Two-field Inflation with Non-minimal Coupling [1]

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[1] Work in progress in collaboration with Ruby Byrne

Introduction

- Primordial non-Gaussianity is reflected in departures from a Gaussian distribution of the Cosmic Microwave Background (CMB) anisotropies.
- Measurements of the non-Gausianity of the CMB therefore put constraints on models of inflation that complement constraints based on measurements of the power spectrum.
- In particular, non-Gaussianity in the CMB anisotropies carries information about interactions of the fields responsible for inflation and the primordial curvature perturbations.
- The dimensionless non-linearity parameter f_{nl} provides a measure for the amplitude of primordial non-Gaussianity; it provides a bridge between the observations of the CMB and modeling of the physics of the early Universe.

- Inspired by the Higgs/Dark Matter inflation model, we investigate f_{nl} in an inflation model with two real scalar fields.
- Depending on the frame in which it is viewed, the scalar fields either have non-minimal couplings to the curvature scalar or non-canonical kinetic terms.

Model

We consider a model with two real scalar fields, h and s, non-minimally coupled to the curvature scalar. The Jordan frame action is

$$\Gamma_{\mathrm{J}} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}f(h,s)\tilde{R} + \frac{1}{2}\partial_\mu s\partial^\mu s + \frac{1}{2}\partial_\mu h\partial^\mu h - \tilde{V}(h,s)\right],$$

with

$$f(h,s) = m_{\mathrm{Pl}}^2 + \xi h^2 + \xi_s s^2$$
$$\tilde{V}(h,s) = \frac{1}{2}m_h^2h^2 + \frac{1}{2}m_s^2s^2 + \frac{1}{4}\lambda h^4 + \frac{1}{2}\kappa h^2s^2 + \frac{1}{4}\lambda_s s^4$$

A conformal transformation $g_{\mu\nu} = f(h,s)\tilde{g}_{\mu\nu}$ removes the non-minimal couplings but introduces a non-trivial field-space metric. The Einstein frame action is

$$\begin{aligned}\Gamma_{\mathrm{E}} &= \int d^4x\sqrt{-g}\left[\frac{1}{2}m_{\mathrm{Pl}}^2 R\right.\\ &\left. +G_{hh}\frac{1}{2}\partial_\mu h\partial^\mu h + G_{sh}\partial_\mu s\partial^\mu h + \frac{1}{2}G_{ss}\partial_\mu s\partial^\mu s - V(h,s)\right],\end{aligned}$$

with

$$\begin{aligned}G_{ss} &= \frac{1}{f(h,s)}(1+6\frac{\xi_s^2 s^2}{f(h,s)})\\ G_{sh} &= 6\frac{\xi\xi_s}{f(h,s)^2}\\ G_{hh} &= \frac{1}{f(h,s)}(1+6\frac{\xi^2 h^2}{f(h,s)})\\ V(h,s) &= \frac{\tilde{V}(h,s)}{f(h,s)^2}.\end{aligned}$$

Approximate $SO(2)$ symmetry

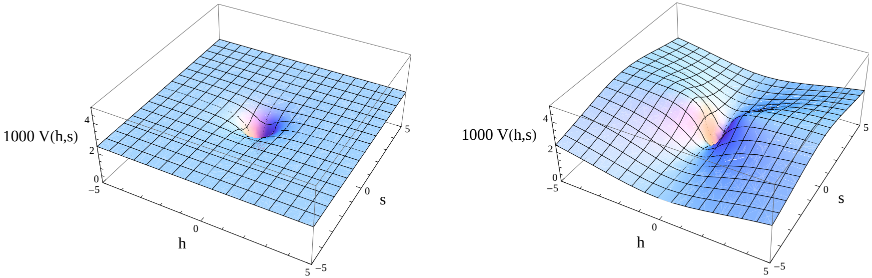
- The model has an exact rotation symmetry if $\lambda = \kappa = \lambda_s$, and $\xi = \xi_s$.
- The parameters δ_1, δ_2, and α quantify the amount and type of symmetry breaking and are defined as

$$\begin{aligned} \delta_1 &\equiv \frac{1}{2}(\lambda - \lambda_s) \\ \delta_2 &\equiv \frac{1}{4}(2\kappa - \lambda - \lambda_s) \\ \alpha &\equiv \frac{1}{2}(\xi - \xi_s) \end{aligned}$$

- The potential is flat in the "radial" direction due to the non-minimal coupling of the scalar fields.
- If the symmetry breaking parameters are small, then the potential is also flat in the "azimuthal" direction.

Illustration of the potential

- Parameters: $\lambda = 1$ and $\xi = 10$.
- Symmetry breaking parameters: $\delta_2 = 0$ and $\alpha = 0$.



$\delta_1 = 0$ $\delta_1 = 0.4$ (exaggerated value)

- Symmetry breaking creates valleys and ridges in the potential.

