Theory of $B_s \to \mu^+ \mu^-$

SUSY 2013
ICTP Trieste
26 April 2013

Based on work with
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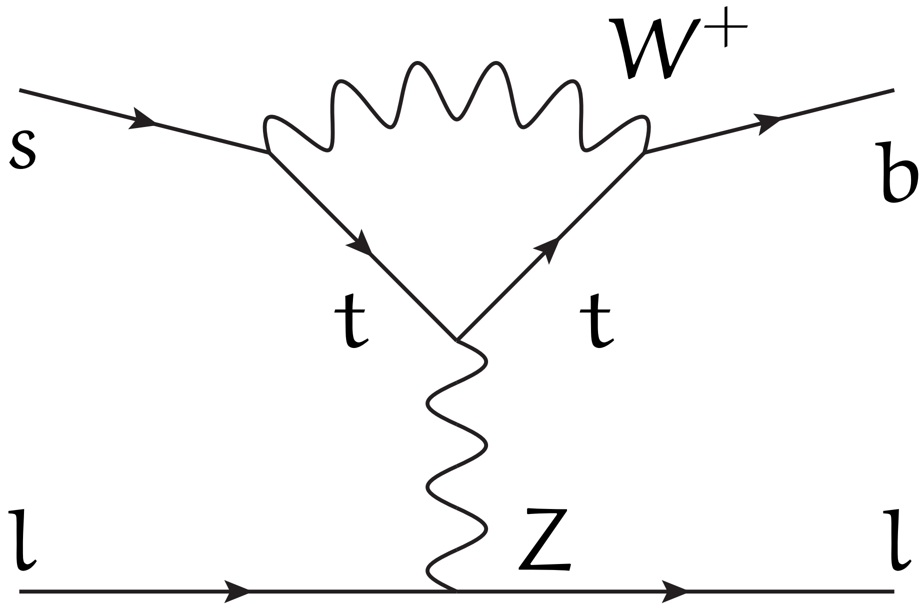
Martin Gorbahn
University of Liverpool

$B_s \to \mu^+ \mu^-$



B_s is pseudoscalar – no photon penguin

$$Q_A = (\bar{b}_L \gamma_\mu q_L)(\bar{l}\gamma_\mu \gamma_5 l)$$

Dominant operator (SM) Wilson

helicity suppression $\left(\propto \frac{m_l^2}{M_B^2}\right)$

Effective Lagrangian in the SM (NP + chirality flipped):

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} (C_S Q_S + C_P Q_P + C_A Q_A) + \text{h.c.}$$

Scalar operators: $Q_S = m_b (\bar{b}_R q_L)(\bar{l} l)$ $\quad Q_P = m_b (\bar{b}_R q_L)(\bar{l}\gamma_5 l)$

Alternative normalisation [Misiak `11]:

$$\mathcal{L}_{\text{eff}} = G_F^2 M_W^2 \, V_{tb}^* V_{ts} \, (C_A Q_A + C_S Q_S + C_P Q_P) + \text{h.c.}$$

MSSM: MFV and Large $\tan\beta$

Lagrangian of 2HDM of type 2

$$H_u \leftrightarrow u_R$$

$$-\mathcal{L} = Y^d_{ij} H_d \bar{d}^i_R q^j + Y^u_{ij} H_u \bar{u}^i_R q^j + \text{h.c.}$$

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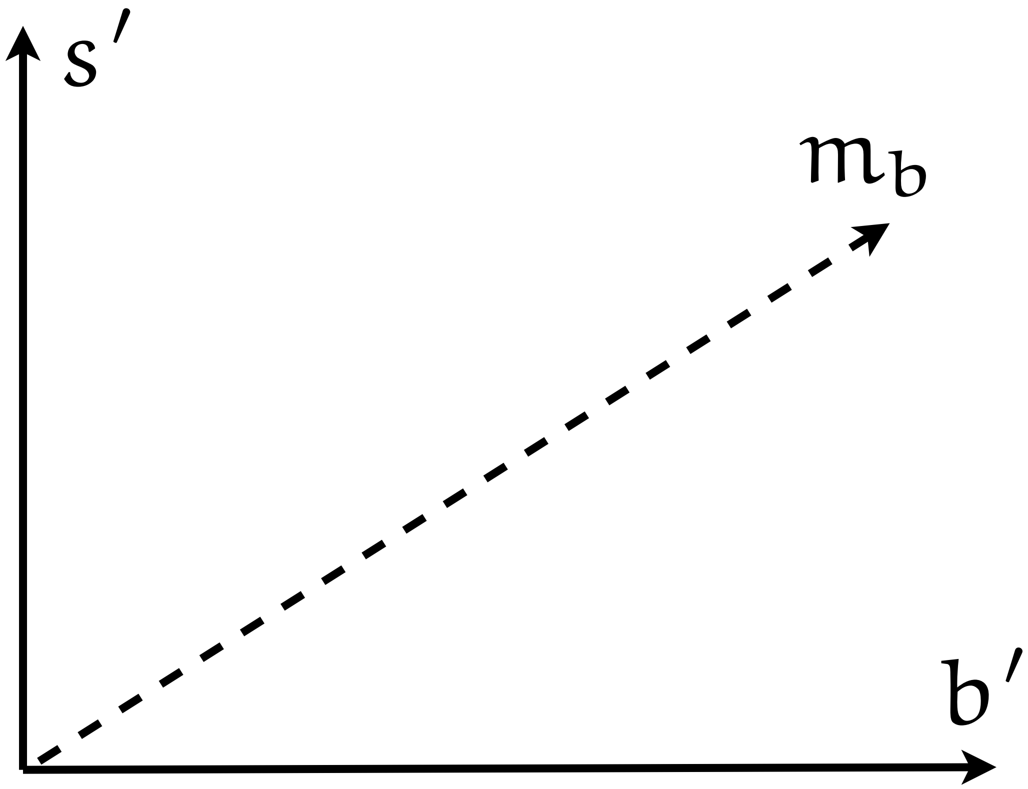

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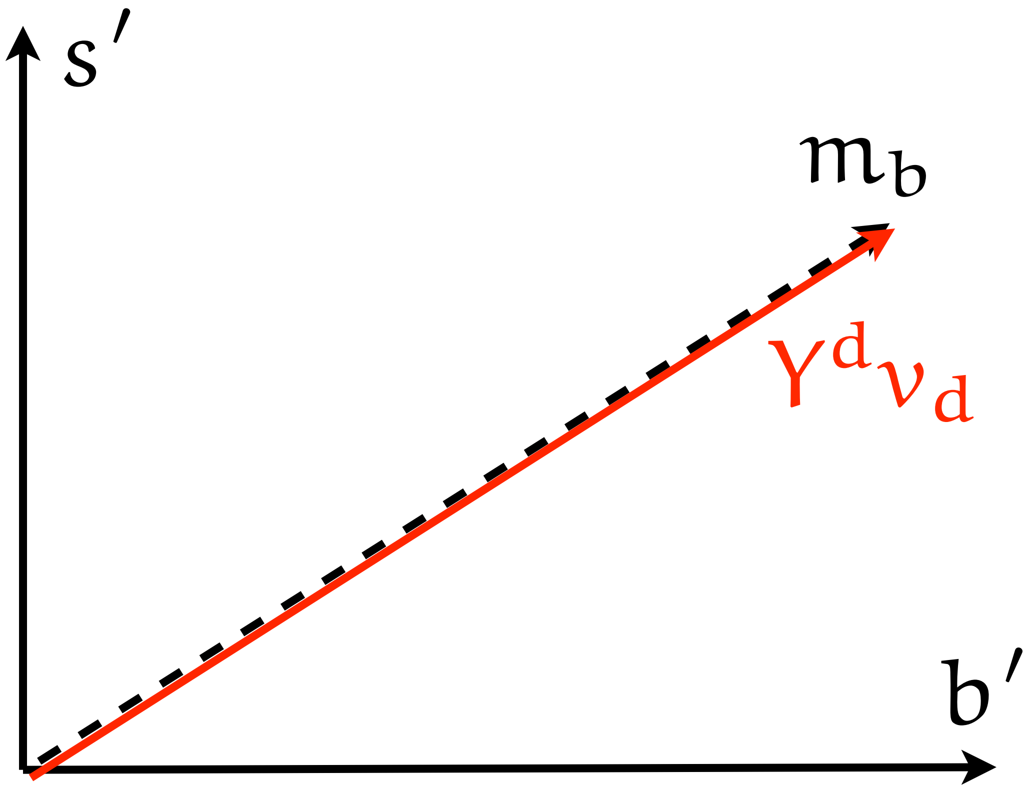

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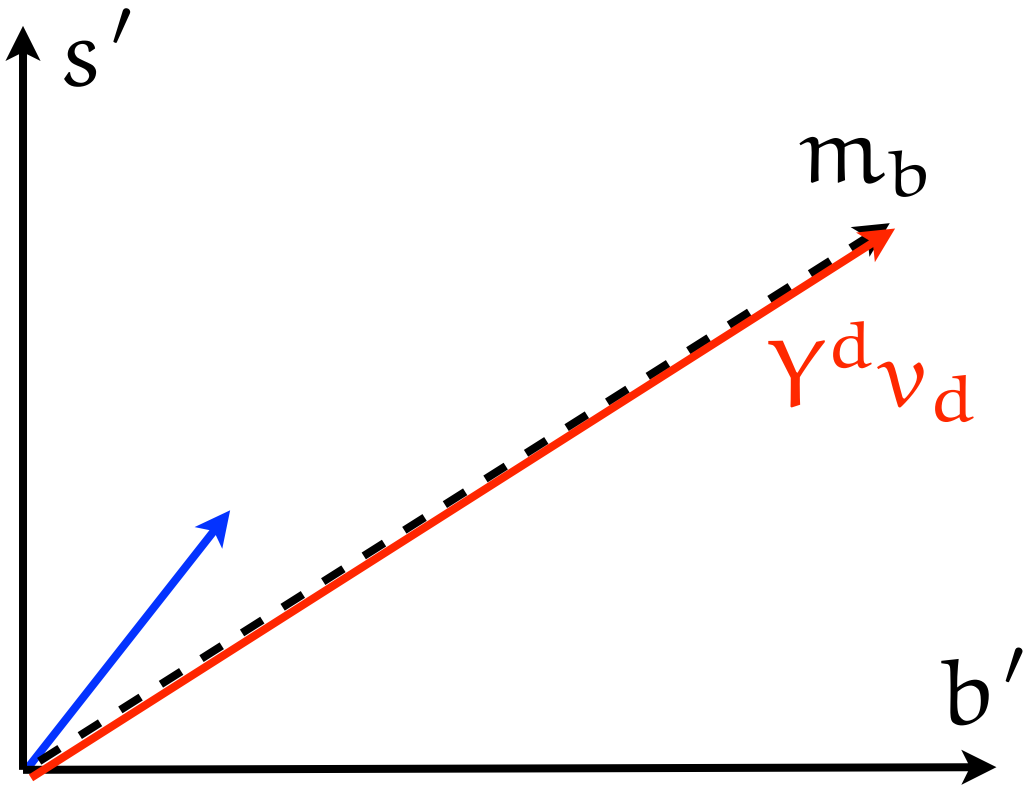

