Studies of CP Violation in B Hadron Decays

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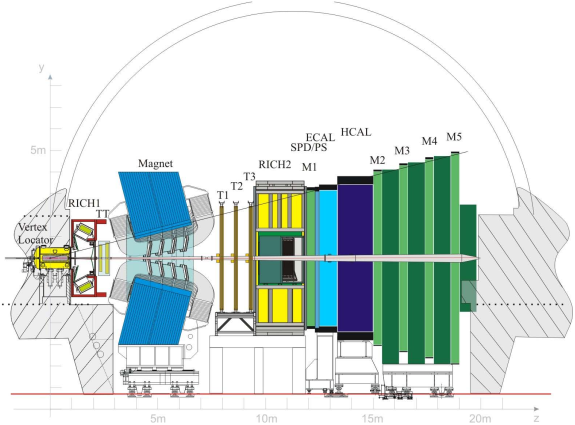
On behalf of the LHCb Collaboration

SUSY 2013
August 26th, 2013

The LHCb Detector



Forward arm spectrometer to study b- and c-hadron decays

- Good impact parameter resolution to identify secondary vertices: 20 μm
- Versatile & efficient trigger for b- and c-hadrons and forward EW signals
- Decay time resolution: 45 fs ($B_s^0 \to J/\psi\phi$)
- Invariant mass resolution: 8 MeV/c^2 ($B \to J/\psi X$) 22 MeV/c^2 ($B \to hh$)
- Excellent particle identification: 95 % K ID efficiency (5 % $\pi \to K$ mis-ID)

Measuring CP Violation: Interfering Paths

CP Violation in Mixing: $\text{Prob}(B_q^0 \to \bar{B}_q^0) \neq \text{Prob}(\bar{B}_q^0 \to B_q^0)$

- Interference through Virtual (loops) and Real (intermediate decay) contributions
- Key Measurements: a_{sl}^s from $B_s^0 \to D_s^- \mu^+ \nu$

Direct CP Violation: $\text{Prob}(B \to f) \neq \text{Prob}(\bar{B} \to \bar{f})$

- Interference between multiple decay paths (for example: Tree + Penguin diagrams)
- Key Measurements: $B_q^0 \to h^+ h^-$; γ from $B^\pm \to D^0 h^\pm$

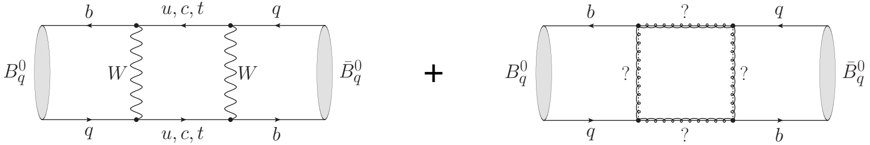
Mixing-Induced CP Violation: $\text{Prob}(B_q^0 \to f) \neq \text{Prob}(B_q^0 \to \bar{B}_q^0 \to f)$

- Interference between direct decay and decay after mixing
- Key Measurements: γ from $B_s^0 \to D_s^\mp K^\pm$; ϕ_s from $B_s^0 \to J/\psi h^+ h^-$

a_{sl}^s from $B_s^0 \to D_s^- \mu^+ \nu$ arxiv 1308.1048

Introduction

- Search for New Physics in neutral B meson mixing.



- Can be probed using wrong-charge asymmetry for flavour specific final state f

$$a_{sl}^q \equiv \frac{\Gamma\left(\bar{B}_q^0(t) \to f\right) - \Gamma\left(B_q^0(t) \to \bar{f}\right)}{\Gamma\left(\bar{B}_q^0(t) \to f\right) + \Gamma\left(B_q^0(t) \to \bar{f}\right)} \xrightarrow{\text{SM}} 0$$

- Decay time independent quantity
- Measured in semi-leptonic $B_q^0 \to D_q^- \mu^+ \nu$ decays
- World averages [HFAG]: including results from CLEO, BABAR, BELLE & D0

$$a_{sl}^d = 0.0007 \pm 0.0027$$
$$a_{sl}^s = -0.0171 \pm 0.0055$$

a_{sl}^s from $B_s^0 \to D_s^- \mu^+ \nu$ arxiv 1308.1048

LHCb Measurement

(time integrated analysis)

