DIMENSIONAL REDUCTION
of
S-CONFINING DUALITIES

Cornell University

work in progress, in collaboration with **C. Csaki, Y. Shirman, and F. Tanedo.**

1- Dimensional reduction of Seiberg dualities

2- S-Confining theories.

3- Dimensional reduction of S-Confining dualities.

In the 90's many 3D dualities were conjectured

Aharony dualities [hep-th/9703215]

Electric (*Theory A*)

$U(N)$ with $F\ (\Box + \overline{\Box})$

$\boxed{W = 0}$

Magnetic (*Theory B*)

$U(F-N)$ with $F\ (\Box + \overline{\Box})$ and F^2 mesons

$\boxed{W = \tilde{q} M q + V_+ \tilde{V}_- + V_- \tilde{V}_+}$

Giveon-Kutusov dualities [hep-th/9802067]

Electric (*Theory A*)

$U(N)_k$ with $F\ (\Box + \overline{\Box})$

$\boxed{W = 0}$

Magnetic (*Theory B*)

$U(|k| + F - N)_{-k}$ with $F\ (\Box + \overline{\Box})$ and F^2 mesons

$\boxed{W = \tilde{q} M q}$

Some of them really looks like Seiberg dualities!

Aharony dualities [hep-th/9703215]

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$$\boxed{W = 0}$$

Magnetic (*Theory B*)

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$$\boxed{W = \tilde{q} M q + V_+ \tilde{V}_- + V_- \tilde{V}_+}$$

Seiberg dualities [arXiv:1112.0938]

Electric (*Theory A*)

$SU(N)$ with $F\ (\square + \overline{\square})$

$$\boxed{W = 0}$$

Magnetic (*Theory B*)

$SU(F-N)$ with $F\ (\square + \overline{\square})$ and F^2 mesons

$$\boxed{W = \tilde{q} M q}$$

Although strong coupling gauge dynamics is very different in 4D and in 3D, this similarity calls for dimensional reduction.

Why did it take so long?

O. Aharony, S. Razamat, N. Seiberg & B. Willet
JHEP 1307 (2013) 149 `[arXiv:1305.3924]`

O. Aharony, S. Razamat, N. Seiberg & B. Willet
JHEP 1307 (2013) 149 `[arXiv:1307.0511]`

Seiberg dualities are IR dualities

In the range of parameters where both theories are asymptotically free, *Theory A* and *Theory B* are equivalent only at low energies

$$\boxed{E \lesssim \Lambda_A \lesssim \Lambda_B}$$

Confinement scale for *Theory A*

$$\Lambda_A^b = \exp(-8\pi^2/g_A^2)$$

Confinement scale for *Theory B*

$$\Lambda_B^b = \exp(-8\pi^2/g_B^2)$$

Such dualities still holds true when we compactify both theories on a circle of radius *r*.

Compactification on a circle.

When we compactify one space dimension to a circle the gauge coupling satisfies:

$$\boxed{g_4^2 = 2\pi r g_3^2}$$

g_3

Compactification on a circle.

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$$\boxed{\Lambda^b = \exp(-4\pi/rg_3^2)}$$

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$$\Lambda^b = \exp(-4\pi / r g_3^2)$$

As in the $r \to 0$ limit should be kept constant

$$\Lambda_A \to 0$$
$$\Lambda_B \to 0$$

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As in the $r \rightarrow 0$ limit should be kept constant

$$\Lambda_A \rightarrow 0$$

$$\Lambda_B \rightarrow 0$$

Straightforward dimensional reduction does not work.

We can take a different limit keeping *r* fixed

$$E \lesssim \Lambda_A \lesssim \Lambda_B < 1/r$$

1- In this limit the effective low-energy behaviour of both theories is three dimensional.

2- *Theory A* and *Theory B* are still dual because of the 4D IR duality.

The 3D duality so obtained from the 4D duality, differs from the naive dimensional reduction.

How do they differ?

1- In the compactified theory, the scalar fields coming from the holonomy are periodic, with period *1/r*. As VEVs of scalar fields which belong to Vector multiplets parametrized the Coulomb branch,

The Coulomb branch is compact.