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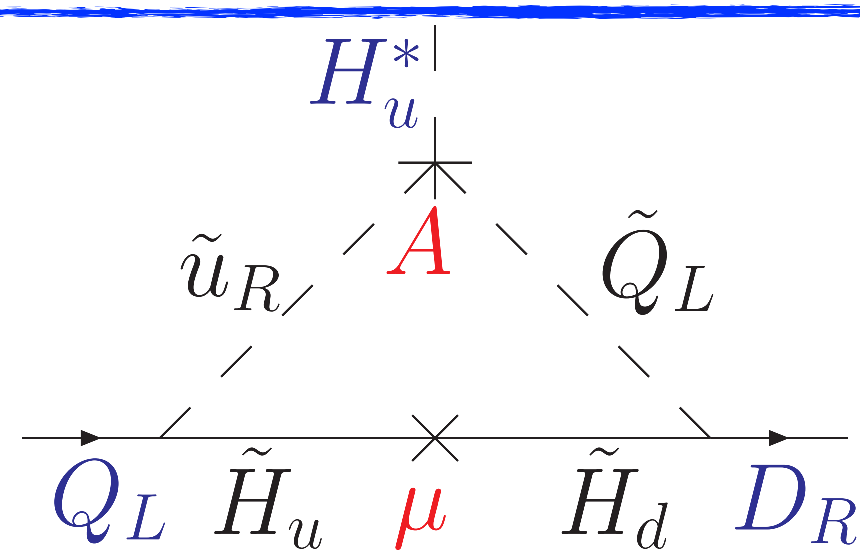

One loop: 2HDM of type 3

$$\Delta\mathcal{L}^Y_{\text{eff}} = \epsilon_Y \bar{d}_R Y^d Y^{u\dagger} Y^u H^*_u \cdot Q_L$$

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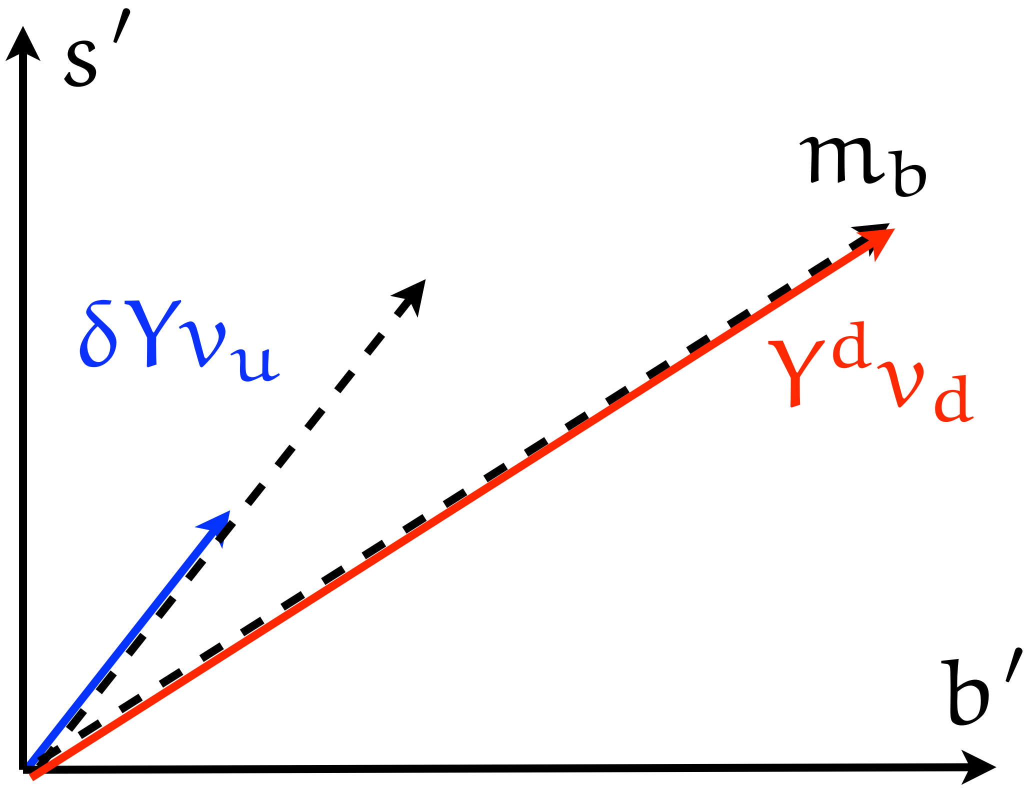

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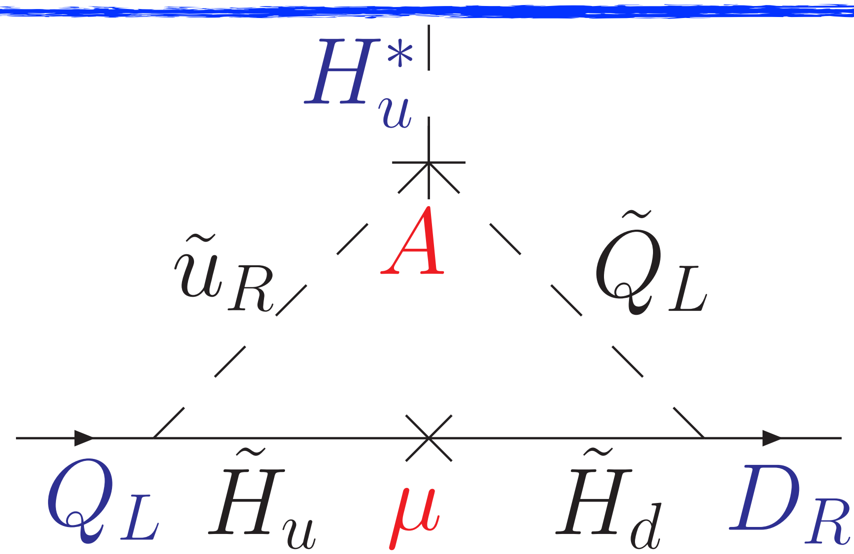

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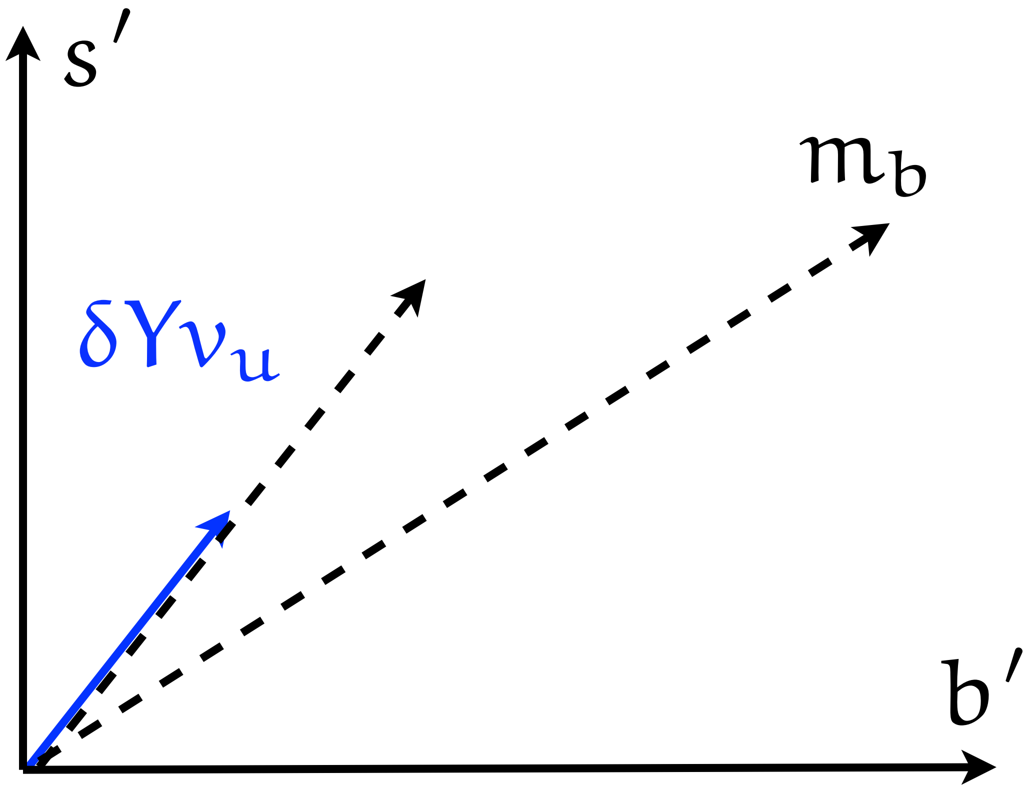

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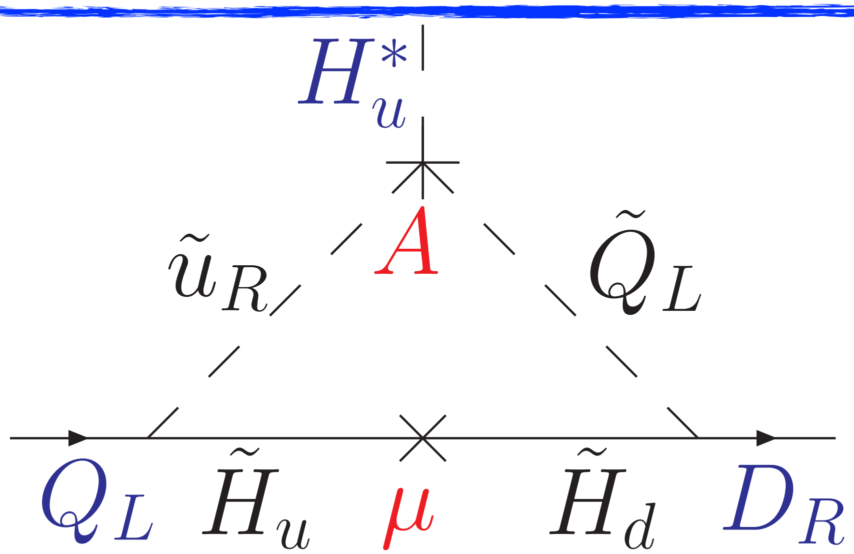