- Data sample: 1 fb^{-1} of 7 TeV data collected in 2011
- Determine

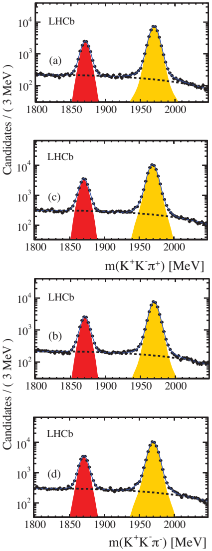
$$A_{\text{meas}} = \frac{\Gamma\left(D_s^- \mu^+\right) - \Gamma\left(D_s^+ \mu^-\right)}{\Gamma\left(D_s^- \mu^+\right) + \Gamma\left(D_s^+ \mu^-\right)} = \frac{1}{2} a_{sl}^s + \mathcal{O}(10^{-4})$$

through measurement of the $B_s^0 \to D_s^- (\to \phi\pi^-)\mu^+\nu$ yields

- Crucial aspect: Asymmetries affecting the measurement of a_{sl}^s

$$A_{\text{meas}} = A_\mu^C - A_{\text{track}} - A_{\text{bkg}}$$

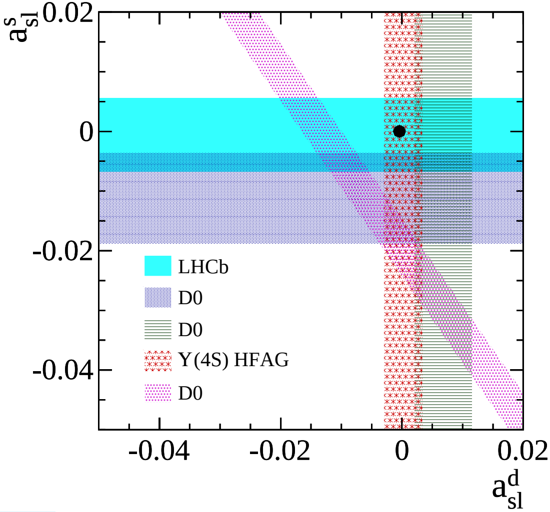
- All asymmetries determined on data
- B meson production asymmetry: at most few percent (impact on $a_{sl}^s < 10^{-4}$)
- Detection charge asymmetry: $A_\mu^C = (0.04 \pm 0.25)$ %
- Track reconstruction asymmetry: $A_{\text{track}} = (0.02 \pm 0.13)$ %
- Background asymmetry: $A_{\text{bkg}} = (0.05 \pm 0.05)$ %



a_{sl}^s from $B_s^0 \to D_s^- \mu^+ \nu$

arxiv 1308.1048

Latest LHCb Results



- Measured CP asymmetry:

$a_{sl}^s = (-0.06 \pm 0.50 \pm 0.36)\%$

- Compatible with the SM

$B_q^0 \to h^- h^+$

Introduction

- Charmless two-body decays offer sensitive probe to search for New Physics
- Problem: presence of hadronic factors in decay amplitude
- Solution: combine multiple measurements using approximate flavour symmetries

$$\Delta \equiv \frac{\mathcal{A}_{\mathrm{CP}}(B_d^0 \to \pi^- K^+)}{\mathcal{A}_{\mathrm{CP}}(B_s^0 \to K^- \pi^+)} + \frac{\mathcal{B}(B_s^0 \to K^- \pi^+)}{\mathcal{B}(B_d^0 \to \pi^- K^+)} \xrightarrow{\text{SM}} 0$$

- Allows extraction of CKM angle γ
- Demonstrate CP violation in B_s system

LHCb Measurements

- Data sample: 1 fb^{-1} of 7 TeV data collected in 2011
- Modes: $B_s^0 \to K^- \pi^+$ and $B_d^0 \to \pi^- K^+$ (time integrated analysis)
- Modes: $B_s^0 \to K^- K^+$ and $B_d^0 \to \pi^- \pi^+$ (time dependent analysis)
- Crucial aspects: particle identification; K–π separation

$B_q^0 \to h^- h^+$

CP Asymmetries

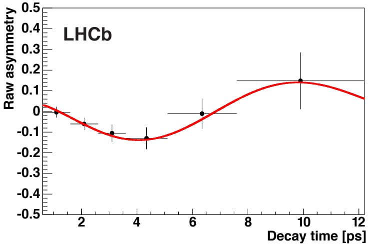
- Decay time distribution

$$\frac{d\Gamma}{dt}\left(B_q^0(t) \to f\right) \propto \cosh\left(\frac{\Delta\Gamma_q}{2} t\right) - D_f \sinh\left(\frac{\Delta\Gamma_q}{2} t\right) + C_f \cos(\Delta m_q t) - S_f \sin(\Delta m_q t)$$

- Leads to an asymmetry

$$\mathcal{A}_{\rm CP} \equiv \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)} = \frac{-C_f \cos(\Delta m_q t) + S_f \sin(\Delta m_q t)}{\cosh\left(\frac{\Delta\Gamma_q}{2} t\right) - D_f \sinh\left(\frac{\Delta\Gamma_q}{2} t\right)}$$