2- Because of the periodicity coming from the holonomy along the compact dimension, a non-perturbative contribution to the super-potential is generated by instantons.

Such term is not generated in the naive 3D reduction.

$$W = W_{3D} + \eta Y$$

This is the 3D SP ovtained by naive dim. reduction.

$\eta \equiv \Lambda^b$

Y is a coordinate of the Coulomb branch.

Summarizing 1/2.

4D

Theory A_4		Theory B_4
$\mathcal{N}=1$	$\longleftrightarrow$	$\mathcal{N}=1$
$W_4=0$		$\tilde{W}_4\neq 0$

3D

Summarizing 1/2.

4D

Theory A_4

$\mathcal{N}=1$

$W_4=0$

Theory B_4

$\mathcal{N}=1$

$\tilde{W}_4\neq 0$

3D

Theory A_3

$\mathcal{N}=2$

$W_3=\eta Y$

Theory B_3

$\mathcal{N}=2$

$\tilde{W}_3=\tilde{W}_4+\tilde{\eta}\tilde{Y}$

Summarizing 2/2.

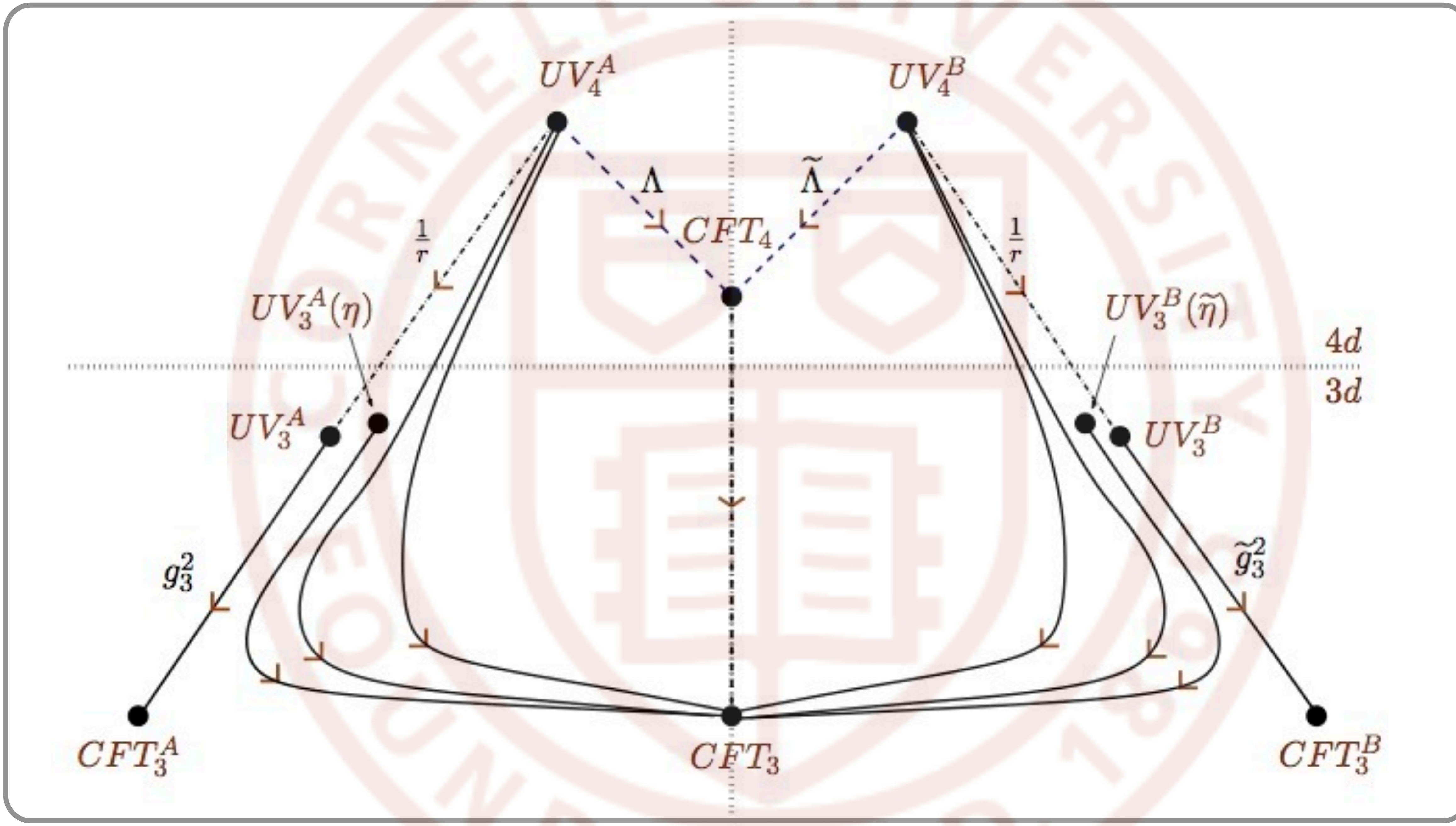

Image taken from [arXiv:1305.3924].

Through dimensional reduction more 3D dualities were conjectured.

$SU(N)$ with F $(\square + \overline{\square})$

$$W = 0$$

$\longleftrightarrow$

$U(F-N)$ with F $(\square + \overline{\square})$ and F^2 mesons

$$W = \tilde{q}Mq + Yb\tilde{b} + \tilde{X}_- + \tilde{X}_+$$

O. Aharony, S. Razamat, N. Seiberg & B. Willet
JHEP 1307 (2013) 149 [arXiv:1305.3924]

$SO(N)$ with F $\square$

$$W = 0$$

$\longleftrightarrow$

$SO(F-N+2)$ with F $\square$ and $F(F+1)/2$ mesons

$$W = \frac{1}{2}Mqq + \frac{i^{F-N}}{4}\tilde{y}Y$$

O. Aharony, S. Razamat, N. Seiberg & B. Willet
[arXiv:1307.0511]