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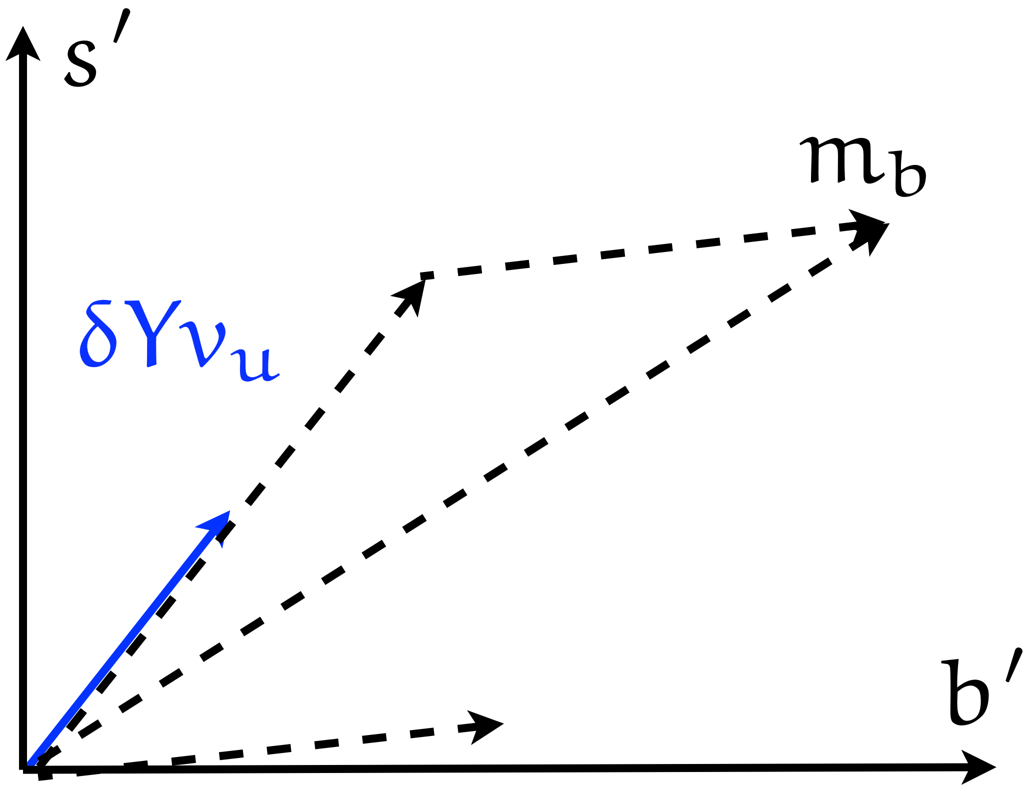

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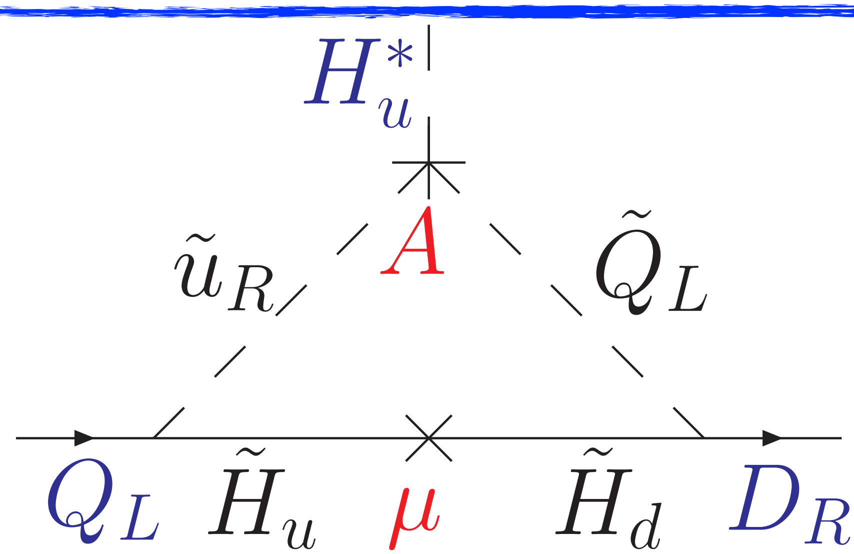

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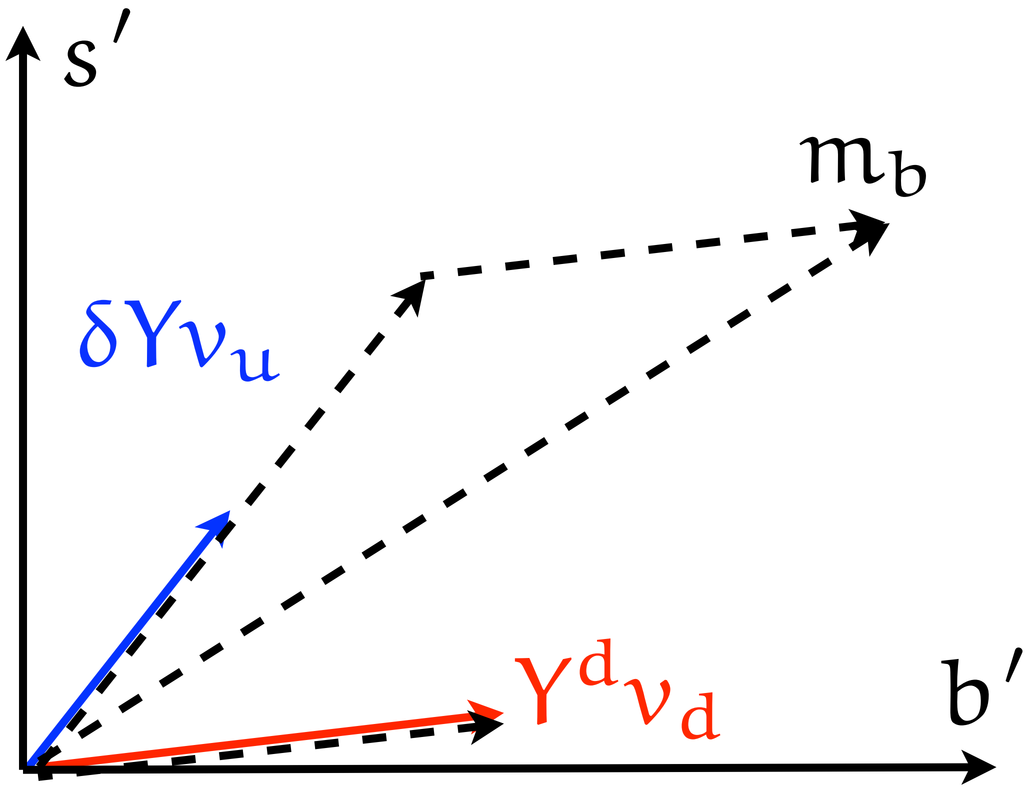

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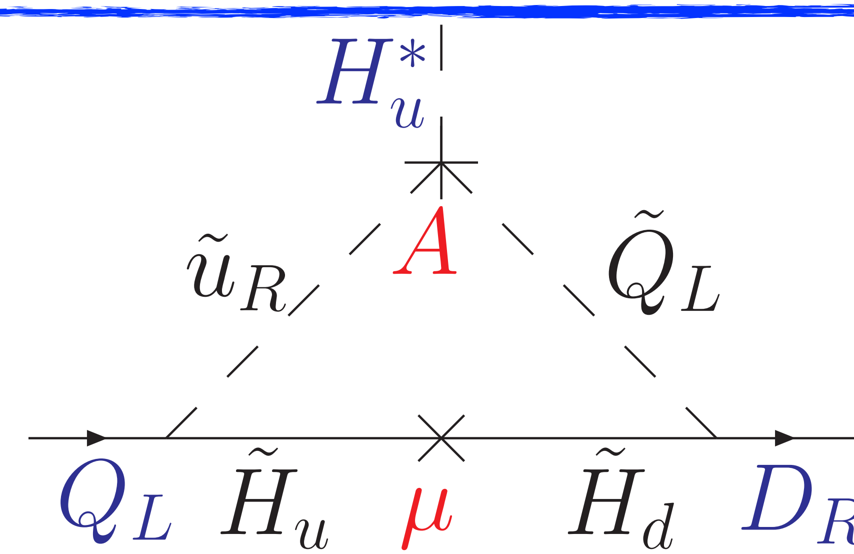

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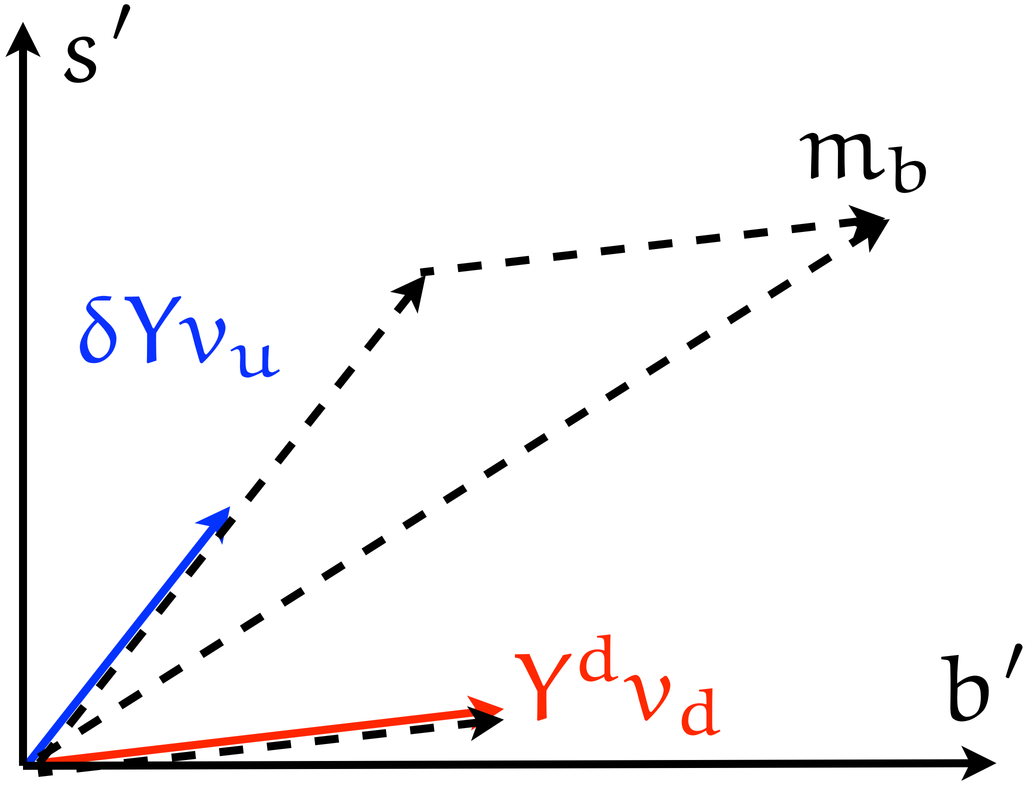