Time evolution

- The time evolution of the Universe is determined by the Einstein equation and the field equations for h and s.
- The scalar fields are expanded around their classical background values

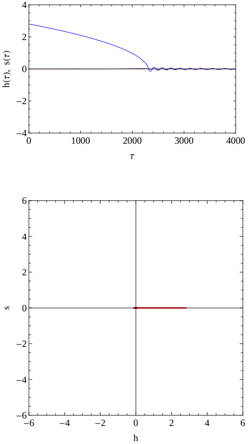
$$\begin{aligned} h(x^\mu) &= h(t) + \delta h(x^\mu) \\ s(x^\mu) &= s(t) + \delta s(x^\mu) \end{aligned}$$

- Similary, the metric is expanded around the FRW metric.
- The time evolution of the background fields is then given by

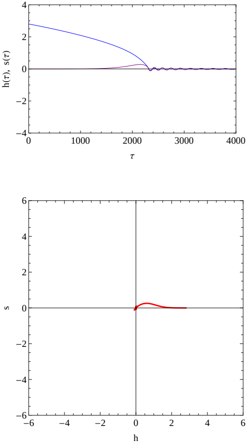
$$\begin{aligned} \ddot{h} &= -\Gamma^h_{hh}\dot{h}^2 - 2\Gamma^h_{hs}\dot{h}\dot{s} - \Gamma^h_{ss}\dot{s}^2 - 3H\dot{h} - G^{-1}_{hh}\frac{\partial V}{\partial h} - G^{-1}_{hs}\frac{\partial V}{\partial s} \\ \ddot{s} &= -\Gamma^s_{ss}\dot{s}^2 - 2\Gamma^s_{sh}\dot{s}\dot{h} - \Gamma^s_{hh}\dot{h}^2 - 3H\dot{s} - G^{-1}_{ss}\frac{\partial V}{\partial s} - G^{-1}_{sh}\frac{\partial V}{\partial h} \\ H &= \sqrt{\frac{1}{3}(\frac{1}{2}G_{hh}\dot{h}^2 + G_{hs}\dot{h}\dot{s} + \frac{1}{2}G_{ss}\dot{s}^2 + V)} \end{aligned}$$

Trajectories

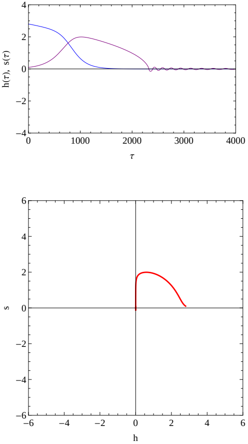
Results for $h_0 = 2.7978$, $\lambda = 1$, $\xi = 10.4$.
Symmetry breaking parameters: $\delta_1 = 0.004$, $\delta_2 = 0$, and $\alpha = 0$.



$s_0 = 0$



$s_0 = 0.00001$



$s_0 = 0.1$

Calculation of f_{nl}

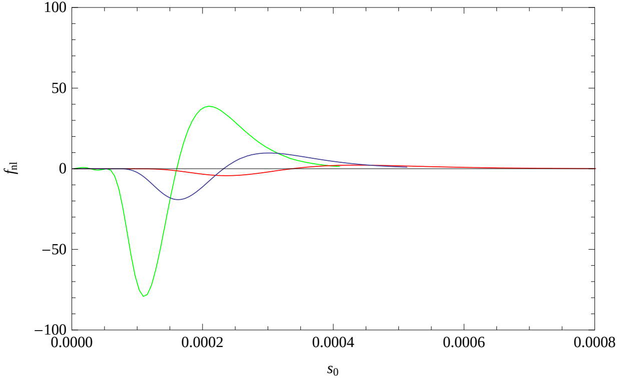
- The parameter f_{nl} characterizes the bi-spectrum of the gauge invariant curvature perturbations. It can be calculated as

$$f_{nl} \quad = \quad -\frac{5}{6}\frac{\mathcal{D}^A N \mathcal{D}^B N \mathcal{D}_A \mathcal{D}_B N}{(\mathcal{D}^C N \mathcal{D}_C N)^2}, \text{ with } A, B, C = h, s.$$

- Twenty-five neighboring trajectories for the evolution of the Universe are calculated numerically.
- The end of inflation for each trajectory is established as the time when the first slow roll parameter first exceeds unity.
- The number of e-folds is determined for each trajectory. A finite differences method is employed to calculate the covariant derivatives of N.

f_{nl} as a function of s_0

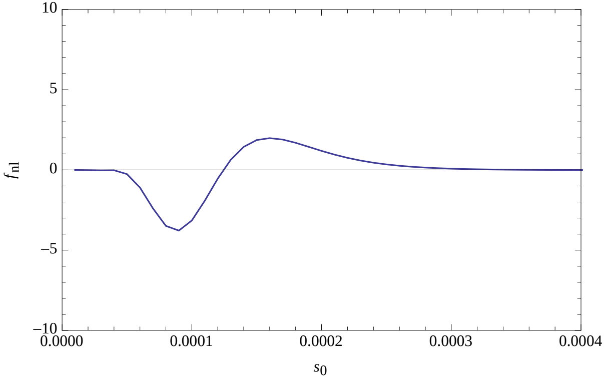
Results for $\lambda = 1.0$.
Symmetry breaking parameters: $\delta_1 = 0.004$, $\delta_2 = 0$, and $\alpha = 0$.