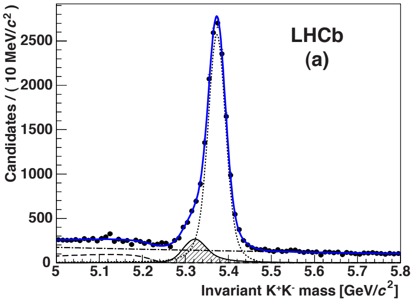
- Measured asymmetry needs to be corrected for detection and production asymmetries
- Time dependent analysis affected by resolution, acceptance, flavour tagging

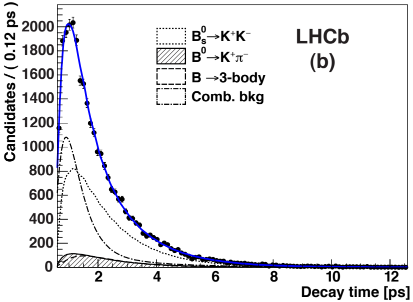


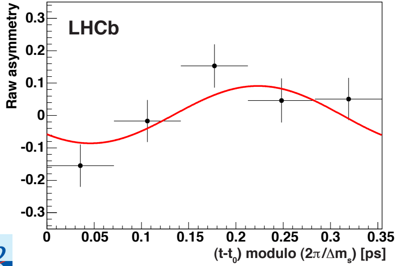
[arxiv 1308.1428]

$B_s^0 \to K^- K^+$ arxiv 1308.1428

Latest LHCb Results







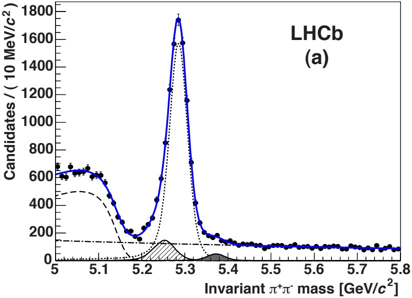
- Measured CP parameters:

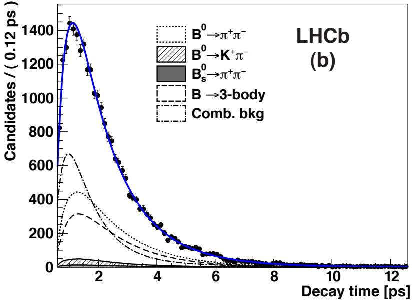
$$C_{KK} = 0.14 \pm 0.11 \pm 0.03$$

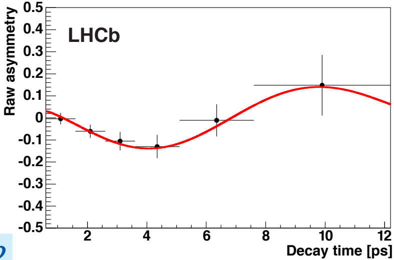
$$S_{KK} = 0.30 \pm 0.12 \pm 0.04$$

$B_d^0 \to \pi^- \pi^+$ arxiv 1308.1428

Latest LHCb Results





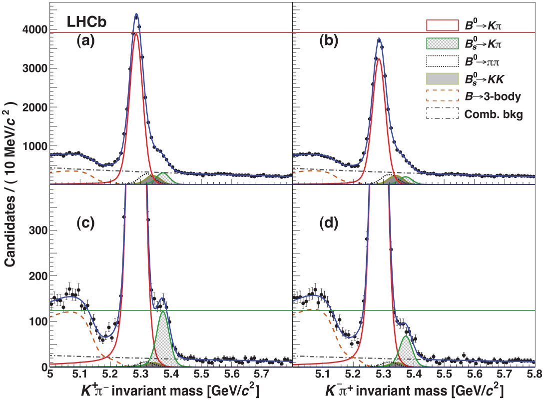


- Measured CP parameters:

$$C_{\pi\pi} = -0.38 \pm 0.15 \pm 0.02$$
$$S_{\pi\pi} = -0.71 \pm 0.13 \pm 0.02$$

$B_s^0 \to K^- \pi^+$ and $B_d^0 \to \pi^- K^+$ arxiv 1304.6173

Latest LHCb Results



- Measured CP asymmetries:

$$\mathcal{A}_{CP}(B_d^0 \to K^+\pi^-) = -0.080 \pm 0.007 \pm 0.003$$

$$\mathcal{A}_{CP}(B_s^0 \to K^-\pi^+) = 0.27 \pm 0.04 \pm 0.01$$

- SM Test:

$$\Delta = -0.02 \pm 0.05 \pm 0.04$$

- First observation of CP violation in the B_s system: $B_s^0 \to K^-\pi^+$
- Compatible with the SM

ϕ_s from $B_s^0 \to J/\psi K^+ K^-$ and $B_s^0 \to J/\psi \pi^+ \pi^-$ arxiv 1304.2600

Introduction

- CP violating phase associated with B_s^0–$\bar{B}_s^0$ mixing
- Accessible due to interference between mixing and decay
- Precise SM prediction