1- Dimensional reduction of Seiberg dualities.

2- **S-Confining theories**

3- Dimensional reduction of S-Confining dualities.

S-Confinement.

"smooth confinement without chiral symmetry breaking and a non-vanishing confining superpotential"

C. Csaki, M. Schmaltz & W. Skiba *Phys. Rev. Lett.* **78** (1997) 799 [hep-th/9610139]

C. Csaki, M. Schmaltz & W. Skiba *Phys. Rev. D* 55 (1997) 7840 [hep-th/9612207]

1- Infrared physics is described everywhere on the moduli space in terms of gauge invariant operators.

2- A non-vanishing superpotential is dynamically generated which is holomorphic function of the confined degrees of freedom.

3- The vacuum of the classical theory, where all the global symmetries are unbroken, is a vacuum of the quantum theory as well.

SU(N) with N+1 flavours.

The magnetic dual has no gauge group.

$$\boxed{W = \frac{1}{\Lambda^{2N-1}}\left(\det M - BM\bar{B}\right)}$$

1- ☐

2- ☐

3- ☐

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1- ☑
2- ☑
3- ☑

SU(N) with *N* flavours.

The magnetic dual has no gauge group.

$$W = \lambda(\det M - B\bar{B} - \Lambda^{2N})$$

1- ☐
2- ☐
3- ☐

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1- ☑
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3- ☐

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The magnetic dual has no gauge group.

$$W = \frac{1}{\Lambda^{2N-1}}(\det M - BM\bar{B})$$

1- ✓

2- ✓

3- ✓

$SU(N)$ with N flavours.

The magnetic dual has no gauge group.