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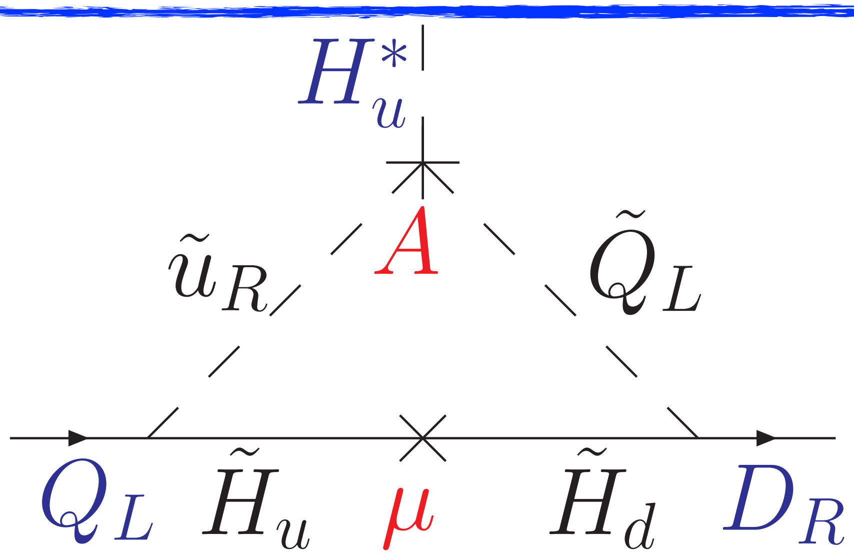
Redefinition

m_b & V_{CKM}

Masses and Yukawas not aligned



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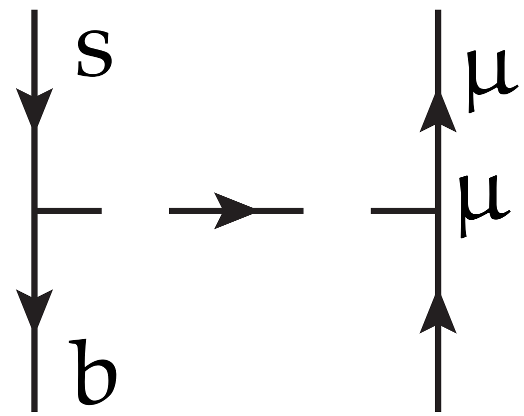
Flavour Violation at large tan β

Large FC scalar interactions: $\kappa_b \bar{b}_R s_L h_d^{0*} \propto Y_b$

[Babu, Kolda '00; ...]



$$Br(B_s \to \mu^+ \mu^-) \propto (\tan\beta)^6 \,/\, (M_A)^4$$

Note, that the tan β sensitivity of the MSSM is unique:

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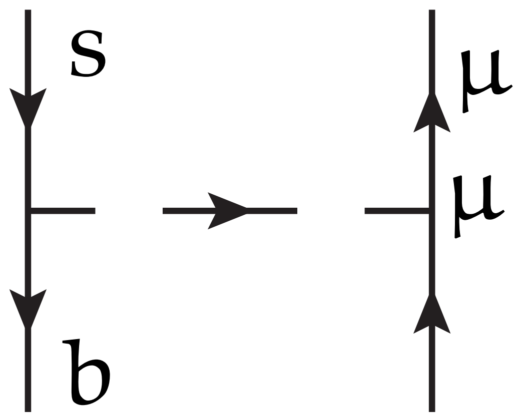


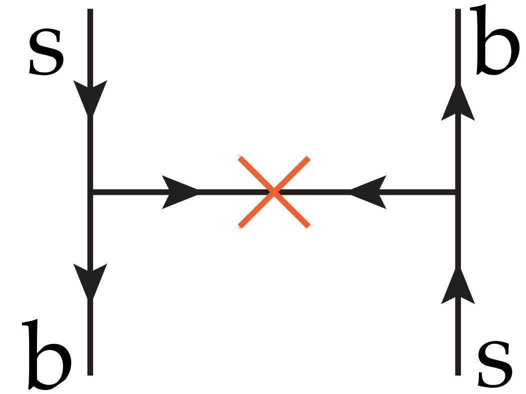
Note, that the $\tan\beta$ sensitivity of the MSSM is unique:

MSSM Higgs sector at $v_d = 0$: a symmetry

$Q(H_d) = 1,\ Q(b_R) = 1$ forbids the operator $(\bar{b}_R s_L)(\bar{b}_R s_L)$

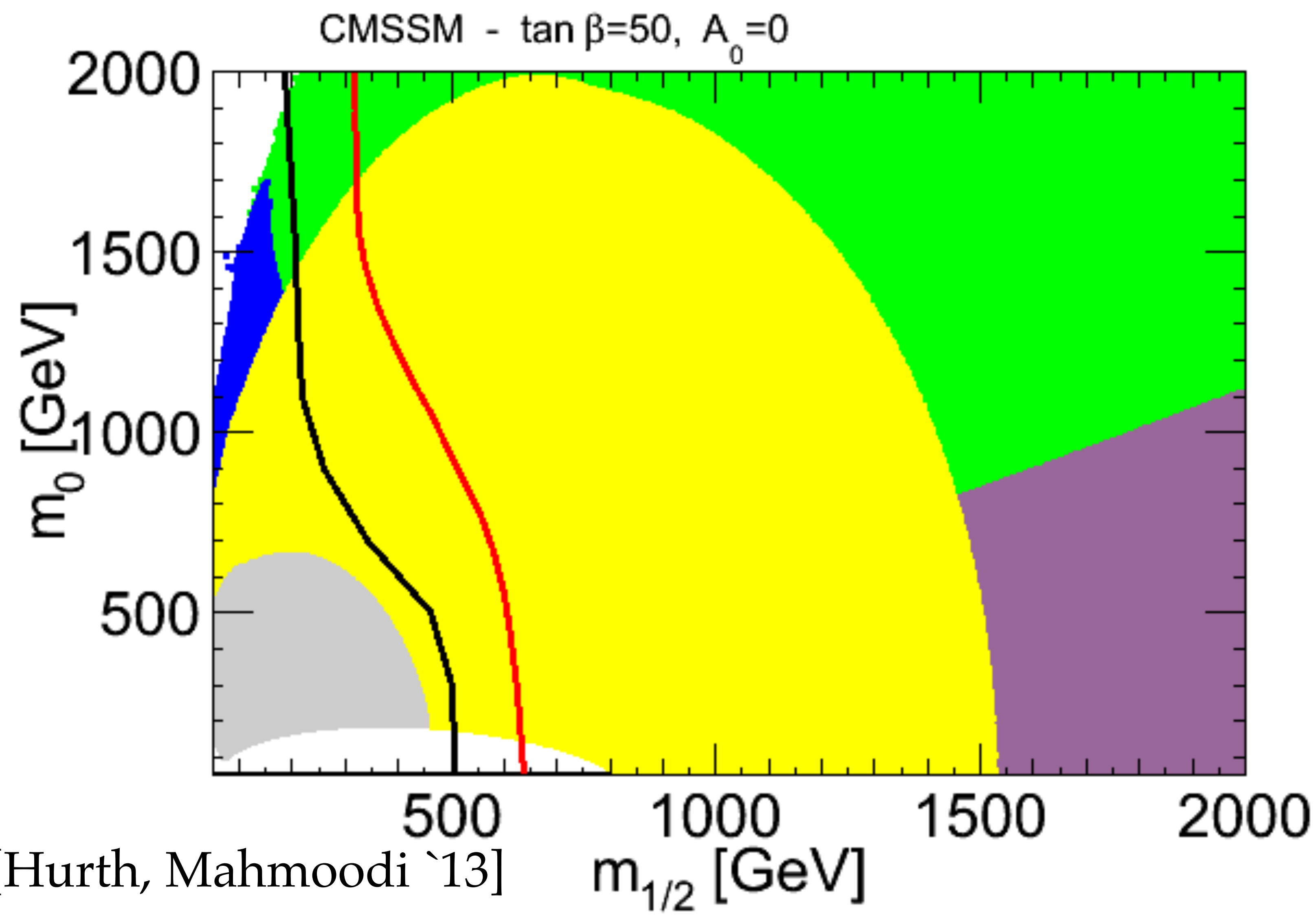
This protects ΔM_s. Contribution of symmetry-breaking terms small

[MG, Jäger, Nierste, Trine '09]



Flavour Violation at large tan β

Measurements of LHCb & ATLAS puts severe constraints on the parameter space at large tan(β)



[Hurth, Mahmoodi `13]

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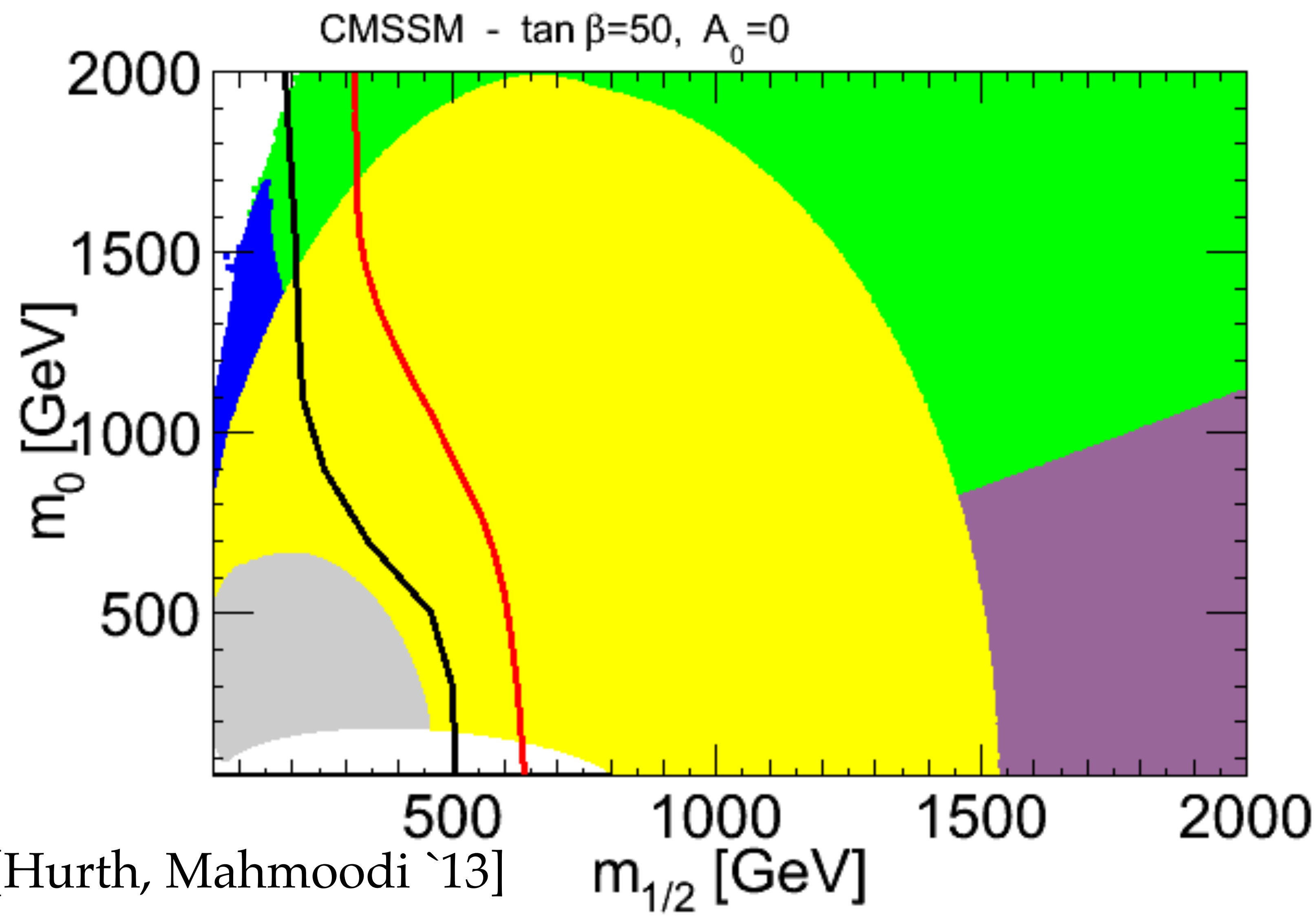

