Discrete *R* Symmetries and Anomalies

arXiv:1212.4371

Angelo Monteux
in collaboration with M. Dine

SCIPP, Physics Department
University of California, Santa Cruz

SUSY 2013 - ICTP Trieste - August 26, 2013

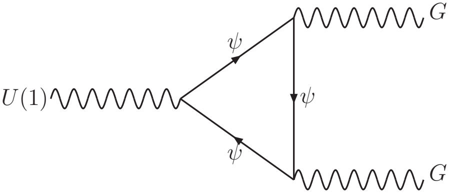
Outline

1. Anomalies
   - Anomaly cancellation
   - Discrete gauge R-symmetries
2. Non-universal anomaly cancellation in type IIB string theory
3. Conclusions

Anomalies in QFT

A symmetry of the classical action can become anomalous at the quantum level: if a global symmetry, it just means that selection rules are broken at the quantum level. If a gauge symmetry, it cannot be anomalous at all.

In four dimensons, anomalies arise through the triangle diagram



In the SM and MSSM, all the anomalies just cancel.

For mixed gauge-$U(1)$ anomalies, it is not strictly necessary that they cancel when considering known SM fields: we can have a Green-Schwarz mechanism, with an axion-like field transforming non-linearly. The gauge symmetry is spontaneously broken and the gauge boson is massive. At low energies, a global $U(1)$ symmetry is left.

4D Green-Schwarz mechanism

$$\mathcal{L} \supset \int d^2\theta \frac{S}{4} \operatorname{Tr} W_\alpha^2 + c.c., \qquad S = s + ia$$

Under a chiral transformation

$$\psi_i \to e^{iq_i \alpha}\psi_i \qquad \mathcal{A}_{GGU(1)} = \sum_f \ell(\mathbf{r}_f) q_f$$

$$S \to S + \frac{i}{2}\delta_{GS}\alpha$$

$$\mathcal{L} \to \mathcal{L} + \frac{\alpha}{32\pi^2} F^a \tilde{F}^a \mathcal{A}_{GGZ_N} - \frac{\alpha}{16}\delta_{GS} F^a \tilde{F}^a$$

With appropriate non-linear axion transformation, the anomaly is cancelled. If multiple gauge groups,

$$\int d^2\theta \, c_i S W_\alpha^{(i)} W_\alpha^{(i)},$$

Non-universal $c_i's \implies$ non-universal anomalies $\mathcal{A}_{G_i G_i U(1)}$. But, if the saxion has a vev, it gives different contributions to $\frac{1}{g_i^2}$ and unification could be spoiled.

But, there can be *multiple axions* S_j, with different linear combinations cancelling anomalies with respect to different gauge groups. In this case, unification can be restored, e.g. if one saxion vev is way larger than the others.

Instanton calculation of anomalies

Look at the instanton effective operator, with all the fermionic zero modes:

$$\langle \psi_0 \psi_0 \ldots \psi_0 \rangle$$

If under the symmetry $\psi \to e^{i\alpha}\psi \equiv \beta\psi$, the instanton operator transforms, the theory is not invariant. If n is the number of zero modes, it transforms as β^n, or, equivalently, the θ angle is shifted, $\theta \to \theta - n\alpha$.

$$\begin{aligned} n = 2\ell(r) &= \left\{ \begin{array}{ll} 1 & \mathbf{N}, \bar{\mathbf{N}} \\ 2N & \mathbf{Adj} \end{array} \right. \quad of\ SU(N) \\ &= \left\{ \begin{array}{ll} 2 & \mathbf{N} \\ 2(N-2) & \mathbf{Adj} \end{array} \right. \quad of\ SO(N) \\ &= \left\{ \begin{array}{ll} 1 & \mathbf{2N} \\ 2(N+1) & \mathbf{Adj} \end{array} \right. \quad of\ Sp(2N) \end{aligned}$$

Discrete gauge symmetries

No global symmetry is expected in quantum gravity. In string theory, it is a theorem. Similar argument believed to hold for discrete symmetries [Krauss, Wilczeck, 1988]

Examples of discrete gauge symmetry:

- $U(1) \to Z_N$ when Φ with charge N gets a vev.
- a discrete subgroup of the d dimensional Lorentz group is left when compactifying extra dimensions

d-dimensional rotation acts differently on spinors and scalars
$\to$ discrete R-symmetries.

Green-Schwarz mechanism:

In particular compactifications of heterotic string theory, there is only one axion which plays a role in anomaly cancellation. This implies some degree of anomaly universality.