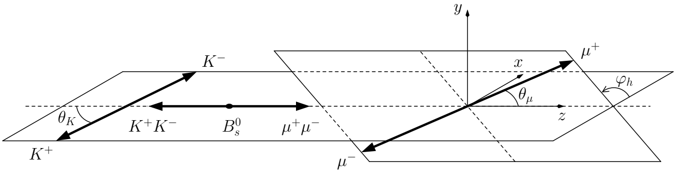
$$\phi_s^{\mathrm{SM}} = 0.0364 \pm 0.0016 \text{ rad}$$

J. Charles et al., [arxiv 1106.4041]

- Small magnitude offers excellent probe to search for New Physics

$$\phi_s = \phi_s^{\mathrm{SM}} + \phi_s^{\mathrm{NP}}$$

- Measured in Vector – Vector final state: $B_s^0 \to J/\psi \phi (\to K^+ K^-)$ (angular analysis)
- Measured in Vector – Pseudo-scalar final state: $B_s^0 \to J/\psi f_0 (\to \pi^+ \pi^-)$
- Simultaneous determination of the B_s lifetime parameters $\Delta\Gamma_s$ and Γ_s



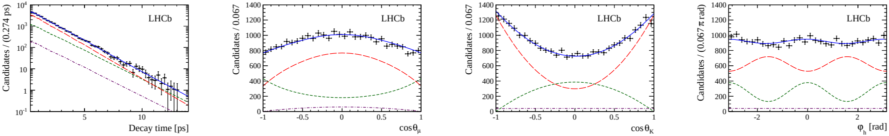
Helicity Frame

ϕ_s from $B_s^0 \to J/\psi K^+ K^-$ and $B_s^0 \to J/\psi \pi^+ \pi^-$ arxiv 1304.2600

LHCb Measurement

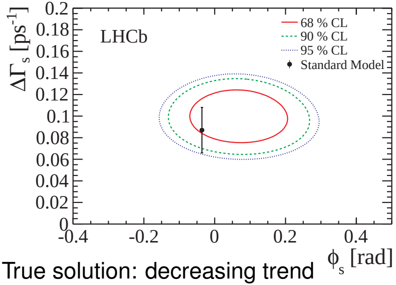
(time integrated analysis)

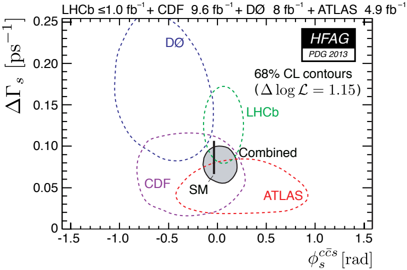
- Data sample: 1 fb^{-1} of 7 TeV data collected in 2011
- Includes both resonant (P-wave) and non-resonant (S-wave) contributions
- Includes both opposite- and same-side tagging
- Per event resolution and mistag model
- Fit to signal candidates only ($_s\mathcal{W}$eighted)
- Unbinned maximum likelihood fit to decay time and decay angles: 8 physics parameters
- $B_s^0 \to J/\psi \pi^+ \pi^-$ uses results on $\Delta\Gamma_s$ and Γ_s from $B_s^0 \to J/\psi K^+ K^-$



ϕ_s from $B_s^0 \to J/\psi K^+ K^-$ and $B_s^0 \to J/\psi \pi^+ \pi^-$ arxiv 1304.2600

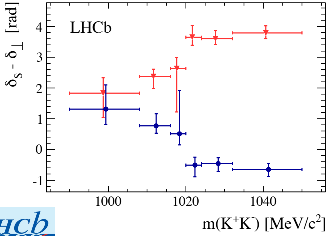
Latest LHCb Results





- True solution: decreasing trend

Red: $\Delta\Gamma_s < 0$ Blue: $\Delta\Gamma_s > 0$



- Measured parameters:

$$\phi_s = 0.01 \pm 0.07 \pm 0.01 \text{ rad}$$
$$\Gamma_s = 0.661 \pm 0.004 \pm 0.006 \text{ ps}^{-1}$$
$$\Delta\Gamma_s = 0.106 \pm 0.011 \pm 0.007 \text{ ps}^{-1}$$

- Compatible with the SM

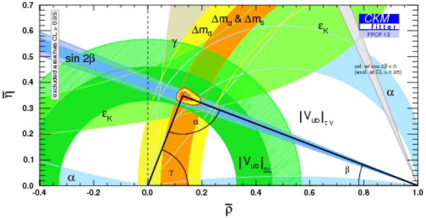
γ from $B^\pm \to D^0 K^\pm$ and $B^\pm \to D^0 \pi^\pm$ arxiv 1305.2050

Introduction

- Precision test of the SM: overconstraining the Unitarity Triangle
- Discrepancies in position of apex form clear sign of New Physics
- Least constrained parameter: angle γ