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With high precision in experiment and theory we can test the large tan β MSSM at the TeV scale

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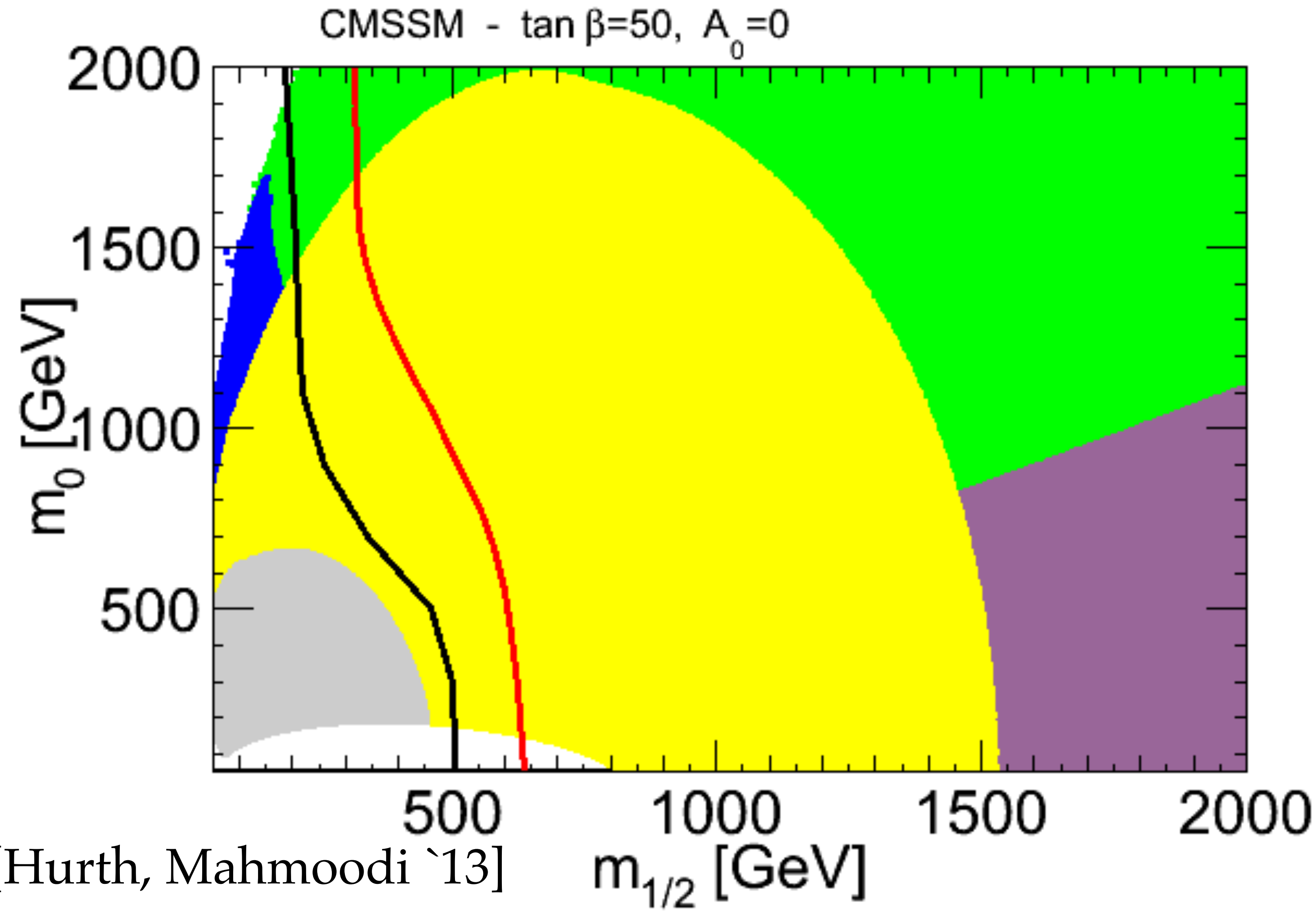

[Hurth, Mahmoodi `13]

With high precision in experiment and theory we can test the large tan β MSSM at the TeV scale

Therefore: discuss the standard model theory prediction

Theory Status (SM)

Standard Model: Scalar Operators are highly suppressed

C_A is known at NLO in QCD [Buras, Buchalla; Misiak, Urban `99]

$C_A(m_t\,/\,M_W)^{NLO} = 1.0113\; C_A(m_t\,/\,M_W)^{LO}$
– for QCD MS-bar $m_t = m_t(m_t)$

The matrix-element <Q_A> is given through the precisely known decay constant f_{Bs}
(f_{Bs} =225(5)MeV [Dowdall `13] - average = 225(3)MeV staggerd twisted mass $N_f$=2: $f_{Bs}$ =228(8)MeV [Carrosa `13])

There will be non-perturbative correction for $\alpha_e \neq 0$

Theory Status (SM)

Soft photon corrections [Buras, Guadagnoli, Isidori `12] :
→ cut on the invariant mass of the lepton pair suppresses the direct photon emission
→ bremsstrahlung affects the branching ratio
→ experimentalists simulate the signal fully inclusive of bremsstrahlung and remove this correction

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The B_s system has a non-zero decay width difference:
→ instantaneous ≠ time integrated branching ratio
[de Bruyn, Fleischer et. al. `12]
→ correction factor can be extracted from experiment
→ B_s mixing allows for additional observables beyond the branching ratio [de Bruyn, Fleischer et. al. `12, Buras et. al. `13]

Electroweak Corrections

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} C_A Q_A + \text{h.c.}$$

$G_F \alpha / \sin^2\theta_W$ does not renormalise under QCD: can be factored out for QCD calculation

Only $G_F \alpha / \sin^2\theta_W \, C_A(m_t / M_W)$ invariant under electroweak scheme change

This combination should always give the same result if we use the same input (G_F, α, M_Z, M_t, M_H) up to higher order corrections

Renormalisation of G_F

We identify G_F with the measured muon lifetime and its theory prediction $G_\mu = G_\mu^{(0)} + G_\mu^{(1)} + \ldots$

G_F can be combined or factored out of the Wilson coefficient

$$\mathcal{H}_{\text{eff}} = \tilde{C}\, Q = G_F C\, Q$$

Since G_F is now an observable and C dimensionless the vacuum expectation dependence cancels in C:

$$C^{(0)} = \frac{\tilde{C}^{(0)}}{G_\mu^{(0)}}, \qquad C^{(EW)} = \frac{\tilde{C}^{(EW)}}{G_\mu^{(0)}} - \frac{\tilde{C}^{(0)} G_\mu^{(EW)}}{\left(G_\mu^{(0)}\right)^2}$$

but other parameters are not so easily fixed from experiment

Electroweak Scheme Uncertainties

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} C_A \left(\frac{m_t}{M_W}\right) Q_A + \text{h.c.}$$

[Buras, et. al. `12]

	MS-bar	OS	unct. $B_s \mu^+ \mu^-$
$\sin \theta_W$	0.231	0.223	4 %
m_t(QCD-MS-bar)	163,2 GeV	164,5 GeV	1 %

Electroweak scheme shift larger then present pure theory error

[Buras, Guadagnoli, Isidori `12]: Follow [Brod, MG, Stamou] and use MS-bar θ_W plus renormalise the masses on-shell (hybrid)

Box contributions contribute differently to $l^+ l^-$ modes than to νν. Are both box and penguin corrections tiny in the hybrid scheme?

Renormalisation Schemes

Calculate in the MS-bar scheme
using tadpole counterterms to produce gauge
independence for intermediate results
fit g_1, g_2, v, λ, m_t from data (G_F, α, M_Z, M_t, M_H)

Use OS scheme: Determine M_W including 1-loop
corrections from input – then $\sin^2\theta_W = 1 - M_W^2 / M_Z^2$

Add finite $\sin\theta_W$, m_t and M_W counterterms to $C_A^{(EW)}$

NLO predictions should agree up to residual scheme
uncertainties if we use the same input data.

