NO GUTS, ALL GLORY:

CHARGE QUANTIZATION IN THE $\mathbb{CP}^1$ NONLINEAR σ-MODEL

ARXIV:1308.????

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  - The role of supersymmetry?

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- Dirac's monopole and GUTs quantize charge, but at a cost:
  - Monopoles, proton decay, doublet-triplet splitting
  - The role of supersymmetry?
- *Can we quantize charge without any of this extra baggage?*

Yes, charge is quantized

in the $\mathbb{CP}^1_{\text{(and other)}}$ nonlinear sigma model

without Grand Unification, monopoles,

proton decay...

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  - **G** is a global symmetry, and **H** will be gauged

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- The group structure is **SU(2)/U(1)**

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- We have a Kähler manifold and holomorphic action
  - Naturally protected with **supersymmetry** (but otherwise we do not rely on it for our derivation)

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- Consistency conditions from the **SU(2)** algebra give differential equations for $F_\pm$
- Require smooth transition to the other hemisphere, $z_+ \rightarrow z_-$
- This yields a **charge quantization** condition: $\alpha \in \mathbb{Z}/2$

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- Another possibility: nothing to see in the low energy theory (all scales $\gg \mathrm{TeV}$)

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- The stability, mass, and charge make the NGB a probe of nuclear structure

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- This and other models which are also possible will be explored in a followup paper

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- $\mathbb{CP}^1$ as the SM hypercharge group has charge quantization and no stomach aches
- **Interesting phenomenology**: DM, catalyze nuclear fusion, nuclear probe (or "see nothing")
- Can **extend** to $\mathbb{CP}^k$ and other models

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