$\gamma = (66 \pm 12)^\circ$ [CKMfitter]

$\gamma = (70.8 \pm 7.8)^\circ$ [UTfit]



LHCb Measurements

- Using decay modes with only Tree diagrams: $B^\pm \to D^0 h^\pm$ (time integrated analysis)
- Using decay modes also with loop diagrams: $B_s^0 \to D_s^\mp K^\pm$ (time dependent analysis)
- Combination of the three $B^\pm \to D^0 h^\pm$ measurements by LHCb

γ from $B^\pm \to D^0 K^\pm$ and $B^\pm \to D^0 \pi^\pm$ arxiv 1305.2050

Combination Method

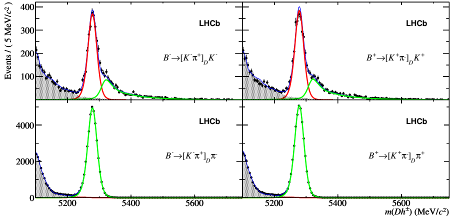
- Frequentist approach (cfr. CKMfitter)
- Maximise combined likelihood of experimental observables
- Includes information from the covariance matrices
- Result is limited by statistics
- Measurements take into account D^0–$\bar{D}^0$ mixing
- Measurements use external input on hadronic parameters describing D meson decay

Input

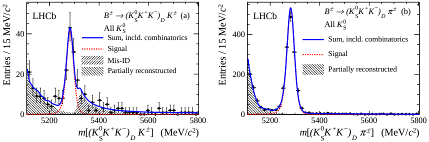
- $B^\pm \to D^0 K^\pm$ and $B^\pm \to D^0 \pi^\pm$ with $D^0 \to K^+ K^-,\ \pi^+\pi^-,\ K^\pm\pi^\mp$ [GLW/ADS Method]
- $B^\pm \to D^0 K^\pm$ with $D^0 \to K_S^0 \pi^+\pi^-,\ K_S^0 K^+ K^-$ [GGSZ Method]
- $B^\pm \to D^0 K^\pm$ and $B^\pm \to D^0 \pi^\pm$ with $D^0 \to K^\pm\pi^\mp\pi^\pm\pi^\mp$ [ADS Method]

γ from $B^\pm \to D^0 K^\pm$ and $B^\pm \to D^0 \pi^\pm$ arxiv 1305.2050

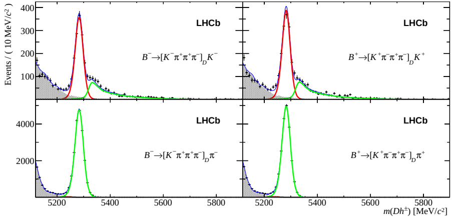
GLW/ADS Method [arxiv 1203.3662]



GGSZ Method [arxiv 1209.5869]



ADS Method [arxiv 1303.4646]



- Data sample:
  1 fb^{-1} of 7 TeV data collected in 2011
- Combination Fit:
  24 input parameters (charge asymmetries, charge averages, ratios of suppressed to favoured final states) + 13 constraints on D^0 decay
- See also Mitesh Patel's talk

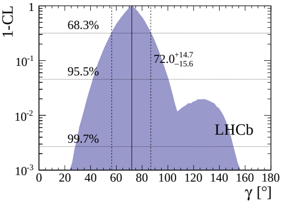
γ from $B^\pm \to D^0 K^\pm$ and $B^\pm \to D^0 \pi^\pm$ arxiv 1305.2050

Latest LHCb Results

Systematic uncertainties assigned for

- neglected correlations between fitted parameters
- undercoverage of the combination method

$B^\pm \to D^0 K^\pm$



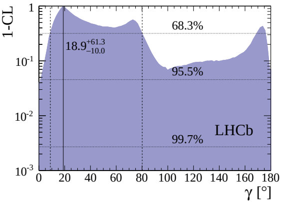
68% C.L.

$\gamma \in [56.4, 86.7]^\circ$

95% C.L.

$\gamma \in [42.6, 99.6]^\circ$

$B^\pm \to D^0 \pi^\pm$



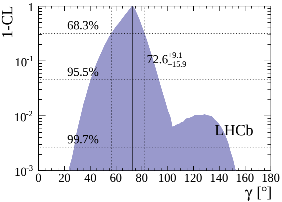
68% C.L.

$\gamma \in [7.4, 99.2]^\circ$

95% C.L.

No Limit

$B^\pm \to D^0 h^\pm$ Combined



68% C.L.

$\gamma \in [55.4, 82.3]^\circ$

95% C.L.