Matching Correction for C_A

There are sizeable shifts going from 1-loop to 2-loop

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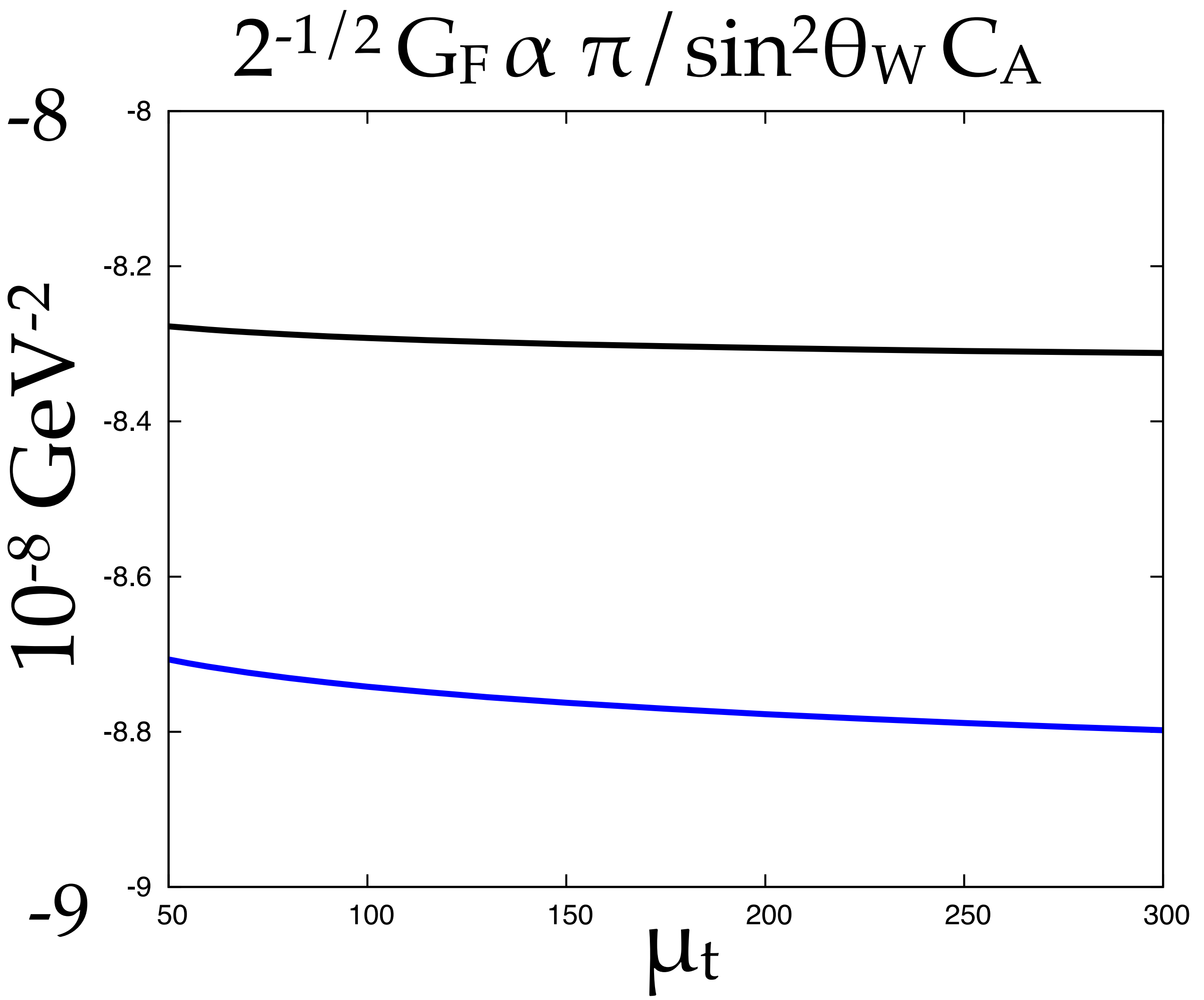
largest shift in the on-shell scheme



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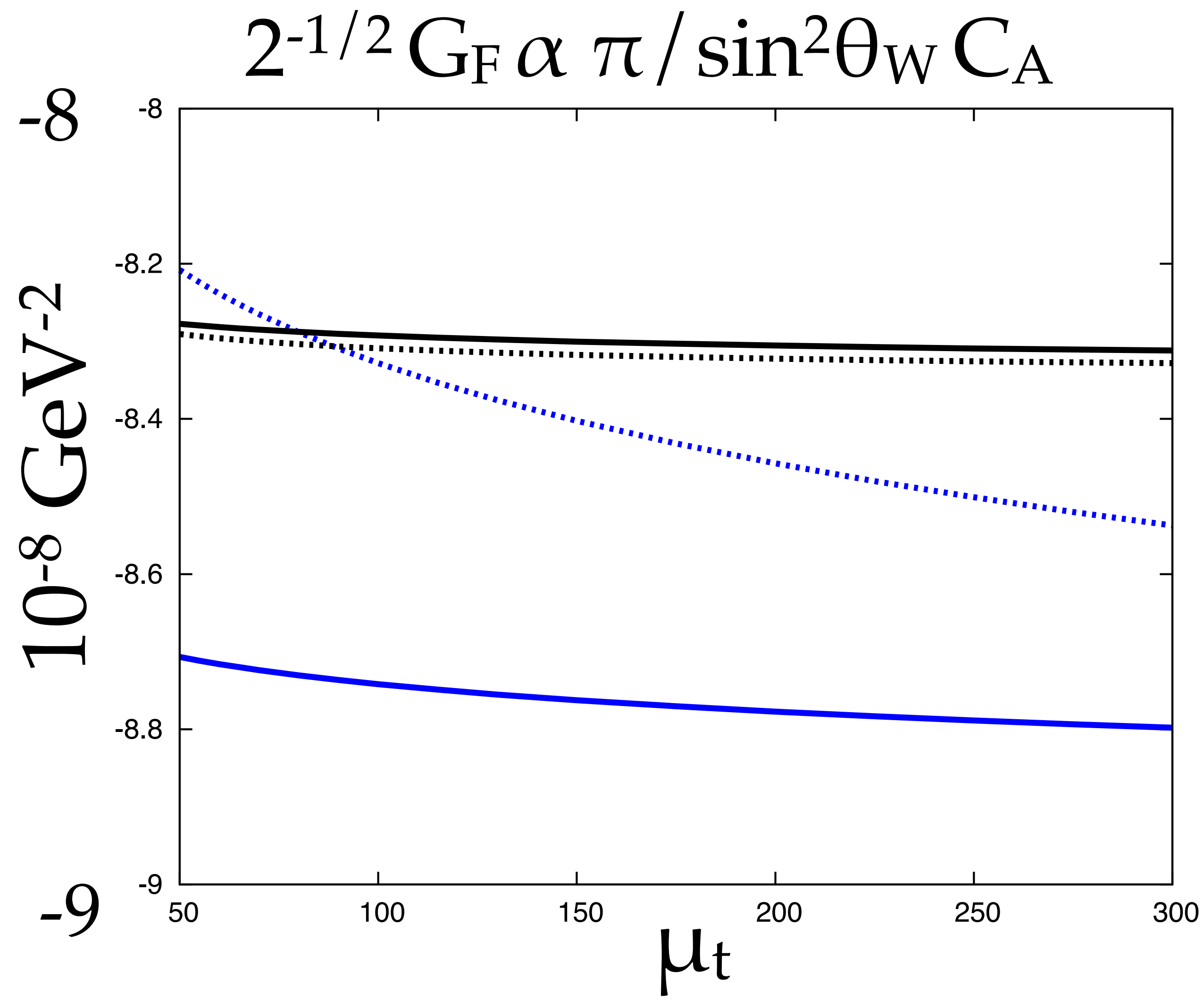
large scale dependence for the MS-bar scheme



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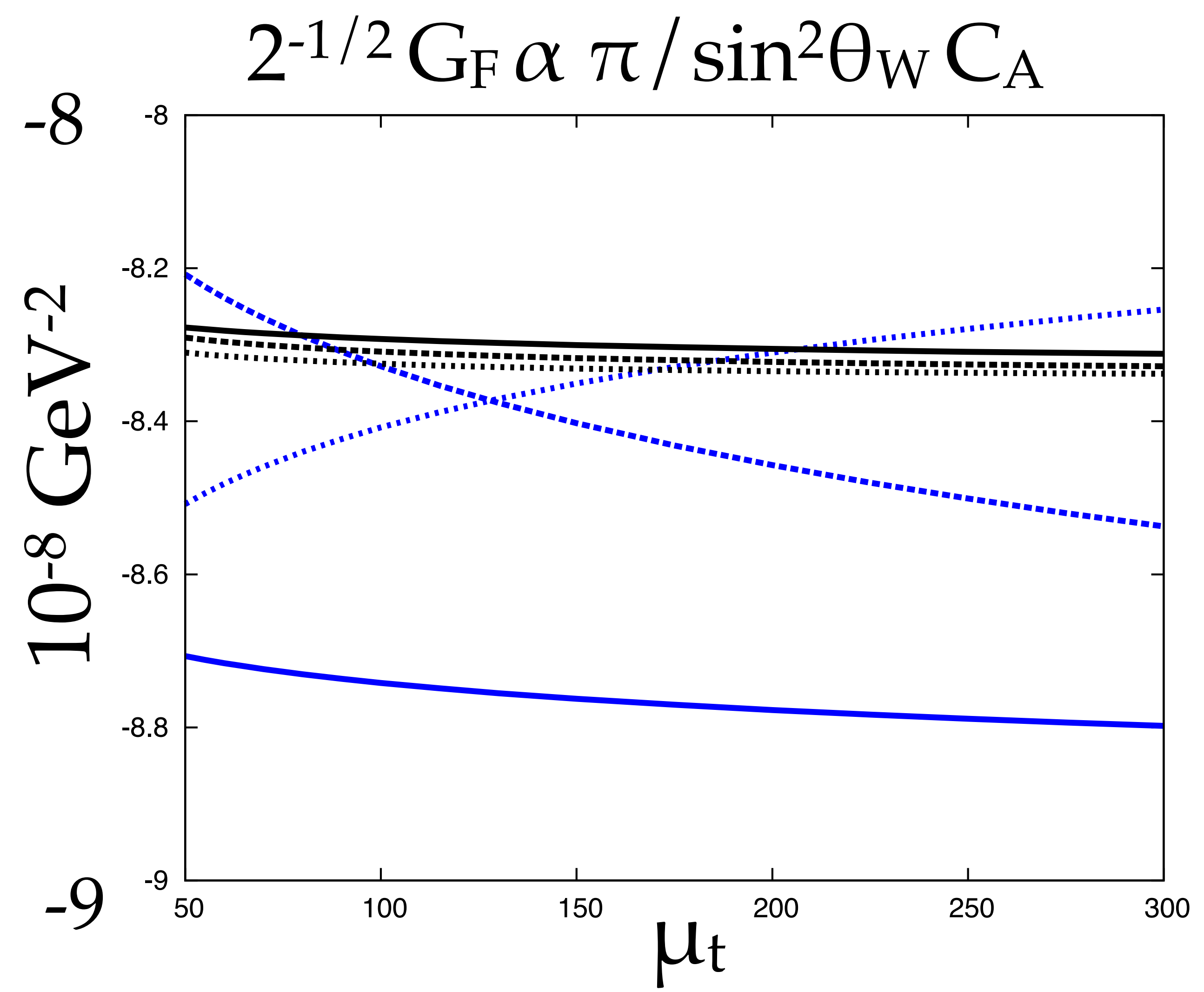