[Green,Schwarz, 1984]

$$\frac{\mathcal{A}_{SU(3)^2 U(1)}}{k_{SU(3)}} = \frac{\mathcal{A}_{SU(2)^2 U(1)}}{k_{SU(2)}} = \frac{\mathcal{A}_{U(1)_Y^2 U(1)}}{k_{U(1)_Y}} = \frac{\mathcal{A}_{U(1)^3}}{3k_{U(1)}} = 2\pi^2 \delta_{GS}$$

where the k_i's are the Kač-Moody levels of the gauge algebras G_i.

Following this GS example of anomaly cancellation in the heterotic string, many have imposed anomaly universality when introducing new symmetries (in many contexts); but this can be questioned:

- R symmetries are necessarily broken at a high energy scale, given the smallness of the observed cosmological constant:

$$V = e^K \left[|W_\phi|^2 - 3|W|^2 \right]$$

  As a result of this breaking, fields in non-vectorlike representations of the symmetry group may gain mass. The low energy theory typically still possesses an approximate discrete R symmetry, with apparent anomalies, even if the microscopic theory was anomaly free.

- in string theory, a GS cancellation involving only one modulus is not generic.

Non universal anomalies in Tyipe IIB

Take type IIB string theory on a T^6/Z_3 orientifold. The gauge group and discrete symmetries are

[Ibanez,Rabadan,Uranga, 1998]

$$SU(12) \times SO(8) \times U(1) \times Z_6^R$$

The field content has the following representations and charges:

[Dine,Graesser, 2005]

$$2(\mathbf{12}, \mathbf{8}, 1; \gamma^{1/2}) + (\mathbf{12}, \mathbf{8}, 1; \gamma^{-1/2}) + 2(\overline{\mathbf{66}}, \mathbf{1}, -2; \gamma^{1/2}) +$$
$$+(\overline{\mathbf{66}}, \mathbf{1}, -2; \gamma^{-1/2}) + (\mathbf{143}, \mathbf{1}, 0; \gamma^{-1/2}) + (\mathbf{1}, \mathbf{28}, 0; \gamma^{-1/2}).$$

with the last term representing the transformation of the fermionic component of the field under Z_6^R, and $\gamma = e^{\frac{2\pi i}{6}}$.

Non-universal gauge anomalies: for $\psi \to e^{i\alpha}\psi = \beta\psi$,the instanton operator transforms as

$$SU(12)^2 U(1): \quad \beta^{-36} \qquad [=3\cdot 8\cdot 1\cdot 1+3\cdot(-2)\cdot 10]$$
$$SO(8)^2 U(1): \quad \beta^{72} \qquad [=3\cdot 12\cdot 1\cdot 2]$$

The anomalies are cancelled by a generalized GS mechanism: 2 couplings of the type

[Ibanez,Rabadan,Uranga, 1998]

$$\mathrm{Tr}(\gamma_k \lambda)\ B_k^{\mu\nu} \cdot F_{\mu\nu}^{U(1)}, \qquad Tr(\gamma_k^{-1} \lambda_G^2)\ \partial^{[\mu} B_k^{\nu\rho]} \cdot W_{\mu\nu\rho}$$

RR (un-)twisted fields B_k, twist matrix $\gamma_k = e^{-2i\pi V\cdot H}$, CP matrices λ, λ_G. Untwisted fields do not contribute to anomaly cancellation.

Discrete anomalies

Field content

$$2(\mathbf{12},\mathbf{8},1;\gamma^{1/2})+(\mathbf{12},\mathbf{8},1;\gamma^{-1/2})+2(\overline{\mathbf{66}},\mathbf{1},-2;\gamma^{1/2})+$$
$$+(\overline{\mathbf{66}},\mathbf{1},-2;\gamma^{-1/2})+(\mathbf{143},\mathbf{1},0;\gamma^{-1/2})+(\mathbf{1},\mathbf{28},0;\gamma^{-1/2}).$$

with $\gamma=e^{\frac{2\pi i}{6}}$. The anomalous transformation of the instanton operator under a discrete rotation is

$$SU(12)^2 Z_6: \quad \gamma^3 \qquad [=2\cdot\tfrac{1}{2}\cdot 8\cdot 1+1\cdot(-\tfrac{1}{2})\cdot 8\cdot 1+(2-1)\cdot\tfrac{1}{2}\cdot 10-\tfrac{1}{2}\cdot 2\cdot 12]$$
$$SO(8)^2 Z_6: \quad \gamma^0 \qquad [=(2-1)\cdot\tfrac{1}{2}\cdot 12\cdot 2-\tfrac{1}{2}\cdot 2\cdot 6]$$

Non-universal discrete anomalies. They can be cancelled by the same twisted fields that cancel the $U(1)$ anomalies.
Fayet-Iliopoulos D-terms follow from supersymmetry and the $B_k\wedge F_{U(1)}$ term:

$$D=6\sqrt{3}(\Phi_1-\Phi_2).$$

where Φ_k are the NS partners of the twisted RR fields in B_k.