$\gamma \in [40.2, 92.7]^\circ$

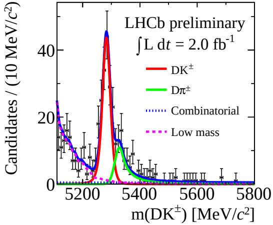
γ from $B^\pm \to D^0(\to K_S^0 h^+ h^-)K^\pm$ LHCb-CONF-2013-004

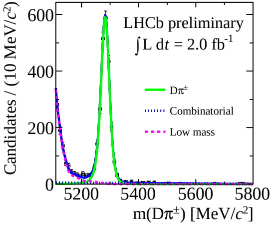
Preliminary Results

- Data sample: 2 fb^{-1} of 8 TeV data collected in 2012
- Same analysis procedure as [arxiv 1209.5869]
- Improved selection strategy
- Result combined with previous GGSZ result

$$\gamma = (57 \pm 16)^\circ$$

- See Mitesh Patel's talk for more details

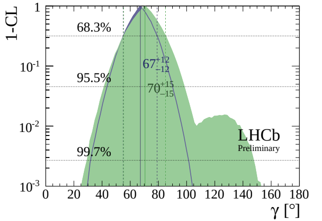




Preliminary Update on Average

$B^\pm \to D^0 K^\pm$ Comparison

[LHCb-CONF-2013-006]



1 fb^{-1} result This update

68% C.L.

$$\gamma \in [55.1, 79.1]^\circ$$

95% C.L.

$$\gamma \in [43.9, 89.5]^\circ$$

Conclusion

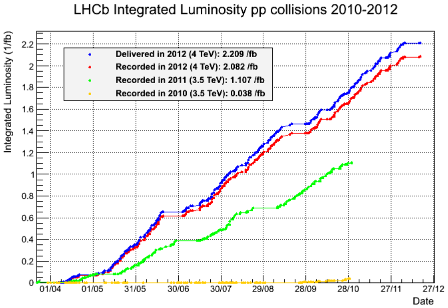
- LHCb has produced many first and world's best CP asymmetry measurements, in many different B decay modes.
- Most of the LHCb results are limited by their statistical uncertainty.
- All results presented here are based on the 1 fb^{-1} dataset collected in 2011. The 2 fb^{-1} dataset collected in 2012 is currently being studied.

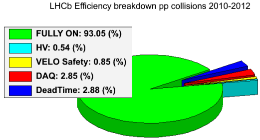
Expect many more updates soon!

Back-up

Performance of the LHCb Detector

Data Taking





- Data taking efficiency: 93.05%
- Percentage of working detector channels: $\approx$ 99%

Efficiencies

- Trigger efficiency:
  Dimuon channels: $\approx$ 90%
  Multibody hadronic channels: $\approx$ 30%
- Track reconstruction efficiency: $>$ 96%

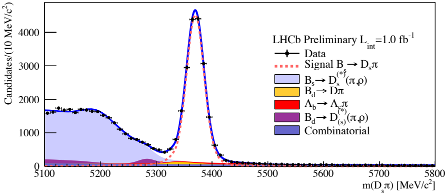
Resolution

- Momentum resolution:
  $\Delta p/p = 0.4\%$ at 5 GeV/c
  $\Delta p/p = 0.6\%$ at 100 GeV/c
- ECAL resolution: 1% $\pm$ 10%

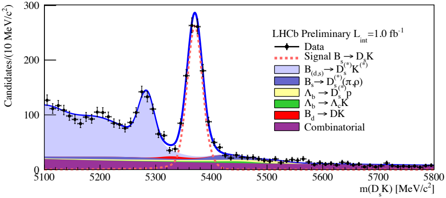
$\gamma - 2\beta_s$ from $B_s^0 \to D_s^\mp K^\pm$ LHCb-CONF-2012-029

LHCb Measurement

- Data sample: 1 fb^{-1} of 7 TeV data collected in 2011
- Reconstructed final states: $D_s^- \to K^- K^+ \pi^-$, $K^- \pi^+ \pi^-$, $\pi^- \pi^+ \pi^-$
- Optimisation and background study done using $B_s^0 \to D_s^\mp \pi^\pm$ mode
- Many partially reconstructed background to consider
- Analysis affected by resolution, acceptance, flavour tagging



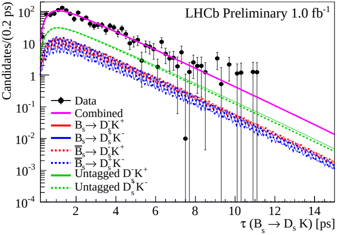
$B_s^0 \to D_s^- \pi^+$



$B_s^0 \to D_s^- K^+$

$\gamma - 2\beta_s$ from $B_s^0 \to D_s^\mp K^\pm$ LHCb-CONF-2012-029

Latest LHCb Results



Decay time distribution for $B_s^0 \to f$

$$\frac{d\Gamma(t)}{dt} \propto \cosh\left(\frac{\Delta\Gamma_s}{2} t\right) - D_f \sinh\left(\frac{\Delta\Gamma_s}{2} t\right) + C_f \cos(\Delta m_s t) - S_f \sin(\Delta m_s t)$$