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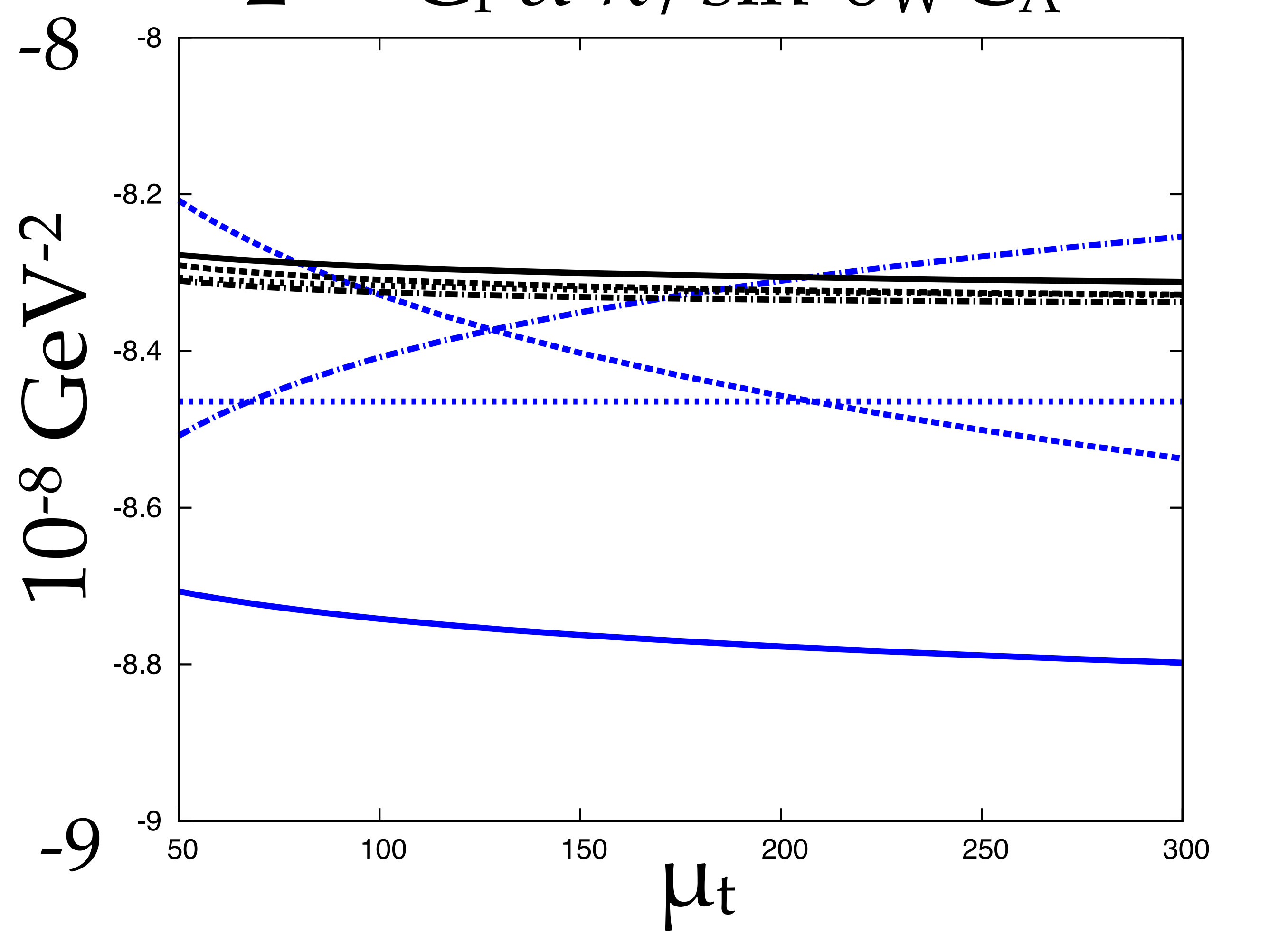
$G_F^2 M_W^2$ removes `artificial´ scale (and parameter dependence)

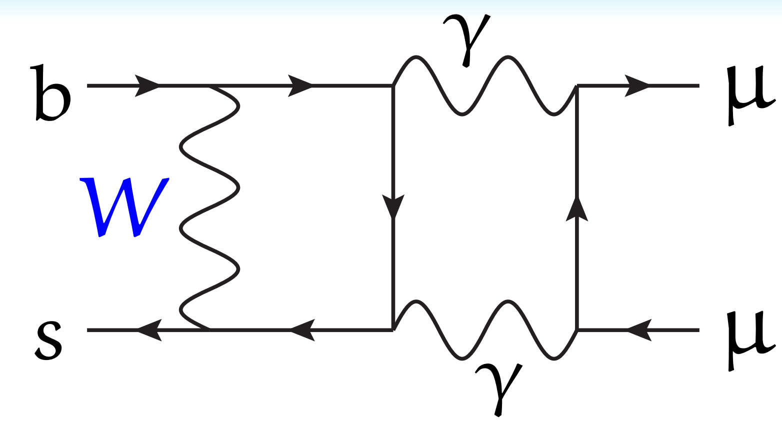
$2^{-1/2} G_F \alpha \pi / \sin^2\theta_W C_A$



2-loop electroweak corrections reduce the modulus of the Wilson coefficient

Renormalisation Group Equation

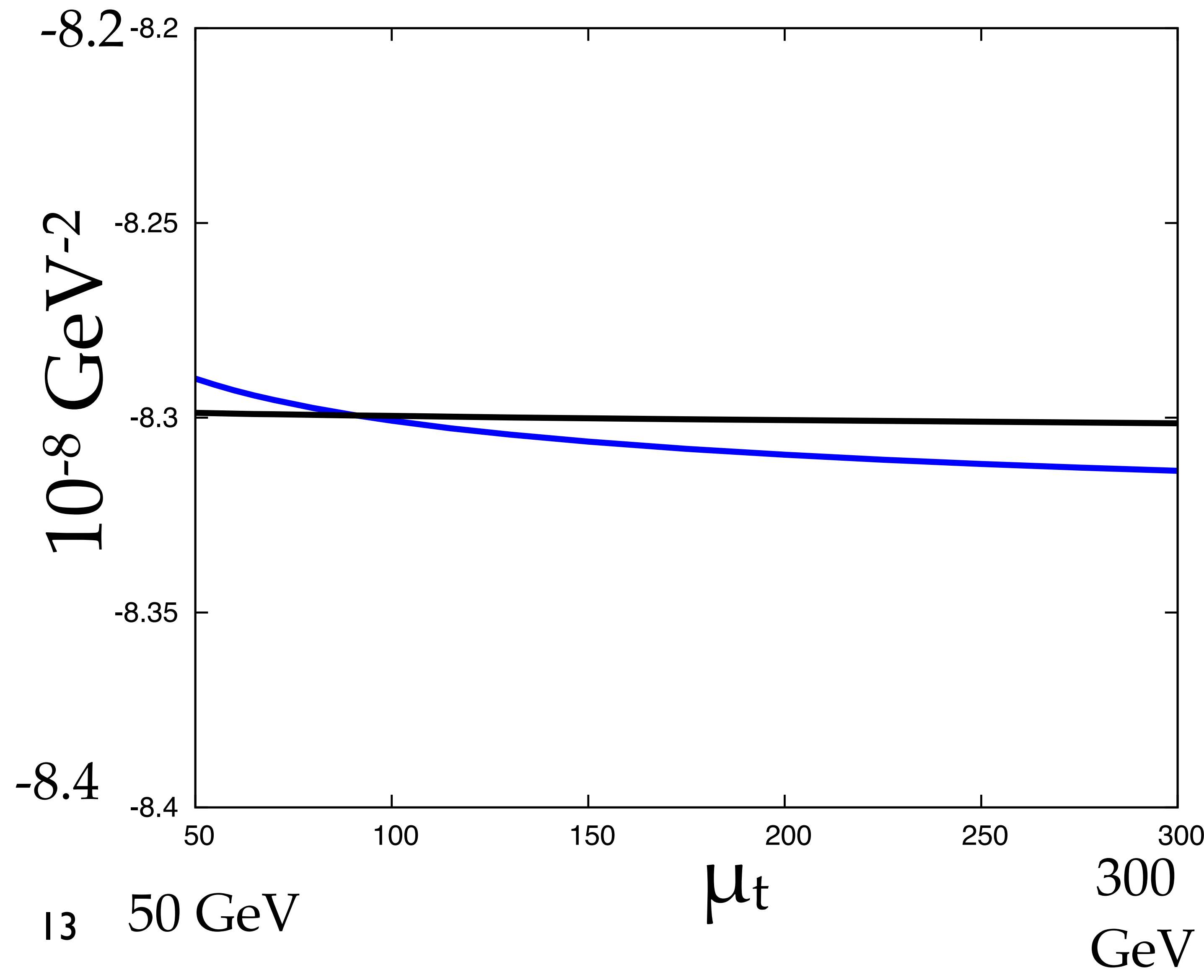
Log enhanced QED corrections known [Bobeth, Gambino, MG, Haisch `03; Huber et. al. `05, Misiak `11]



Study residual scale dependence for the G_F^2 M_W^2 normalised results

$G_F^2 M_W^2 C(\mu_0)$ is scale dependent, while $U(M_Z, \mu_0)\, G_F^2 M_W^2 C(\mu_0)$ is only residually scale dependent.