Red: $\xi = 9.6, \quad h_0 = 2.9101, \quad N = 60.$
Blue: $\xi = 10.0, \quad h_0 = 2.8523, \quad N = 60.$
Green: $\xi = 10.4, \quad h_0 = 2.7978, \quad N = 60.$

f_{nl} as a function of s_0, continued

Results for $\lambda = 1.0$.
Symmetry breaking parameters: $\delta_1 = 0$, $\delta_2 = 0.001$, and $\alpha = 0$.



$\xi = 10.0, \quad h_0 = 2.8523, \quad N = 60.$

Scaling of $f_{nl}(\xi, s_0)$

The function $f_{nl}(\xi, s_0)$ is observed to obey the approximate scaling relation

$$f_{nl}(\xi + \delta\xi, s_0) = (1 + \delta x_1) f_{nl}(\xi, (1 + \delta x_2) s_0).$$

This scaling relation implies that f_{nl} satisfies the differential equation

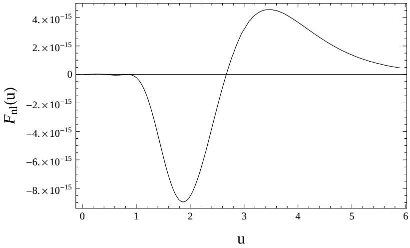
$$\frac{\partial f_{nl}}{\partial \xi} = \frac{\delta x_1}{\delta \xi} f_{nl} + s_0 \frac{\delta x_2}{\delta \xi} \frac{\partial f_{nl}}{\partial s_0}.$$

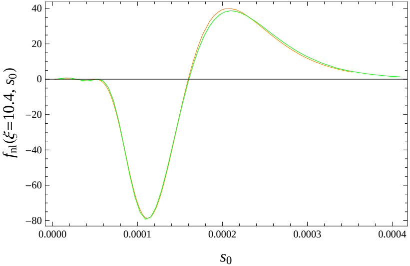
The solution to this differential equation takes the form

$$f_{nl}(\xi, s_0) = e^{\frac{\delta x_1}{\delta \xi} \xi} F_{nl}(e^{\frac{\delta x_2}{\delta \xi} \xi} s_0).$$

How well does this scaling relation work?

$$\frac{\delta x_1}{\delta \xi} = 3.52$$
$$\frac{\delta x_2}{\delta \xi} = 0.935$$

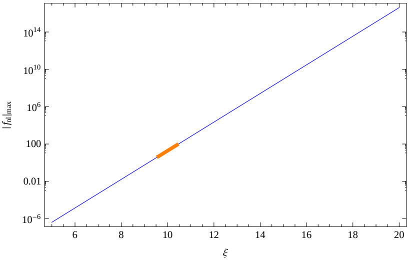


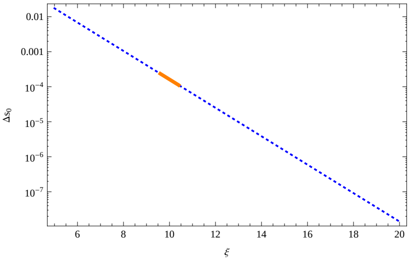


Green: directly calculated

Orange: via scaling relation

Extrapolation of scaling behavior





- Δs_0 is the difference between the values of s_0 for which f_{nl} is maximal and minimal (large in magnitude but negative). It is a measure of the range in s_0 for which non-Gaussianity is significant.
- Increasing ξ produces an exponentially increasing maximum value of f_{nl}.
- At the same time, the range of s_0 for which a significant value of f_{nl} is obtained is exponentially suppressed.

Implications for Higgs-Dark Matter inflation

- In a very minimal scenario, a combination of the Standard Model Higgs field and a singlet scalar dark matter field can double as the inflaton fields of slow roll inflationary models.
- Consistency with the observed power spectrum requires large non-minimal couplings ($\xi \approx 10^4$) of the Higgs and dark matter scalars to the gravitational scalar curvature.
- Unitarity violation provides a challenge to this scenario and may require the introduction of additional fields or higher dimension interaction terms for its resolution.
- Applying our preliminary analysis of f_{nl} to this scenario hints that non-Gaussianity will be small except in highly fine-tuned slices of parameter space.