$$\begin{aligned}
C &= 1.01 \pm 0.50 \pm 0.23 && = (1 - r^2)/(1 + r^2) \\
S_f &= -1.25 \pm 0.56 \pm 0.24 && = 2r \sin\left(\delta_s - (\gamma - 2\beta_s)\right)/(1 + r^2) \\
S_{\bar{f}} &= 0.08 \pm 0.68 \pm 0.28 && = 2r \sin\left(\delta_s + (\gamma - 2\beta_s)\right)/(1 + r^2) \\
D_f &= -1.33 \pm 0.60 \pm 0.26 && = 2r \cos\left(\delta_s - (\gamma - 2\beta_s)\right)/(1 + r^2) \\
D_{\bar{f}} &= -0.81 \pm 0.56 \pm 0.26 && = 2r \cos\left(\delta_s + (\gamma - 2\beta_s)\right)/(1 + r^2)
\end{aligned}$$

CP Violation in $D^0 \to K^- K^+$ and $D^0 \to \pi^- \pi^+$ LHCb-CONF-2013-003

LHCb Measurement

- Data sample: 1 fb^{-1} of 7 TeV data collected in 2011
- Use $D^{*+} \to D^0 \pi_s^+$ to tag the intial flavour of the D^0
- Or use semileptonic B decays: $\bar{B} \to D^0 \mu^- \bar{\nu}_\mu X$

$$\mathcal{A}_{\mathrm{CP}}(f) \equiv \frac{\Gamma\left(D^0(t) \to f\right) - \Gamma\left(\bar{D}^0(t) \to f\right)}{\Gamma\left(D^0(t) \to f\right) + \Gamma\left(\bar{D}^0(t) \to f\right)}$$

- To cancel detector asymmetries (as much as possible) and eliminate the contribution of indirect CP violation, measure

$$\Delta\mathcal{A}_{\mathrm{CP}} \equiv \mathcal{A}_{\mathrm{CP}}(K^- K^+) - \mathcal{A}_{\mathrm{CP}}(\pi^- \pi^+)$$

Latest LHCb Results

$$\begin{aligned}
\Delta\mathcal{A}_{\mathrm{CP}} &= (-0.34 \pm 0.15 \pm 0.10)\% \quad \text{from } D^{*0} \\
&= (+0.49 \pm 0.30 \pm 0.14)\% \quad \text{from } \bar{B} \to D^0 \mu^- \bar{\nu}_\mu X \\
&= (-0.15 \pm 0.16)\% \quad \text{Combination}
\end{aligned}$$

$B_s^0 \to J/\psi K^+ K^-$ and $B_s^0 \to J/\psi \pi^+ \pi^-$ Analysis arxiv 1304.2600

Symmetry Transformation

$$(\phi_s, \Delta\Gamma_s, \delta_0, \delta_\parallel, \delta_\perp, \delta_S) \leftrightarrow (\pi - \phi_s, -\Delta\Gamma_s, -\delta_0, -\delta_\parallel, \pi - \delta_\perp, -\delta_S)$$

Flavour Tagging Performance

Taggin power: $\epsilon_{\text{tag}} \mathcal{D}^2 \equiv \epsilon_{\text{tag}} (1 - 2\omega)^2$

- Opposite-side Tagger Only:

$$\begin{aligned} \epsilon_{\text{tag}} &= (33.00 \pm 0.28)\% \\ \langle \omega \rangle &= (36.83 \pm 0.15)\% \\ \epsilon_{\text{tag}} \mathcal{D}^2 &= (02.29 \pm 0.06)\% \end{aligned}$$

- Overlap region (OS+SS):

$$\begin{aligned} \epsilon_{\text{tag}} &= (03.90 \pm 0.11)\% \\ \epsilon_{\text{tag}} \mathcal{D}^2 &= (00.51 \pm 0.03)\% \end{aligned}$$

- Same-side Tagger Only:

$$\begin{aligned} \epsilon_{\text{tag}} &= (10.26 \pm 0.18)\% \\ \epsilon_{\text{tag}} \mathcal{D}^2 &= (00.89 \pm 0.17)\% \end{aligned}$$

- Total:

$$\begin{aligned} \epsilon_{\text{tag}} &= (39.36 \pm 0.32)\% \\ \langle \omega \rangle &= 35.9\% \\ \epsilon_{\text{tag}} \mathcal{D}^2 &= (03.13 \pm 0.13 \pm 0.20)\% \end{aligned}$$