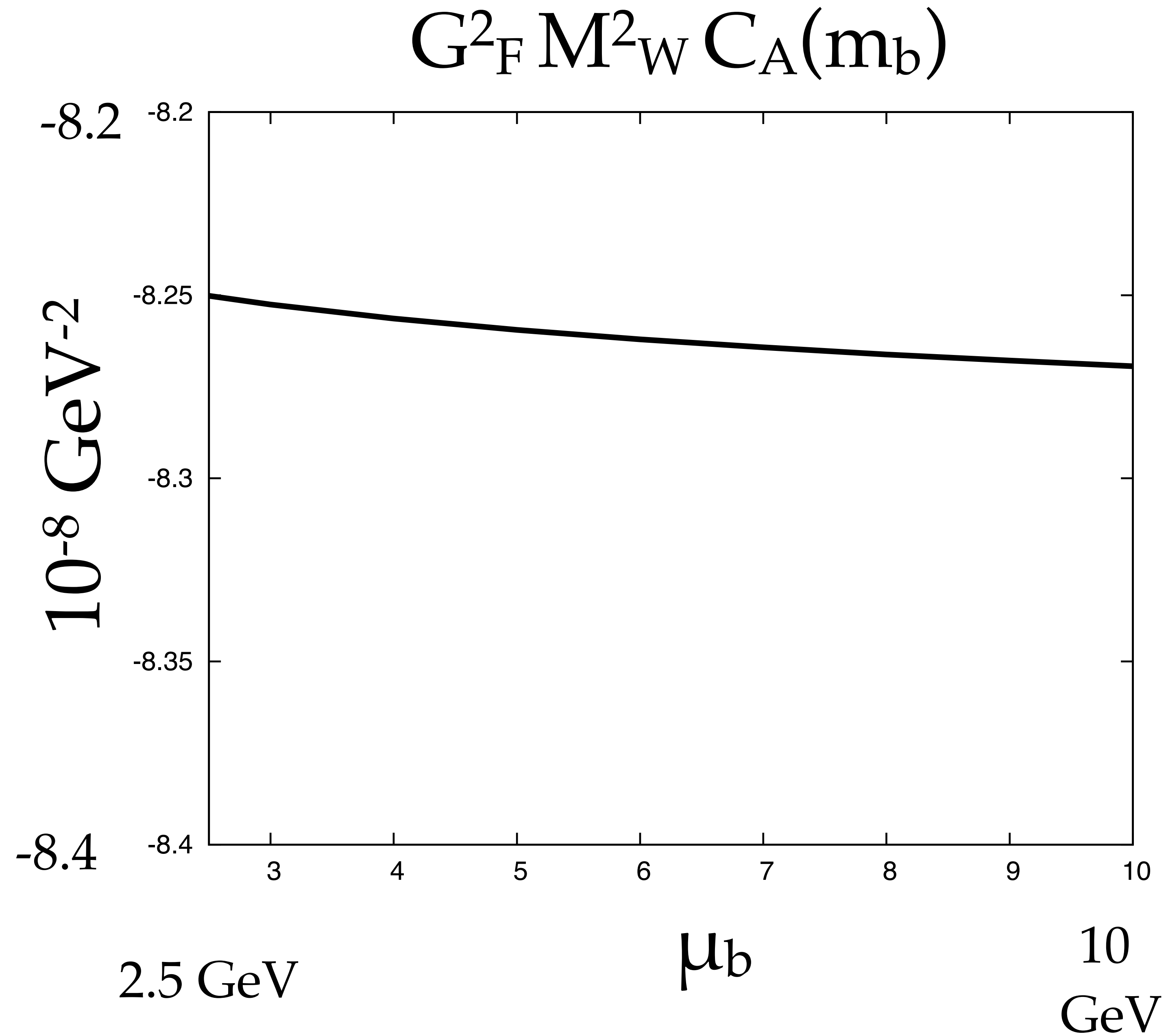
$G^2_F M^2_W C_A(M_Z)$



Wilson Coefficient at m_b

The log enhanced QED corrections further reduce the modulus of the Wilson coefficient

Varying μ_b in $U(\mu_b, m_t)\, G_F^2\, M_W^2\, C(m_t)$ gives a measure of uncertainty regarding the contributions of virtual QED corrections at m_b.



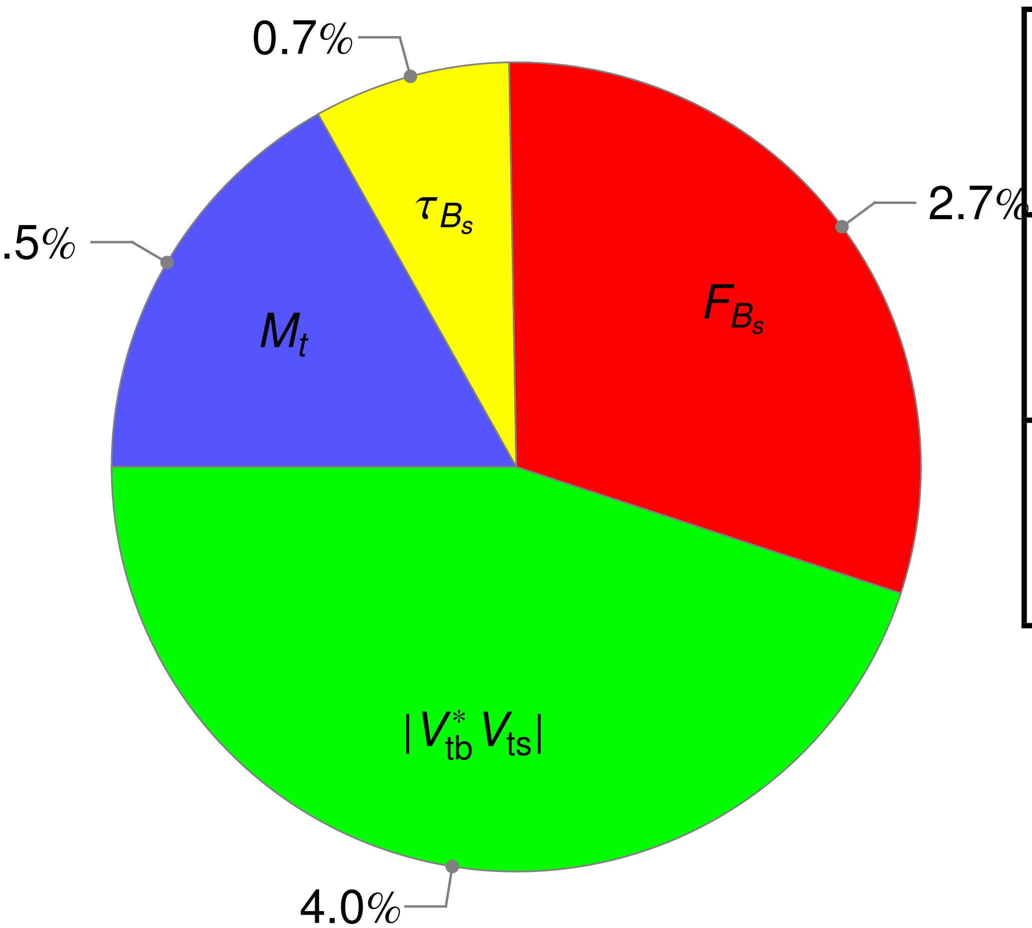
there are also uncertainties from non-perturbative QED corrections

Preliminary Theory Prediction

Electroweak corrections shift BFGK value 3.25 10^{-9} → 3.16 10^{-9}

$$\mathcal{B}^{(0)}_{\mathrm{B}_s} = 3.16 \times 10^{-9} \left(\frac{M_t}{173.2\mathrm{GeV}}\right)^{3.07} \left(\frac{F_{B_s}}{225\mathrm{MeV}}\right)^{2} \left(\frac{\tau_{B_s}}{1.500\mathrm{ps}}\right) \left|\frac{V^*_{tb} V_{ts}}{0.0405}\right|^2$$

$$\overline{\mathcal{B}}_{B_s} = 1.096(17) \times \mathcal{B}^{(0)}_{B_s}$$



[Buras, Fleischer, Girrbach, Knegjens `13]

	$Br(B_s \to \mu^+ \mu^-)$
CMS	$3.0\ ^{+0.9}_{-0.8}$ (stat) $^{+0.6}_{-0.4}$ (syst) 10^{-9}
LHCb	$2.9\ ^{+1.1}_{-1.0}$ (stat) $^{+0.3}_{-0.1}$ (syst) 10^{-9}

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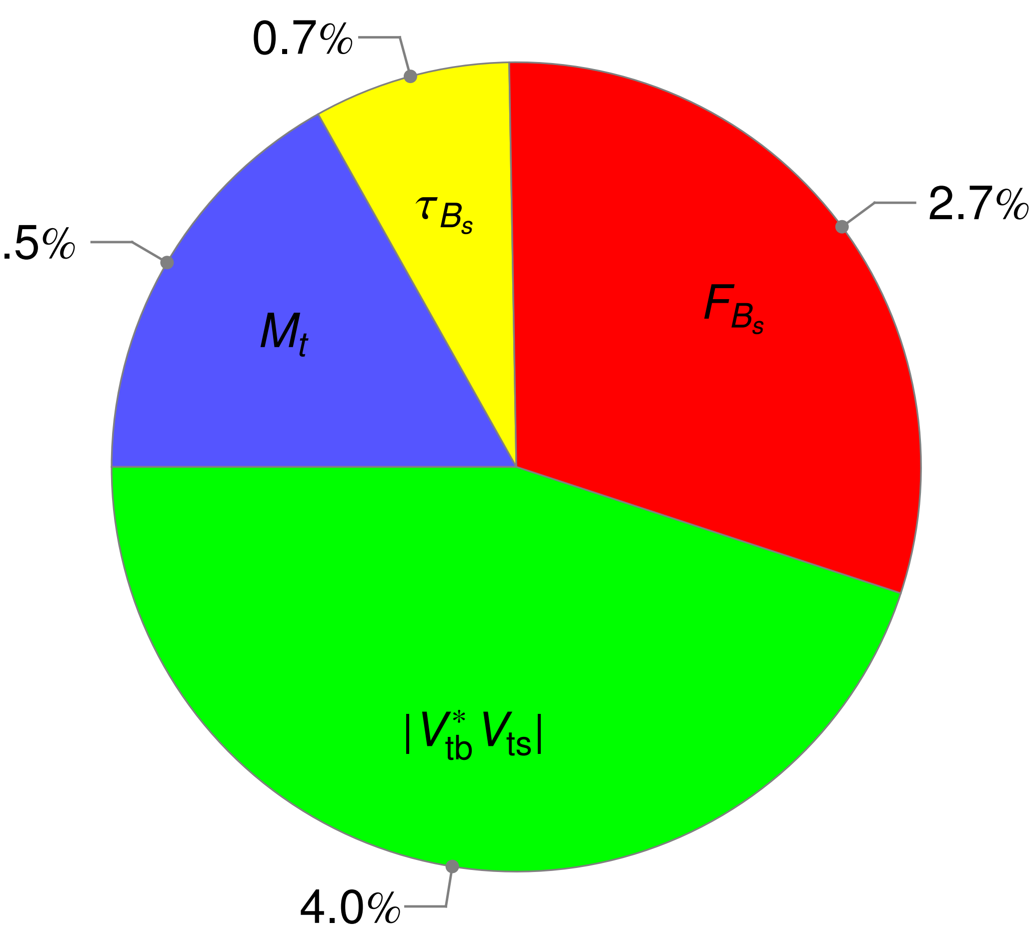

[Buras, Fleischer, Girrbach, Knegjens `13]

Pie chart is now free of electroweak scheme uncertainties

But: These numerics use multi-loop OS to OCD MS-bar shift for m_t
→ NNLO QCD Matching removes

1% uncertainty from QED corrections

Conclusions

$B_s \to \mu^+ \mu^-$ is highly sensitive to the MSSM Higgs sector at large tan β

No more electroweak scheme ambiguities in $B_s \to \mu^+ \mu^-$

Corrections small w.r.t. experimental error, significant w.r.t. to theory uncertainty

$B_s \to \mu^+ \mu^-$ can provide a precision probe of scalar, but also axial vector coupling interactions