde{\tau}_L}^2)^2\right) - B_\mu v_d v_u + \frac{1}{2}\left(|\mu|^2(v_d^2 + v_u^2) + m_{H_d}^2 v_d^2 + m_{H_u}^2 v_u^2 + m_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 v_{\tilde{\tau}_R}^2\right) + \frac{1}{4}\left(Y_\tau^2(v_d^2 v_{\tilde{\tau}_L}^2 + v_d^2 v_{\tilde{\tau}_R}^2 + v_{\tilde{\tau}_L}^2 v_{\tilde{\tau}_R}^2\right) + \frac{Y_\tau}{\sqrt{2}} v_{\tilde{\tau}_L} v_{\tilde{\tau}_R} (A_\tau v_d - \mu v_u) + \ldots$$

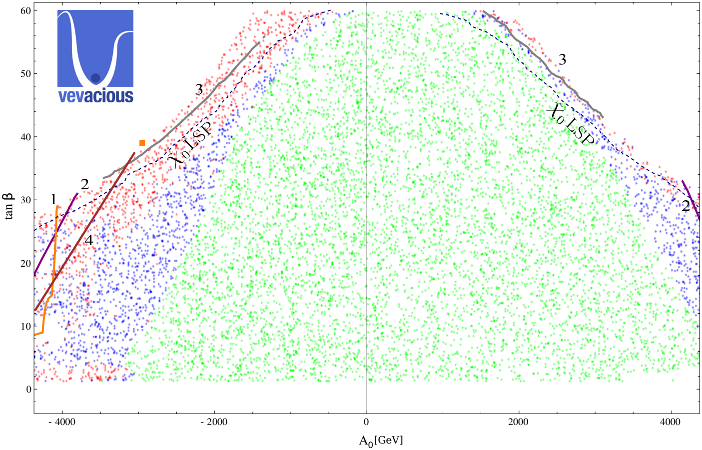
Some conditions in the literature have often been (mis-)used:

- $A_\tau^2 < 3(m_{H_d}^2 + |\mu|^2 + m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2)$ ["A_τ"]
- $A_t^2 < 3(m_{H_u}^2 + |\mu|^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2)$ ["A_t"]
- $|(Y_\tau v_u \mu)/(\sqrt{2} m_\tau)| < 56.9\sqrt{m_{\tilde{\tau}_L} m_{\tilde{\tau}_R}} + 57.1(m_{\tilde{\tau}_L} + 1.03 m_{\tilde{\tau}_R}) - 1.28 \times 10^4 \text{GeV} + \frac{1.67\times 10^6 \text{GeV}^2}{m_{\tilde{\tau}_L} + m_{\tilde{\tau}_R}} - 6.41 \times 10^6 \text{GeV}^3 (\frac{1}{m_{\tilde{\tau}_L}^2} + \frac{0.983}{m_{\tilde{\tau}_R}^2})$ ["numeric"]

("A_τ", "A_t": L. Alvarez-Gaumé, J. Polchinski, M. Wise, Nucl. Phys. B221;
"numeric": Kitahara, Yoshinaga, arXiv:1303.0461, JHEP)

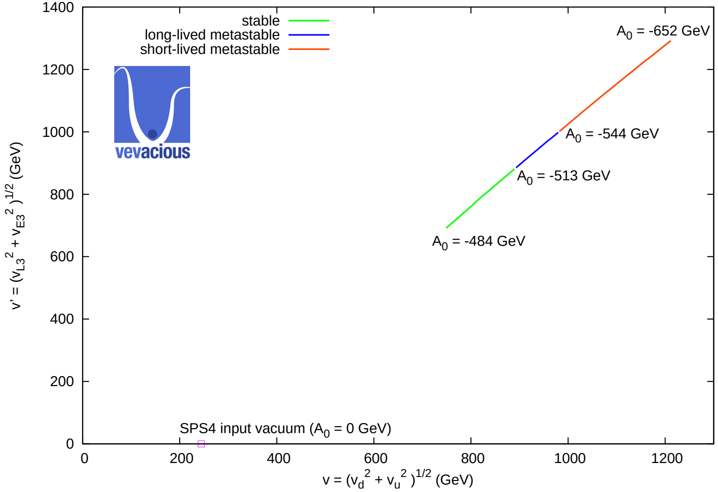
Existing formulae rather weak

$M_{1/2} = 1000$ GeV, $m_0 = 1000$ GeV, $\mu > 0$



Purple: "A_τ"; Orange: "A_t"; Grey: "numeric"
(Dark red: variant on "A_τ")

Evolution of a CCB minimum



$$m_0 = 400\ \text{GeV},\ M_{1/2} = 300\ \text{GeV},\ \tan\beta = 50,\ \mu > 0$$

Minimizing potentials not trivial:

vevacious

Minimizing potentials not trivial:

- Vitally important if extending Higgs sector with extra scalars

Minimizing potentials not trivial:

- Vitally important if extending Higgs sector with extra scalars
- Multiple VEV-ing fields $\rightarrow$ rich vacuum structure

Minimizing potentials not trivial:

- Vitally important if extending Higgs sector with extra scalars
- Multiple VEV-ing fields → rich vacuum structure
- Difficult, but feasible with modern techniques

Minimizing potentials not trivial:

- Vitally important if extending Higgs sector with extra scalars
- Multiple VEV-ing fields $\rightarrow$ rich vacuum structure
- Difficult, but feasible with modern techniques
- Automated by `SARAH` + `Vevacious`

Minimizing potentials not trivial:

- Vitally important if extending Higgs sector with extra scalars
- Multiple VEV-ing fields → rich vacuum structure
- Difficult, but feasible with modern techniques
- Automated by `SARAH` + `Vevacious`

The CMSSM is an excellent example:

Minimizing potentials not trivial:

- Vitally important if extending Higgs sector with extra scalars
- Multiple VEV-ing fields → rich vacuum structure
- Difficult, but feasible with modern techniques
- Automated by `SARAH` + `Vevacious`

The CMSSM is an excellent example:

- Non-trivial VEV structure

Minimizing potentials not trivial:

- Vitally important if extending Higgs sector with extra scalars
- Multiple VEV-ing fields $\rightarrow$ rich vacuum structure
- Difficult, but feasible with modern techniques
- Automated by `SARAH` + `Vevacious`

The CMSSM is an excellent example:

- Non-trivial VEV structure
- Often has CCB global minimum

Minimizing potentials not trivial:

- Vitally important if extending Higgs sector with extra scalars
- Multiple VEV-ing fields $\rightarrow$ rich vacuum structure
- Difficult, but feasible with modern techniques
- Automated by `SARAH` + `Vevacious`

The CMSSM is an excellent example:

- Non-trivial VEV structure
- Often has CCB global minimum

Thank you for your attention!

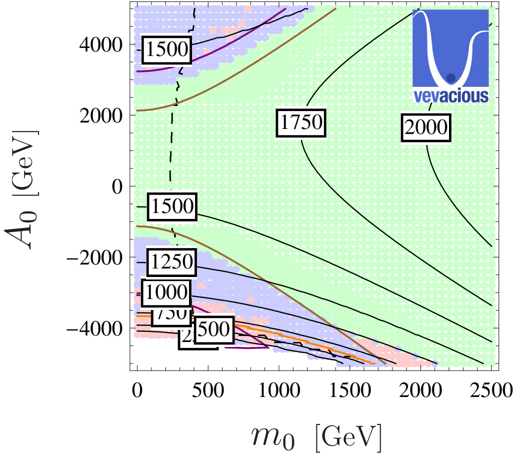
Backup slides

$M_{1/2} = 1000$ GeV, $\tan\beta = 10$, $\mu > 0$; $m_{\tilde{t}_1}$ (GeV) contours

orange line:
A_t condition border

purple line:
A_τ condition border

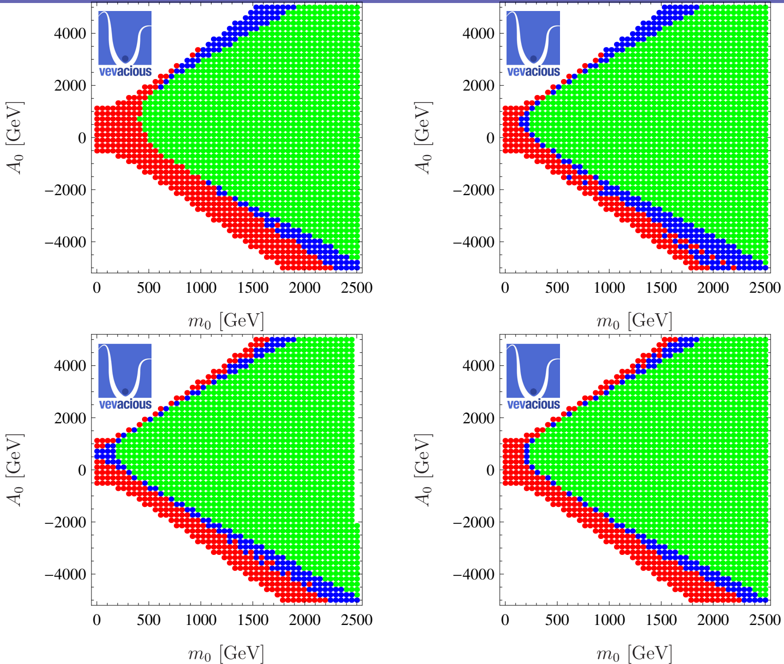
dashed black line:
neutralino LSP border



Tunneling times

- $\Gamma/$ volume $= Ae^{-B/\hbar}(1 + \mathcal{O}(\hbar))$
- A is solitonic solution, should be $\sim$ energy scale of potential
- $B \sim$ ([surface tension]/[energy density difference])^3
- typically TeV-scale energy barriers, energy depth differences $\Rightarrow$ roughly tunneling times of (factors of $16\pi^2$ *etc.*)/TeV $\ll$ age of Universe

Scale and loop order dependence: halving Q



Scale and loop order dependence: doubling Q