$$W = \lambda(\det M - B\bar{B} - \Lambda^{2N})$$

1- ✓

2- ✓

3- ✗

$SU(N)$	$(N+1)(\Box+\overline{\Box})$	s-confining
$SU(N)$	$\ydiagram{1,1}+N\overline{\Box}+4\Box$	s-confining
$SU(N)$	$\ydiagram{1,1}+\overline{\ydiagram{1,1}}+3(\Box+\overline{\Box})$	s-confining
$SU(N)$	$\mathrm{Adj}+\Box+\overline{\Box}$	Coulomb branch
$SU(4)$	$\mathrm{Adj}+\ydiagram{1,1}$	Coulomb branch
$SU(4)$	$3\ydiagram{1,1}+2(\Box+\overline{\Box})$	$SU(2)$: $8\Box$
$SU(4)$	$4\ydiagram{1,1}+\Box+\overline{\Box}$	$SU(2)$: $\ydiagram{2}+4\Box$
$SU(4)$	$5\ydiagram{1,1}$	Coulomb branch
$SU(5)$	$3(\ydiagram{1,1}+\overline{\Box})$	s-confining
$SU(5)$	$2\ydiagram{1,1}+2\Box+4\overline{\Box}$	s-confining
$SU(5)$	$2(\ydiagram{1,1}+\overline{\ydiagram{1,1}})$	$Sp(4)$: $3\ydiagram{1,1}+2\Box$
$SU(5)$	$2\ydiagram{1,1}+\overline{\ydiagram{1,1}}+2\overline{\Box}+\Box$	$SU(4)$: $3\ydiagram{1,1}+2(\Box+\overline{\Box})$
$SU(6)$	$2\ydiagram{1,1}+5\overline{\Box}+\Box$	s-confining
$SU(6)$	$2\ydiagram{1,1}+\overline{\ydiagram{1,1}}+2\overline{\Box}$	$SU(4)$: $3\ydiagram{1,1}+2(\Box+\overline{\Box})$
$SU(6)$	$\ydiagram{1,1,1}+4(\Box+\overline{\Box})$	s-confining
$SU(6)$	$\ydiagram{1,1,1}+\ydiagram{1,1}+3\overline{\Box}+\Box$	$SU(5)$: $2\ydiagram{1,1}+\overline{\ydiagram{1,1}}+2\overline{\Box}+\Box$
$SU(6)$	$\ydiagram{1,1,1}+\ydiagram{1,1}+\overline{\ydiagram{1,1}}$	$Sp(6)$: $\ydiagram{1,1,1}+\ydiagram{1,1}+\Box$
$SU(6)$	$2\ydiagram{1,1,1}+\Box+\overline{\Box}$	$SU(5)$: $2(\ydiagram{1,1}+\overline{\ydiagram{1,1}})$
$SU(7)$	$2(\ydiagram{1,1}+3\overline{\Box})$	s-confining
$SU(7)$	$\ydiagram{1,1,1}+4\overline{\Box}+2\Box$	$SU(6)$: $\ydiagram{1,1,1}+\ydiagram{1,1}+3\overline{\Box}+\Box$
$SU(7)$	$\ydiagram{1,1,1}+\overline{\ydiagram{1,1}}+\Box$	$Sp(6)$: $\ydiagram{1,1,1}+\ydiagram{1,1}+\Box$

A complete classification.

$Sp(2N)$	$(2N+4)\Box$	s-confining
$Sp(2N)$	$\ydiagram{1,1}+6\Box$	s-confining
$Sp(2N)$	$\ydiagram{2}+2\Box$	Coulomb branch
$Sp(4)$	$2\ydiagram{1,1}+4\Box$	$SU(2)$: $8\Box$
$Sp(4)$	$3\ydiagram{1,1}+2\Box$	$SU(2)$: $\ydiagram{2}+4\Box$
$Sp(4)$	$4\ydiagram{1,1}$	$SU(2)$: $2\ydiagram{2}$
$Sp(6)$	$2\ydiagram{1,1}+2\Box$	$Sp(4)$: $2\ydiagram{1,1}+4\Box$
$Sp(6)$	$\ydiagram{1,1,1}+5\Box$	$Sp(4)$: $2\ydiagram{1,1}+4\Box$
$Sp(6)$	$\ydiagram{1,1,1}+\ydiagram{1,1}+\Box$	$SU(2)$: $\ydiagram{2}+4\Box$
$Sp(6)$	$2\ydiagram{1,1,1}$	$SU(3)$: $\ydiagram{2}+\overline{\ydiagram{2}}$
$Sp(8)$	$2\ydiagram{1,1}$	$Sp(4)$: $5\ydiagram{1,1}$

1- Dimensional reduction of Seiberg dualities.

2- S-Confining theories.

3- Dimensional reduction of S-Confining dualities.