Now we look at the low energy theory and see if the discrete anomalies can be cancelled by appropriate axion transformation laws.

zero D-term

The U(1) becomes massive and disappears from the low-energy theory, which is left with $SU(12), SO(8), Z_6$.
The light linear combination of Φ_1, Φ_2 left is $\Phi = \Phi_1 + \Phi_2$ and it cancels the remaining discrete anomalies $(\gamma^3, 1)$ together with the model-independent dilaton, through the gauge kinetic functions

$$f_{SU(12)} = S - 6(\Phi_1 + \Phi_2) \qquad f_{SO(8)} = S + 4(\Phi_1 + \Phi_2).$$

Assigning appropriate non-linear transformation laws to S and Φ the non-universal discrete anomalies can be cancelled.

non-zero D-terms

Positive D term: can be cancelled by giving vev to $(\overline{\mathbf{66}}, \mathbf{1}, -2; \gamma^{1/2})$:

$$\langle \overline{\mathbf{66}} \rangle = \begin{pmatrix} \sigma_2 & & & & & \\ & \sigma_2 & & & & \\ & & \sigma_2 & & & \\ & & & \sigma_2 & & \\ & & & & \sigma_2 & \\ & & & & & \sigma_2 \end{pmatrix}$$

$$SU(12) \times SO(8) \longrightarrow Sp(12) \times SO(8)$$

The scalar component of the $\mathbf{66}$ is neutral under the R-symmetry,

$$q^R_\phi = q^R_\psi + q^R_\lambda = 0$$

The unbroken low-energy discrete symmetry is the same Z_6. The $G^2 \times Z_6$ anomalies are:

$$Sp(12)^2 Z_6 : \quad \gamma^3 \qquad SO(8)^2 Z_6 : \quad \gamma^0$$

There are three light moduli remaining in the low energy theory, more than enough to cancel the discrete anomaly

Alternatively, if $(\overline{\mathbf{66}}, \mathbf{1}, -2; \gamma^{-1/2})$ gets a vev, the low-energy discrete symmetry is different: $Z_6' = Z_6 \times U(1)_{\alpha = 2\pi i/6}$. The same results hold.

Conclusions

Similar results hold for many other orientifolds. Also, for orbifold blow-ups, multiple moduli take part in the cancellation of non-universal (continuous) Abelian anomalies.

[Lüdeling,Ruehle and Wieck, 2012]

Non-universality seems to be the rule and universality the exception.

From the low-energy perspective, there is no strong rationale for enforcing any (discrete) Abelian anomaly constraints. Only if one is committed to some particular microscopic framework (e.g. heterotic strings compactified on orbifolds), or the assumption that there is no small parameter in the microscopic theory, one can justify such constraints.

Thank you for listening

Backups

negative D-term

give a vev to the $(\mathbf{12}, \mathbf{8}, 1; \gamma^{\pm 1/2})$ fields

$$\langle(\mathbf{12}, \mathbf{8})\rangle = \left(\begin{array}{cccccccc|cccc} v & & & & & & & & 0 & 0 & 0 & 0 \\ & v & & & & & & & & & & \\ & & v & & & & & & & & & \\ & & & v & & & & & & & & \\ & & & & v & & & & & & & \\ & & & & & v & & & & & & \\ & & & & & & v & & & & & \\ & & & & & & & v & & & & \end{array}\right)$$

The low energy gauge group is $SO(8)$ and the discrete symmetry is anomaly free.

A dynamical model for R-breaking

Take SQCD with N colors and F_1 quarks Q_f and F_2 quarks Q_a:

$$W = S_1 \bar{Q}_f Q_f + S_1^3 + \frac{S_2^2}{M_p} \bar{Q}_a Q_a + S_1^3 + \frac{1}{M_p} S_2^4.$$

At low energies one can integrate out the quarks and the gauge fields and is left with an effective theory for the singlets, which can have comparable vevs.

S_1, S_2 have charges $\frac{2}{3}, \frac{1}{2}$. They can be coupled to SM fields

$$S_1 \bar{q} q + S_2 \bar{\ell} \ell,$$

where q, ℓ charges are $\frac{2}{3}, \frac{3}{4}$. It is possibe for q and ℓ to gain approximately the same masses (and not spoil unification), but as they have different R-charges, the low- and high-energy theories will have different R-anomalies