Input for the γ Combination arxiv 1305.2050

GGSZ Method

$$
\begin{aligned}
x_- &= r_B^K \cos(\delta_B^K - \gamma) \\
y_- &= r_B^K \sin(\delta_B^K - \gamma) \\
x_+ &= r_B^K \cos(\delta_B^K + \gamma) \\
y_+ &= r_B^K \sin(\delta_B^K + \gamma)
\end{aligned}
$$

ADS & GLW Method

- Ignoring corrections due to D^0–$\bar{D}^0$ mixing

$$
R^f_{K/\pi} = \frac{\Gamma(B^- \to D[\to f]K^-) + \Gamma(B^+ \to D[\to \bar{f}]K^+)}{\Gamma(B^- \to D[\to f]\pi^-) + \Gamma(B^+ \to D[\to \bar{f}]\pi^+)} = R_{\rm cab} \frac{1 + (r_B^K)^2 + 2 r_B^K \cos\delta_B^K \cos\gamma}{1 + (r_B^\pi)^2 + 2 r_B^\pi \cos\delta_B^\pi \cos\gamma}
$$

$$
A^f_h = \frac{\Gamma(B^- \to D[\to f]h^-) - \Gamma(B^+ \to D[\to \bar{f}]h^+)}{\Gamma(B^- \to D[\to f]h^-) + \Gamma(B^+ \to D[\to \bar{f}]h^+)} = \frac{2 r_B^h \sin\delta_B^h \sin\gamma}{1 + (r_B^h)^2 + 2 r_B^h \cos\delta_B^h \cos\gamma}
$$

$$
R^\pm_h = \frac{\Gamma(B^\pm \to D[\to f_{\rm sup}]h^\pm)}{\Gamma(B^\pm \to D[\to f]h^\pm)} = \frac{r_f^2 + (r_B^h)^2 + 2 r_B^h r_f \kappa \cos(\delta_B^h + \delta_f \pm \gamma)}{1 + (r_B^h r_f)^2 + 2 r_B^h r_f \kappa \cos(\delta_B^h - \delta_f \pm \gamma)}
$$

Input for the γ Combination arxiv 1305.2050

GGSZ Method [arxiv 1209.5869]

$$
\begin{aligned}
x_- &= (0.0 \pm 4.3 \pm 1.5 \pm 0.6) \times 10^{-2} \\
y_- &= (2.7 \pm 5.2 \pm 0.8 \pm 2.3) \times 10^{-2} \\
x_+ &= (-10.3 \pm 4.5 \pm 1.8 \pm 1.4) \times 10^{-2} \\
y_+ &= (-0.9 \pm 3.7 \pm 0.8 \pm 3.0) \times 10^{-2}
\end{aligned}
$$

ADS Method [arxiv 1303.4646]

$$
\begin{aligned}
R_{K/\pi}^{K3\pi} &= 0.0765 \pm 0.0017 \pm 0.0026 \\
A_{\pi}^{K3\pi} &= -0.006 \pm 0.005 \pm 0.010 \\
A_{K}^{K3\pi} &= -0.026 \pm 0.020 \pm 0.018 \\
R_{K-}^{K3\pi} &= 0.0071 \pm 0.0034 \pm 0.0008 \\
R_{K+}^{K3\pi} &= 0.0155 \pm 0.0042 \pm 0.0010 \\
R_{\pi-}^{K3\pi} &= 0.00400 \pm 0.00052 \pm 0.00011 \\
R_{\pi+}^{K3\pi} &= 0.00316 \pm 0.00046 \pm 0.00011
\end{aligned}
$$

GLW/ADS Method [arxiv 1203.3662]

$$
\begin{aligned}
R_{K/\pi}^{K\pi} &= 0.0774 \pm 0.0012 \pm 0.0018 \\
R_{K/\pi}^{KK} &= 0.0773 \pm 0.0030 \pm 0.0018 \\
R_{K/\pi}^{\pi\pi} &= 0.0803 \pm 0.0056 \pm 0.0017 \\
A_{\pi}^{K\pi} &= -0.0001 \pm 0.0036 \pm 0.0095 \\
A_{K}^{K\pi} &= 0.0044 \pm 0.0144 \pm 0.0174 \\
A_{K}^{KK} &= 0.148 \pm 0.037 \pm 0.010 \\
A_{K}^{\pi\pi} &= 0.135 \pm 0.066 \pm 0.010 \\
A_{\pi}^{KK} &= -0.020 \pm 0.009 \pm 0.012 \\
A_{\pi}^{\pi\pi} &= -0.001 \pm 0.017 \pm 0.010 \\
R_{K}^{-} &= 0.0073 \pm 0.0023 \pm 0.0004 \\
R_{K}^{+} &= 0.0232 \pm 0.0034 \pm 0.0007 \\
R_{\pi}^{-} &= 0.00469 \pm 0.00038 \pm 0.00008 \\
R_{\pi}^{+} &= 0.00352 \pm 0.00033 \pm 0.00007
\end{aligned}
$$