Flowing down 1/2

The 3D dualities obtained reducing 4D ones, contain a non-perturbative contribution to the Super-potential we need to get rid off.

Matching Quantum Numbers

1- *Complex Masses*

$m Q\bar{Q}$ | $Y_F = m Y_{F-1}$

2- *VEVs*

$\langle Q\bar{Q}\rangle = v^2$ | $Y_F = \frac{Y_{F-1}}{v^2}$

3- *Real Masses*

Real mass deformations are a "novelties" of 3D theories. As they can be related to ***weakly gauge global symmetry***, they can be easily mapped across the duality.

$$\eta Y \to 0$$

Flowing down 2/2

While "decoupling" the instanton term Chern-Simons terms might be generated.

Ex.

Gauge group	# flavours	S-Confining
$SU(N)$	$N+1$	*yes* ☐ *no* ☐
$SU(N)$	N	*yes* ☐ *no* ☐
$SU(N)$	$N-1$	*yes* ☐ *no* ☐

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As it comes from reducing an *SU(N)* theory with *N*+2 flavours

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As it comes from reducing a *SU(N)* theory with *N*+1 flavours

Flowing down 2/2

While "decoupling" the instanton term Chern-Simons terms might be generated.

Ex.

Gauge group	# flavours	S-Confining
$SU(N)$	$N+1$	yes ☐ no ☒
$SU(N)$	N	yes ☒ no ☐
$SU(N)$	$N-1$	yes ☐ no ☒

As it comes from reducing an *SU(N)* theory with *N+2* flavours

As it comes from reducing a *SU(N)* theory with *N+1* flavours

Flowing down 2/2

While "decoupling" the instanton term Chern-Simons terms might be generated.

Ex.

Gauge group	# flavours	S-Confining
$SU(N)$	$N+1$	yes ☐ no ☒
$SU(N)$	N	yes ☒ no ☐
$SU(N)$	$N-1$	yes ☐ no ☒

As it comes from reducing an *SU(N)* theory with *N*+2 flavours

As it comes from reducing a *SU(N)* theory with *N*+1 flavours

As it comes from reducing a *SU(N)* theory with *N* flavours. We obtained in fact a ***Quantum modified constraint***.

Not all "flows" of 4D S-Confining dualities lead to 3D S-Confining dualities

$$SU(4) \text{ with } 3\,(\Box + \overline{\Box}) \;\&\; \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array} + \overline{\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}}$$

Not all "flows" of 4D S-Confining dualities lead to 3D S-Confining dualities

Real Masses

$$SU(4) \ \text{with} \ 3\ (\Box + \overline{\Box}) \ \ \& \ \ \begin{array}{c}\Box\\[-4pt]\Box\end{array} \cancel{+} \begin{array}{c}\Box\\[-4pt]\Box\end{array}$$

Not all "flows" of 4D S-Confining dualities lead to 3D S-Confining dualities

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Not all "flows" of 4D S-Confining dualities lead to 3D S-Confining dualities

Real Masses

$SU(4)$ with $3\ (\square + \overline{\square})$ & $\cancel{\begin{smallmatrix}\square\\\square\end{smallmatrix} + \begin{smallmatrix}\square\\\square\end{smallmatrix}}$

SU *Not S-Confining!* $\overline{\square})$

Not all "flows" of 4D S-Confining dualities lead to 3D S-Confining dualities

Real Masses

$$SU(4) \text{ with } 3\ (\Box + \overline{\Box})\ \&\ \cancel{\boxminus + \boxminus}$$

$$SU \quad + \overline{\Box})$$

Not S-Confining!

We want to come up with a complete classification of allowed deformations and thus 3D S-Confining dualities!

CONCLUSIONS

1- Naive dimensional reduction of 4D dualities does not work. A more involved procedure is needed to obtain 3D dualities from 4D.

2- In the process a non-perturbative contribution to the Super-Potential is generated which we need to deal with.

3- Flowing down to different theories with less flavours or exploring the moduli space allows to decouple the ηY term and flow to S-Confining theories.

4- In 4D, exploring the moduli space of S-Confining theories provide more S-Confining dualities. We expect the same to happen in 3D, is it true?

THANKS!